Implementing DL Systems

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  - **BUT** even simple domain encoding is **disastrous** with large numbers of roles

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  - Recent DL research has addressed this problem (with considerable success)

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  - But often generally applicable to search based algorithms

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$\{\text{HappyFather}, \neg\text{HappyFather}\} \longrightarrow$ **clash**

$\{\forall\text{has-child}.(\text{Doctor} \sqcup \text{Lawyer}), \exists\text{has-child}.(\neg\text{Doctor} \sqcap \neg\text{Lawyer})\} \longrightarrow$ **search**

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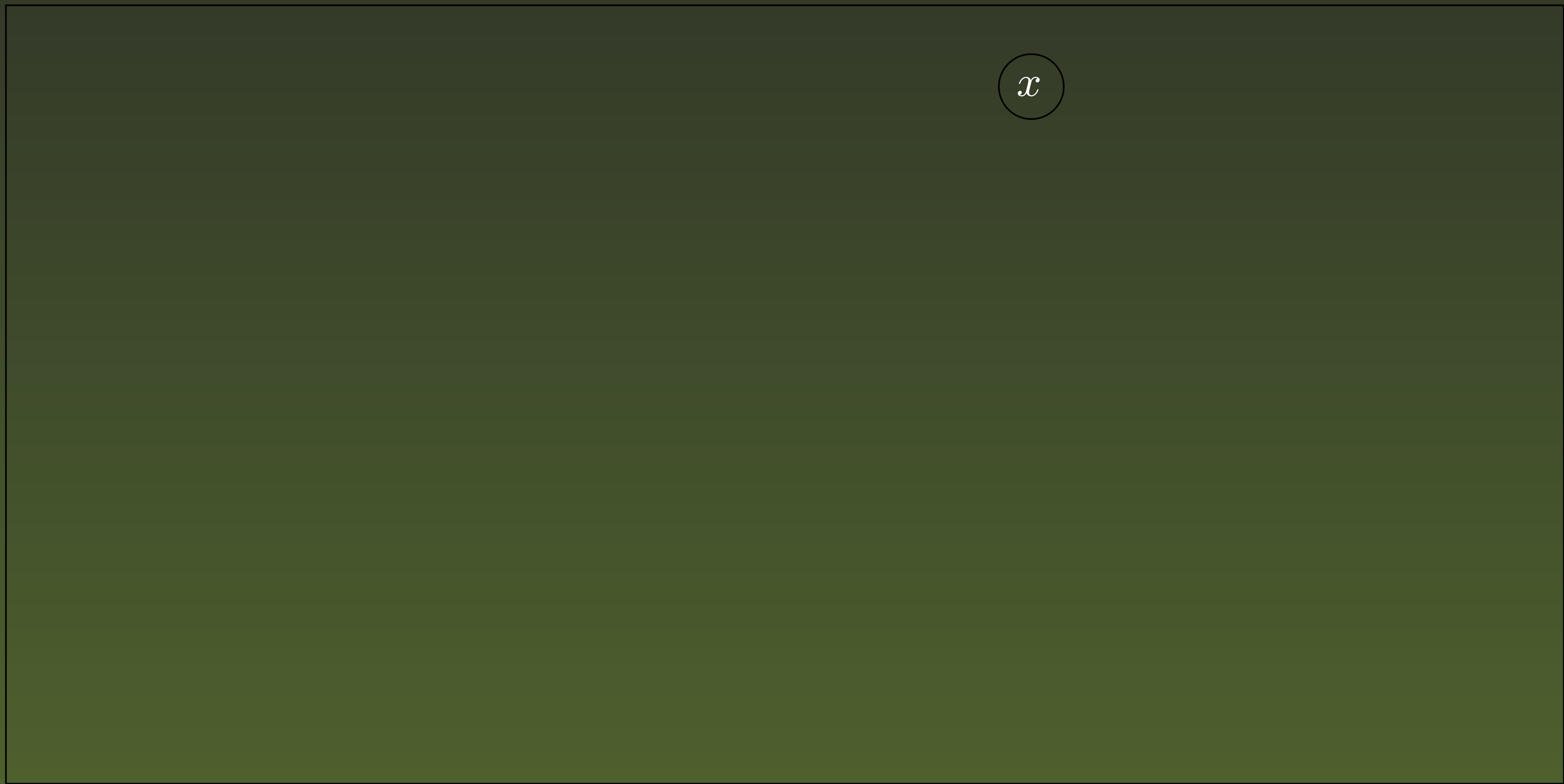

