Testing a Saturation-Based Theorem Prover: Experiences and Challenges

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First-order Automatic Theorem Proving:

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- example systems: E, SPASS, Vampire

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- SMT-COMP 2016: 79 answers classified as incorrect

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Quite complex piece of software (≈194000 lines of C++)

➥ easy to introduce incorrectness when adding a new feature

Outline

Theorem proving basics

Standard form of the input:

$$F \quad := \quad (Axiom_1 \wedge \ldots \wedge Axiom_n) \rightarrow Conjecture$$

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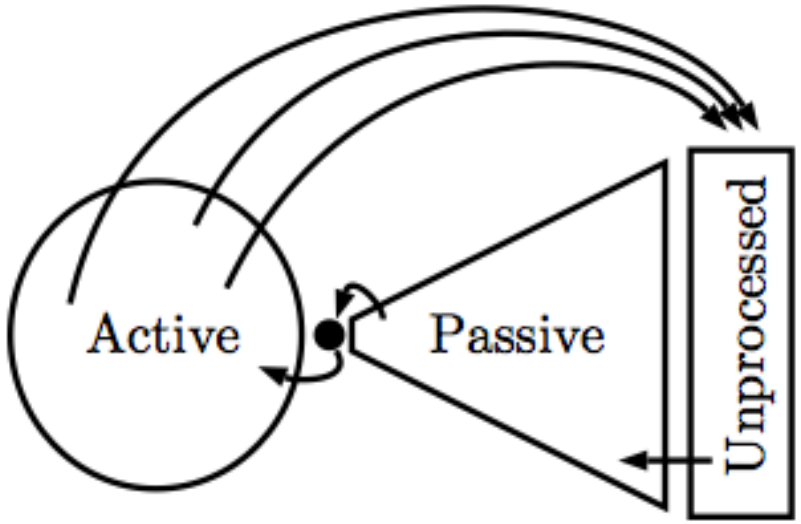
Example inference rule: $\dfrac{C_1 \vee P \quad C_2 \vee \neg P}{C_1 \vee C_2}$

The Saturation Process

Saturation = fixed-point (closure) computation



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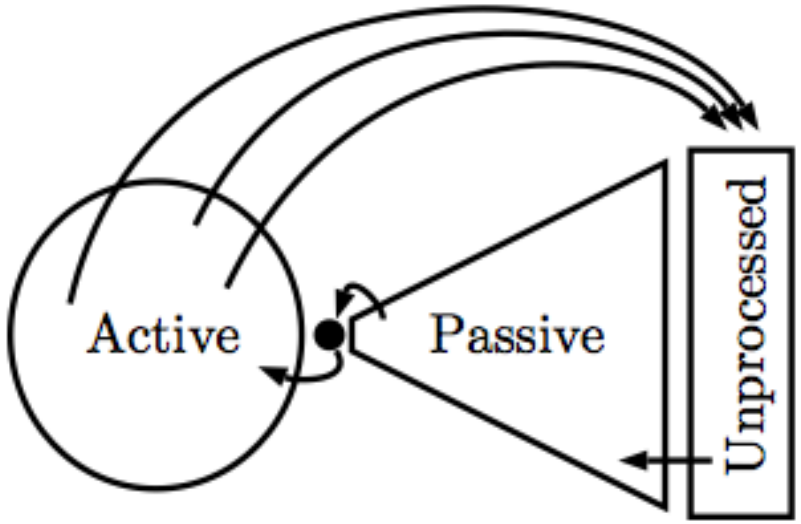

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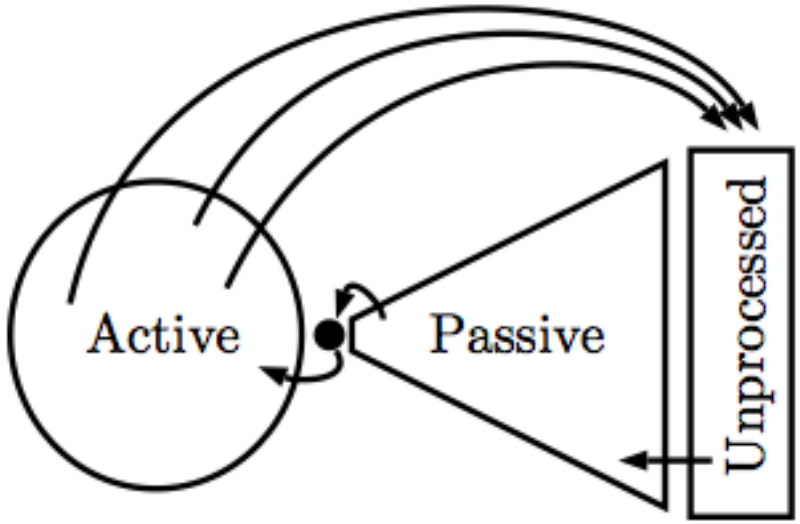

