Reasoning Procedures II

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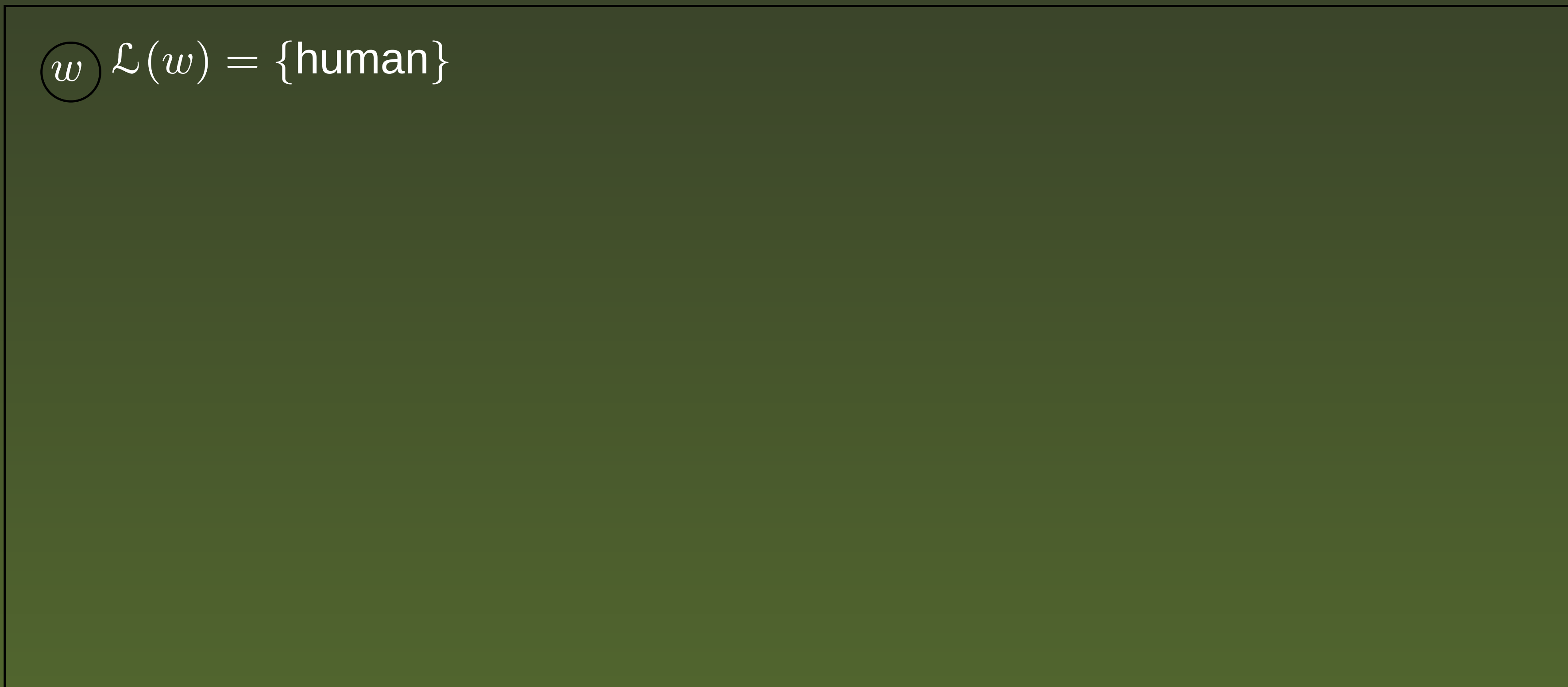