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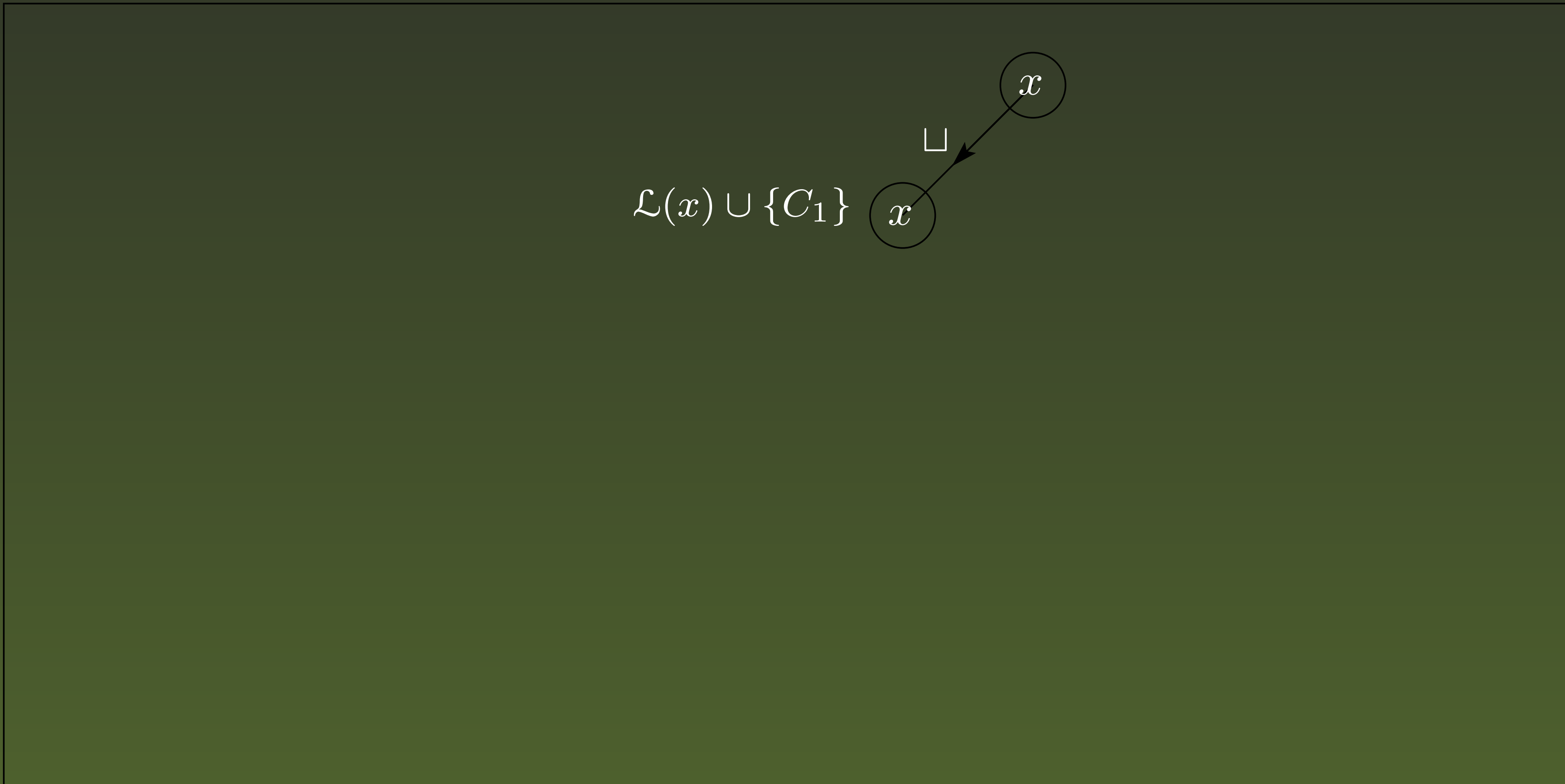