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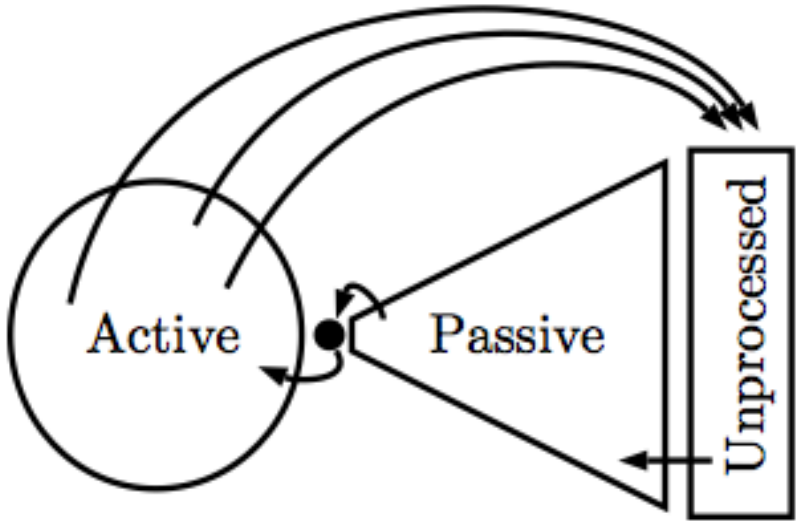

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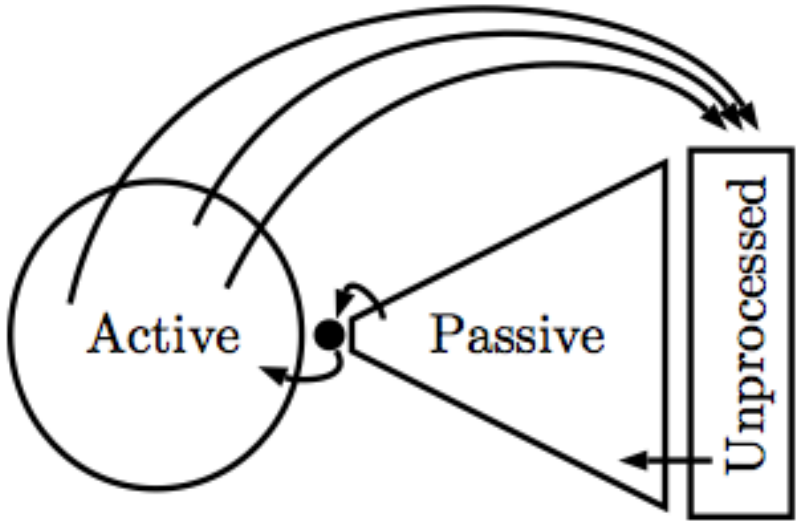

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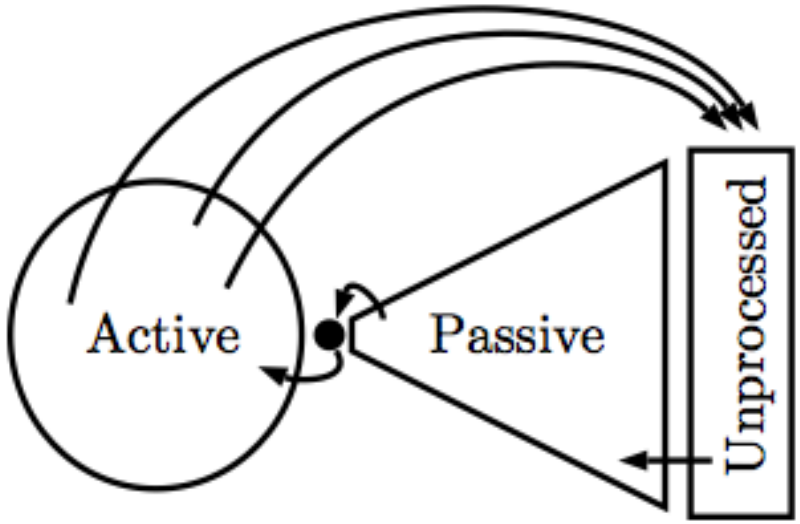

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- explosive in nature
- may not terminate
- various tricks to mitigate the explosion

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  2. incomplete strategy failed

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unsoundness: Reports `Theorem` for an invalid F.
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- never (strictly) violated after finitely many steps

Violating the Contract of Proper Behaviour

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assertion violation defensive development via assertions

- around 2500 assertions in total;
  (one per 77 lines on average)
- potential errors detected early on

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Input Search Space

Problem input space:

- infinite, in principle
- in practice, we sample representative benchmarks, e.g.:
  - the TPTP library (∼20k problems)
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Configuration space:

- around 75 proof search parameters
  (boolean, multi-valued, numeric)
- the search space $> 2^{75}$
  (too large to explore systematically)

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silent memory issues and segmentation faults

- one of the most difficult kinds to resolve
- Valgrind is usually of great help here

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3. ~p(b) [input]
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5. ~p(a) [backward demodulation 4,3]
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"`vampire -p proofcheck`" for step 5:

```
fof(pr4,axiom, a = b ).
fof(pr3,axiom, ~p(b) ).
fof(r5,conjecture, ~p(a) ).
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Ideally, ...

...strive for a standalone proof format with formal semantics!

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➥ turned into a response property

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problem space coverage

- little work done so far
- libraries may lack inputs for testing a new feature
- experiment with fuzzing?

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