Propositions

Statements

> **"The sky is blue"**
>
> **"The moon is made of green cheese"**
>
> **"The cat sat on the mat"**

Represented (usually) by single upper case letters, P, Q, R, S…

Linked by

- **And** $\wedge$
- **Or** $\vee$
- **not** $\neg$
- **implication** $\rightarrow$

Logic

Never draw a false conclusion from a true premise

- **All that is guaranteed**

  From a false conclusion, anything follows

$P \rightarrow Q \equiv \neg(P \wedge \neg Q)$

It is never the case that P is true and Q is false

$P \rightarrow Q \equiv (\neg P \vee Q)$

Either P must be false or Q is true (since P must then be true).

Examples

If the moon is made of green cheese, then unicorns exist.

It is not the case that both:
a) the moon is made of green cheese and
b) unicorns do not exist.

Either the moon is not made of green cheese or unicorns exist.

Important

To prove $P \rightarrow Q$

prove that $P \wedge \neg Q$ is impossible

Assume $P \wedge \neg Q$ and derive a contradiction.

Combining operators: truth values

P∧Q

P ↓ Q→	**T**	**F**
T	***T***	***F***
F	***F***	***F***

P∨Q

P ↓ Q→	**T**	**F**
T	***T***	***T***
F	***T***	***F***

P→Q

P ↓ Q→	**T**	**F**
T	***T***	***F***
F	***T***	***T***

Practice in Truth Tables

Create a truth table for ‘not and’ (NAND)

Create a truth table for ‘exclusive or’, *i.e.* for ‘P or Q but not both’

Practice in Reading it

What is the equivalent in English of:

$\neg(P \vee Q)$

$\neg(P \wedge Q)$

Predicates

A way of describing something:

- **Green, Cloudy, Tall, Handsome,…**
- **Usually written as uppercase letter followed by a variable or word followed by a variable**

  G(x), C(x),T(mary), H(john)

  Gx, Cx, Tmary, Hjohn,

  Green x, Cloudy x, Tall mary, Handsome john …

Computational notations

Prolog: variables in upper case, predicates and constants in lower case

green(X), cloudy(X), tall(mary), handsome(john)

Lisp derivatives: variables preceded by '?'

(green ?x), (cloudy ?y), (tall mary), (handsome john)

Quantifiers

for_all $\forall$

All men are mortal

$$\forall x.\ Man\ x \rightarrow Mortal\ x$$

For all x, If x is a man, then x is mortal

Notice: '$\forall$ always goes with '$\rightarrow$'

there_exists $\exists$

Some men are tall

$$\exists x\ .\ Man\ x \wedge Tall\ x$$

There exists something which is both man and tall

Notice: '$\exists$' always goes with '$\wedge$'

Examples with quantifiers

All men who play basket ball are tall

$\forall x$ **. Man x $\wedge$ PlaysBasketBall x**
$\rightarrow$ Tall x

How does this relate to 'All tall men play basket ball'?

Some tall men play basket ball

$\exists x$ **. Man x $\wedge$ Tall x $\wedge$**
PlaysBasketBall x

How does this relate to 'Some basket ball players are tall'?

More examples

No short men play basket ball

$\neg\exists$ **x . Man x $\wedge$ Short x $\wedge$ PlaysBasketBall x**

All men who play basket ball are not short

$\forall$**x . Man x $\wedge$ PlaysBasketBall x $\rightarrow \neg$ Short x**

Are these the same? Can you prove it?

More examples to try

Some computer science exams are difficult

All good students pass exams

All lectures in CS242 take place in LT104.

All lecture theatres seats are uncomfortable

All mammals have fur

All mammals except marsupials give birth live.

And some more

Some large birds fly.

Some flying birds are large.

Some large flying animals are birds.

All large flying animals are birds

Predicates with two or more arguments (Arity ≥ 2)

Parent(x, y) **Parent x y**

Member_of(x, y) **Member_of x y**

Married_to(x, y) **…**

Marriage_date(x, y, z)

…

Standard transformations

$\forall x . Px \equiv \neg\exists x . \neg Px$

Something is always true if and only if there is no case in which it is false.

$\exists x . Px \equiv \neg\forall x . \neg Px$

Something may be true if and only if it is not always false

Important: To prove $\forall x . Px$

Assume $\exists x. \neg Px$ and try to derive a contradiction.

Computationally: Try to construct an x such that $\neg Px$ and show that it always leads to an impossibility.

More on paradoxes of material implications

What's wrong with the following definition:

A good parent is someone all of whose children are good.

GoodParent x ≡ $\forall$y . Parent(x, y) → Good y

General Principle

$\neg\exists$ x . Px → ($\forall$y z . Py → Qz)

If no Ps exist, then being a P implies anything.

How does this relate to 'good parents' ?

Optional Advanced Point
An important idiom:

All humans have at least two fingers:

$\forall$x . Human x $\rightarrow$
$\exists$ y z . Hand y $\wedge$ Hand z $\wedge$
has(x, y) $\wedge$ has(x, z)

Why is this not enough?

What must be added?

How would you say:
'All humans have exactly two hands'?

More Examples to put into logic

All bright students who study hard pass exams

CS341 requires each student to write two essays

If a student taking CS341 writes two essays and gets 50% or more on each, then the student passes C341

All birds which eat insects fly.

No very large bird except the albatross flies

All urban foxes who raise all their cubs have safe dens.

Some bright students are lazy

Some bright students who do not pass exams are lazy

Some bright lazy students pass exams

A set which is not a member of itself is a Russell set

-----------------optional----------------------

All dogs have four legs

All mammals have at least two legs

All octopus have eight legs.

All eight legged creatures are octopus