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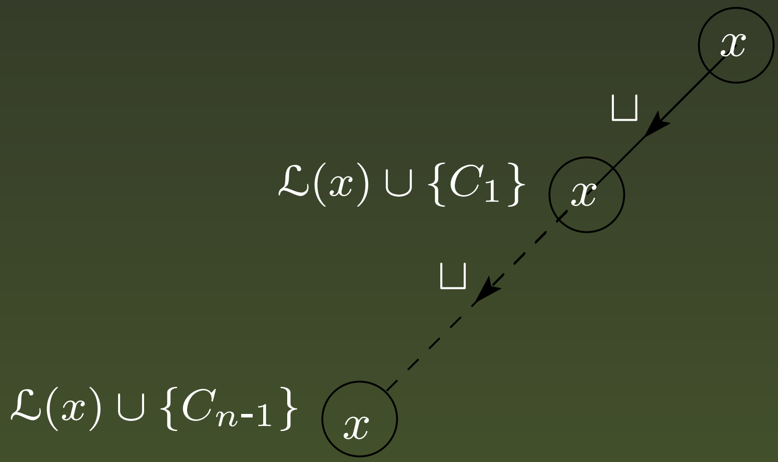

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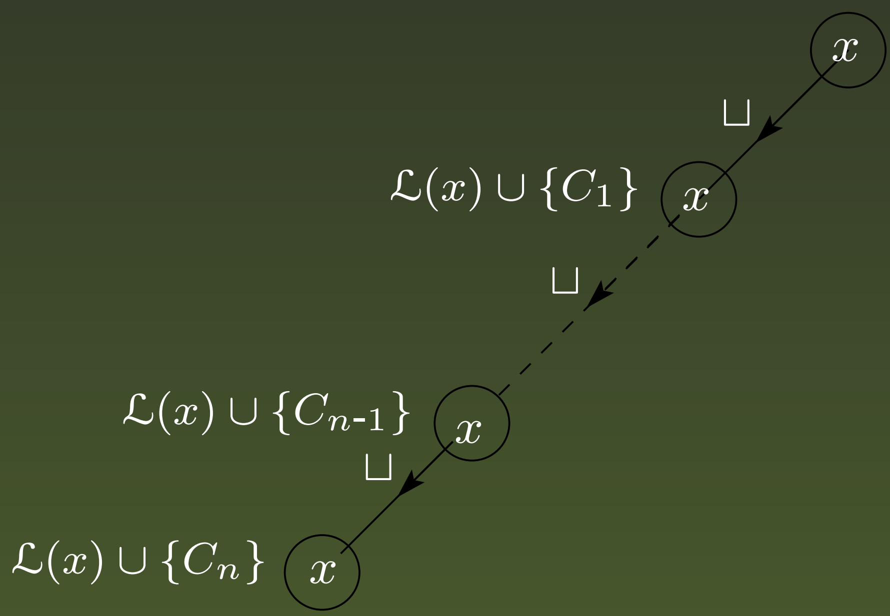