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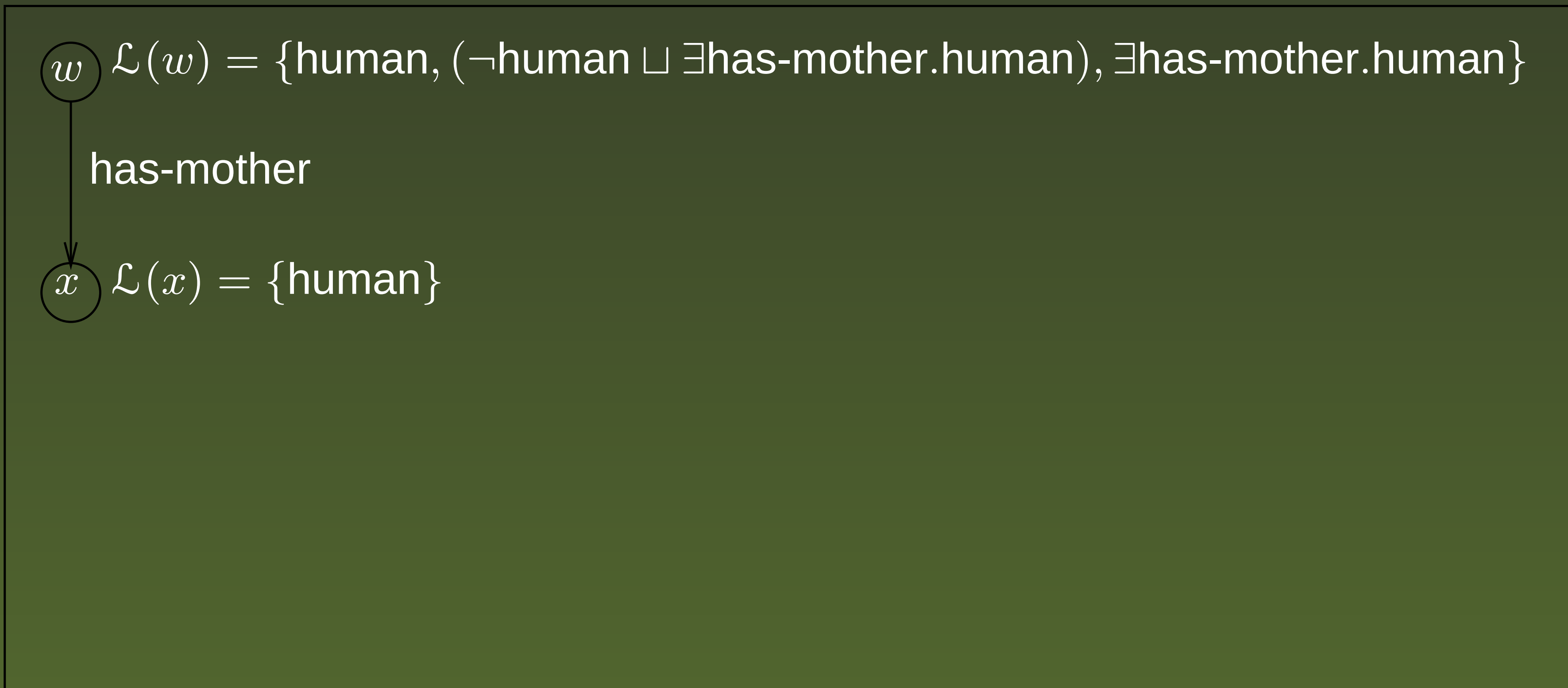

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