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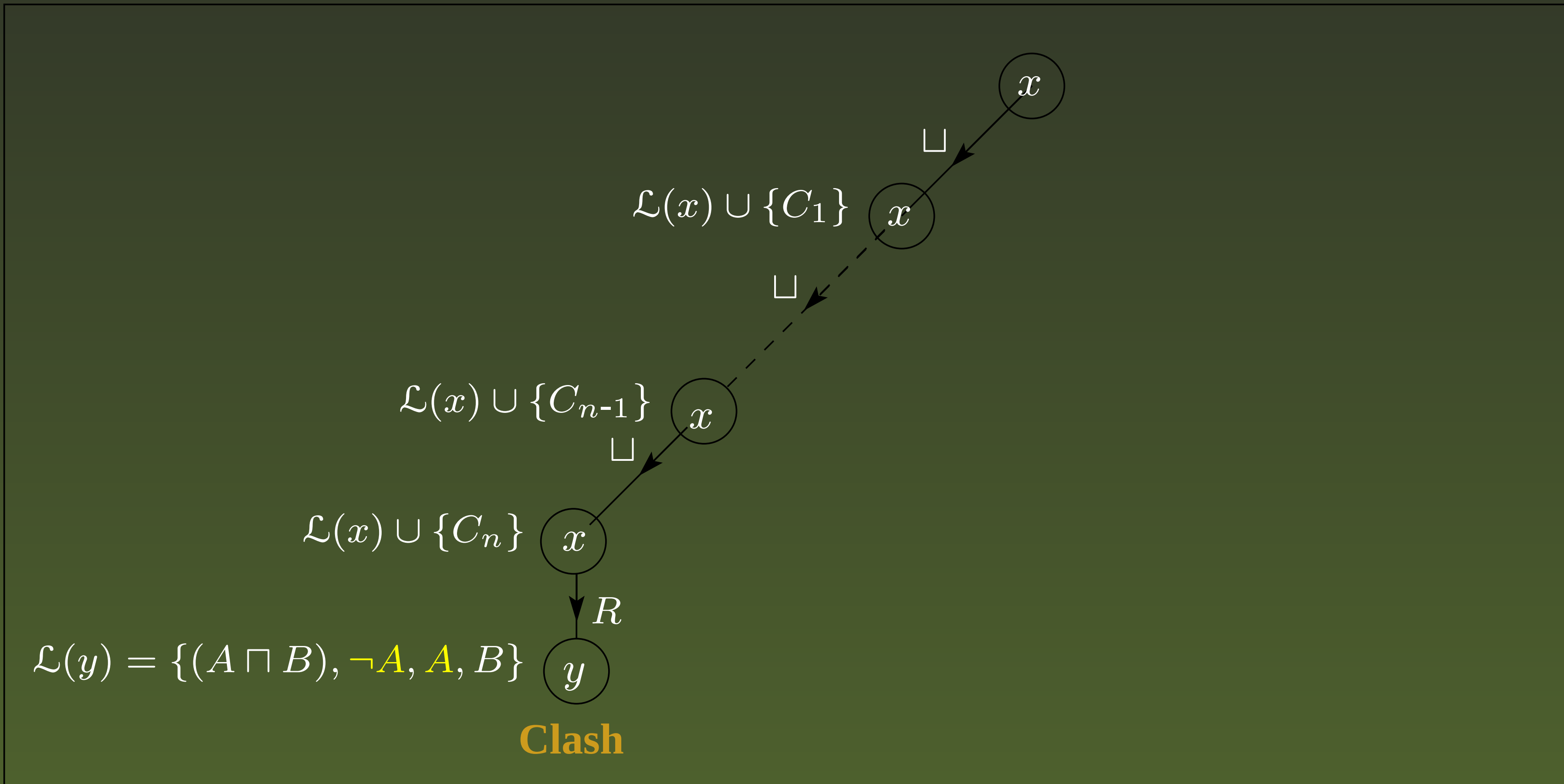

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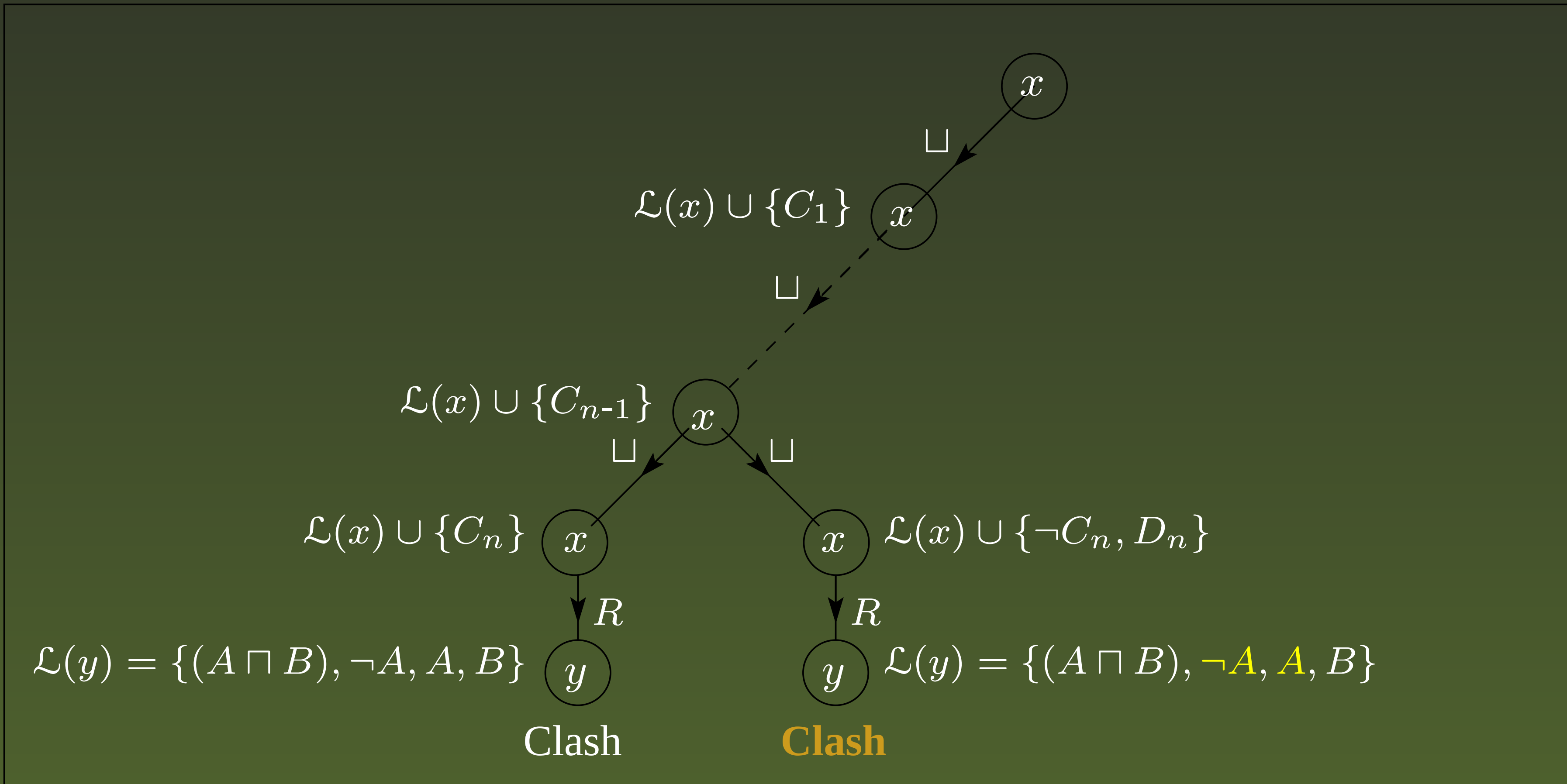