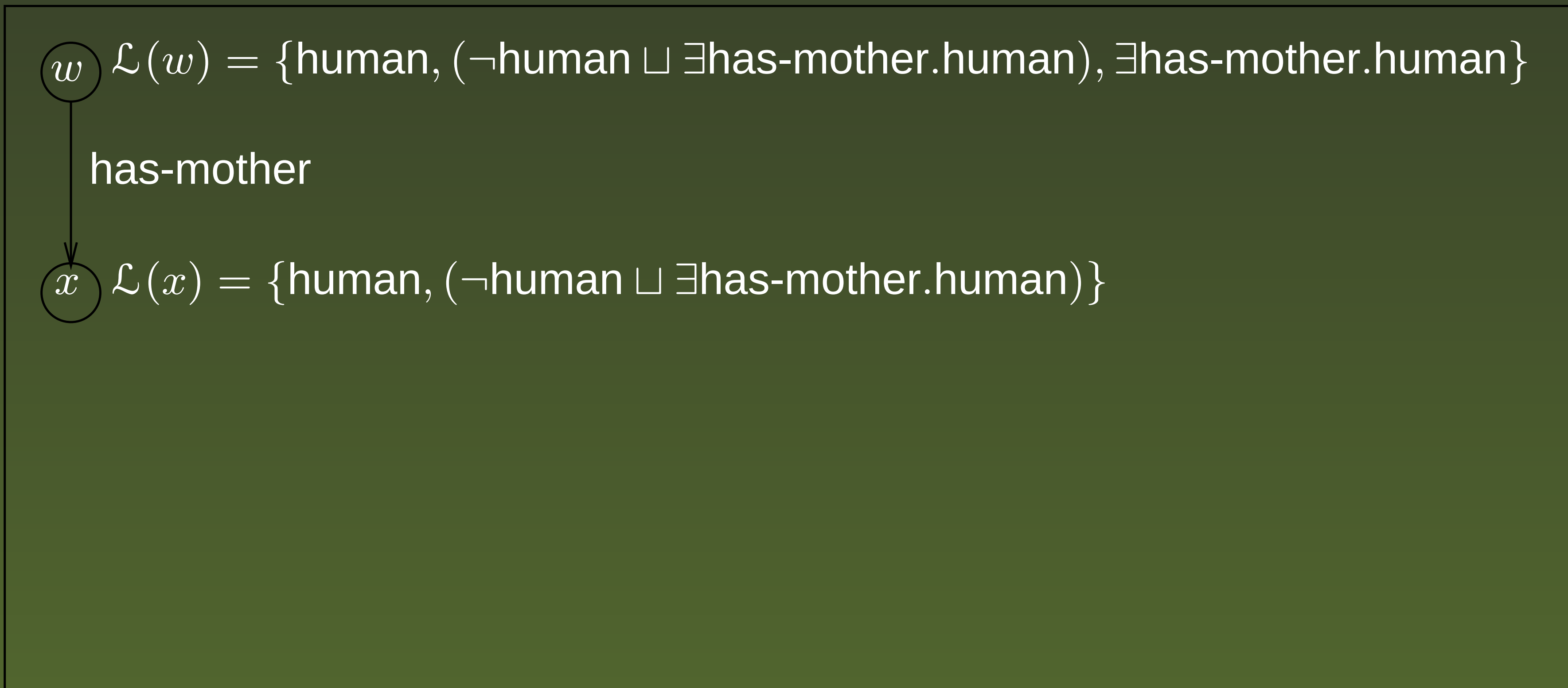

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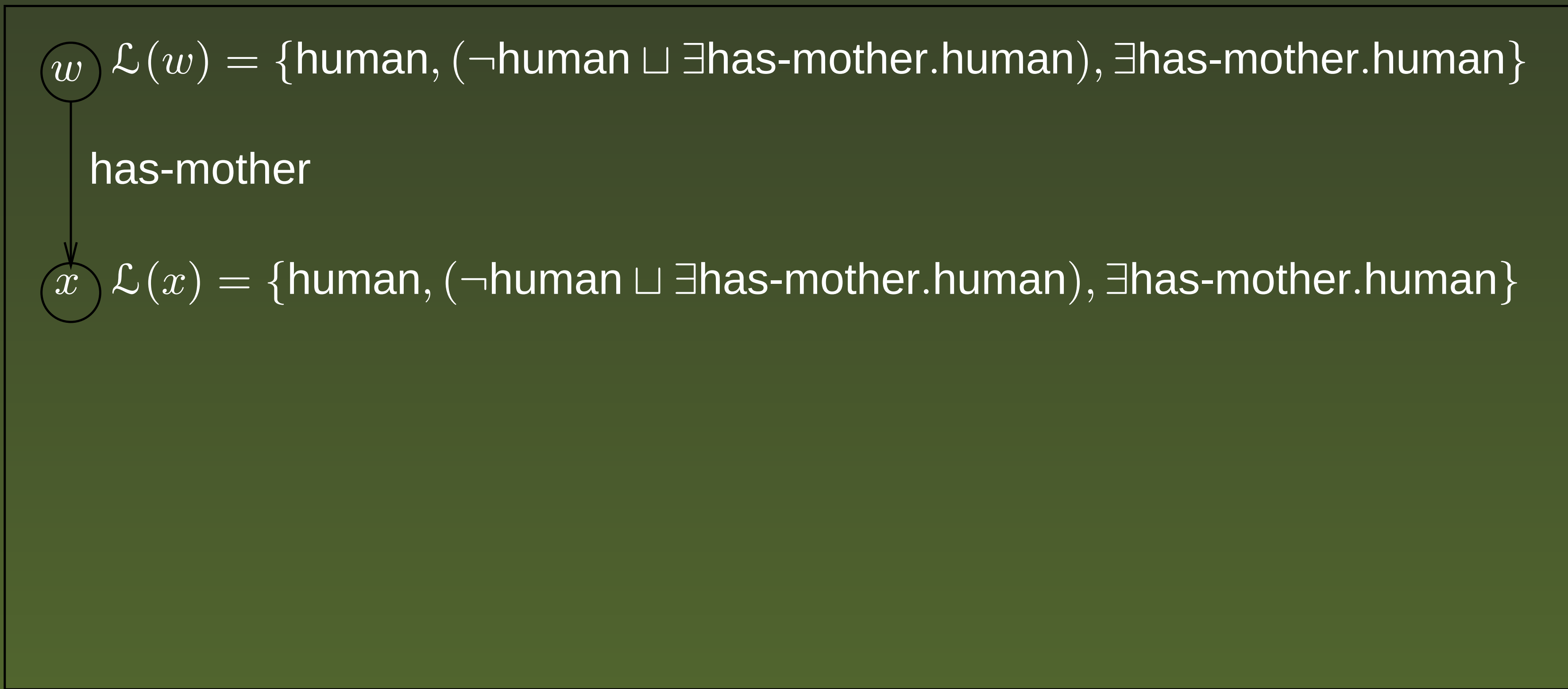