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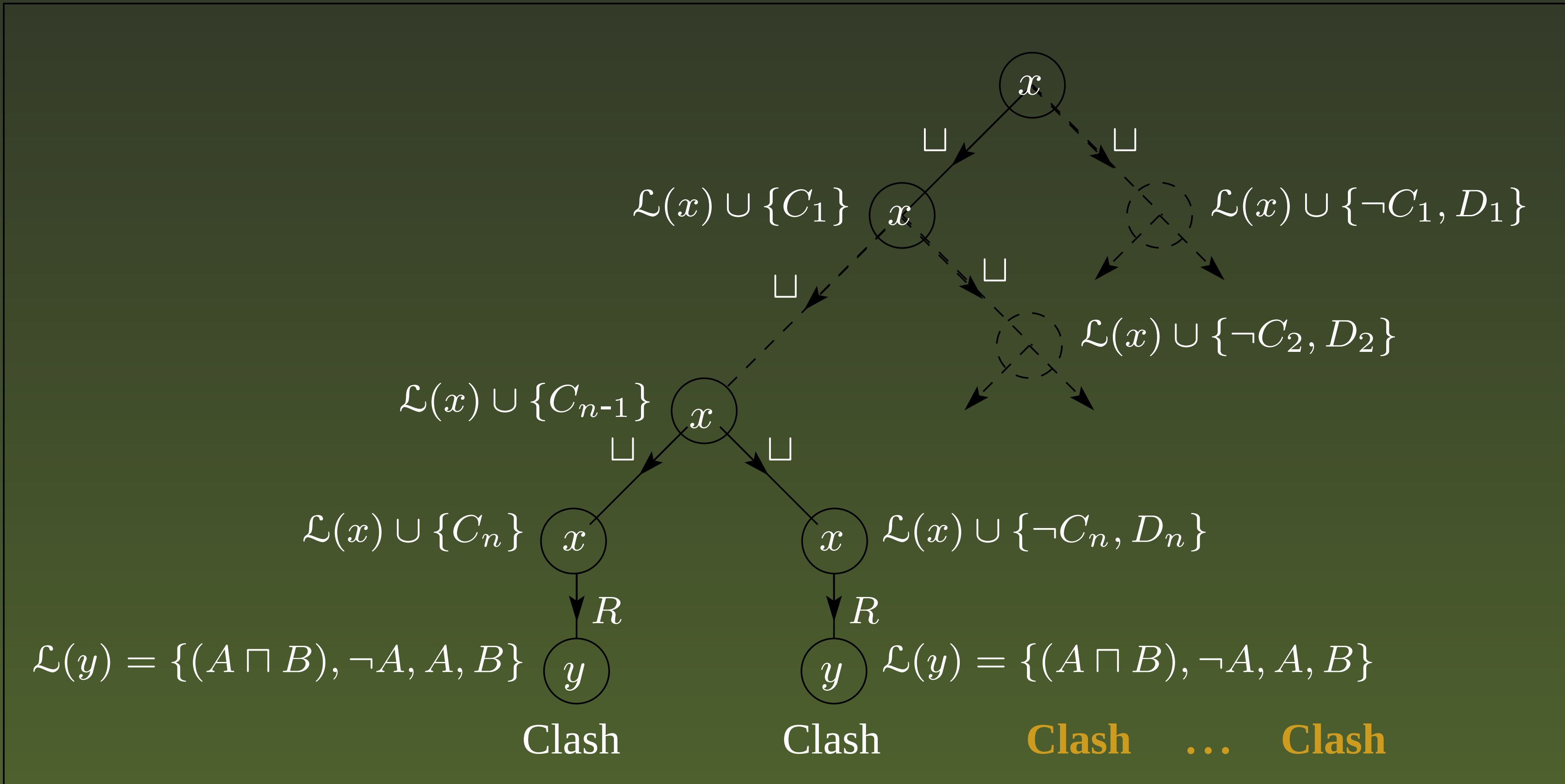

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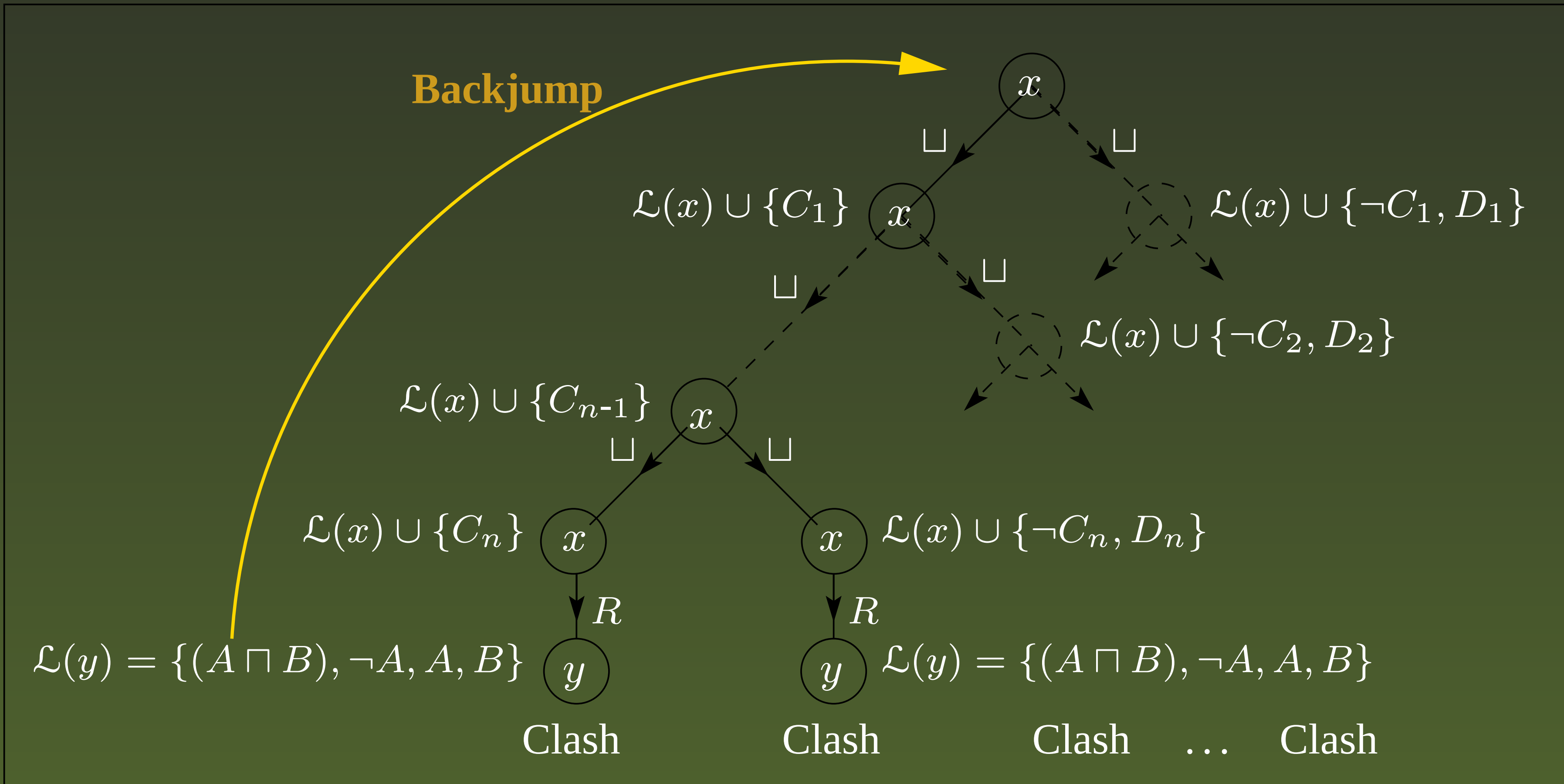

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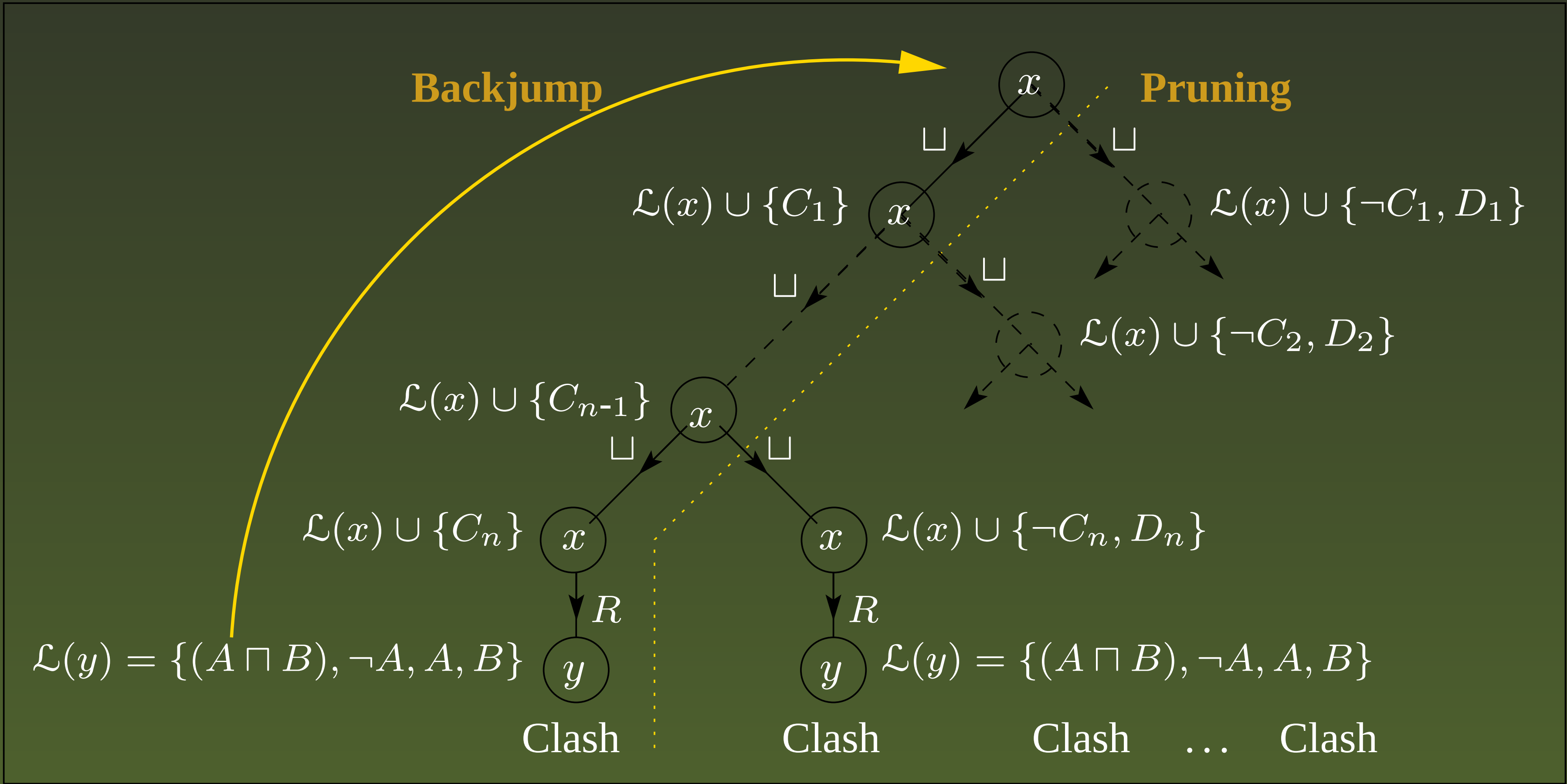

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