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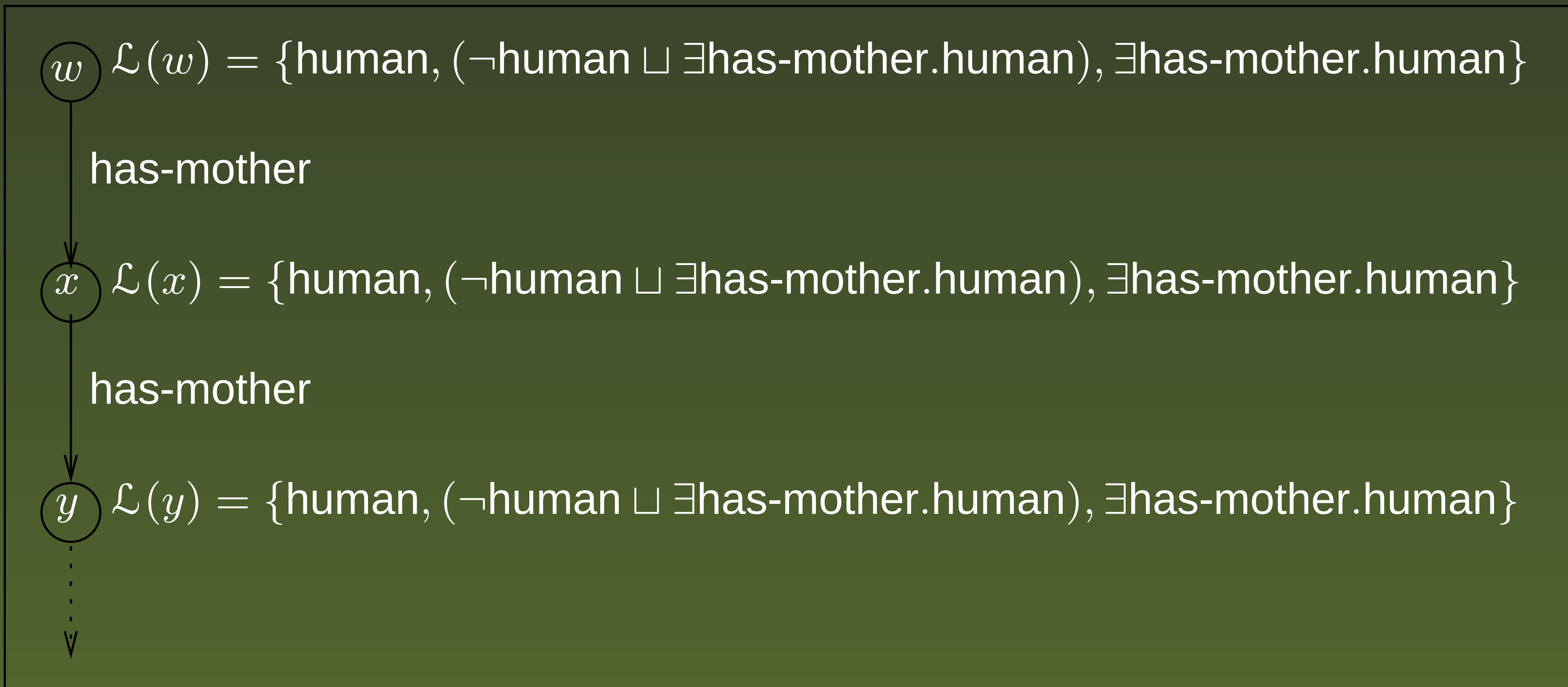

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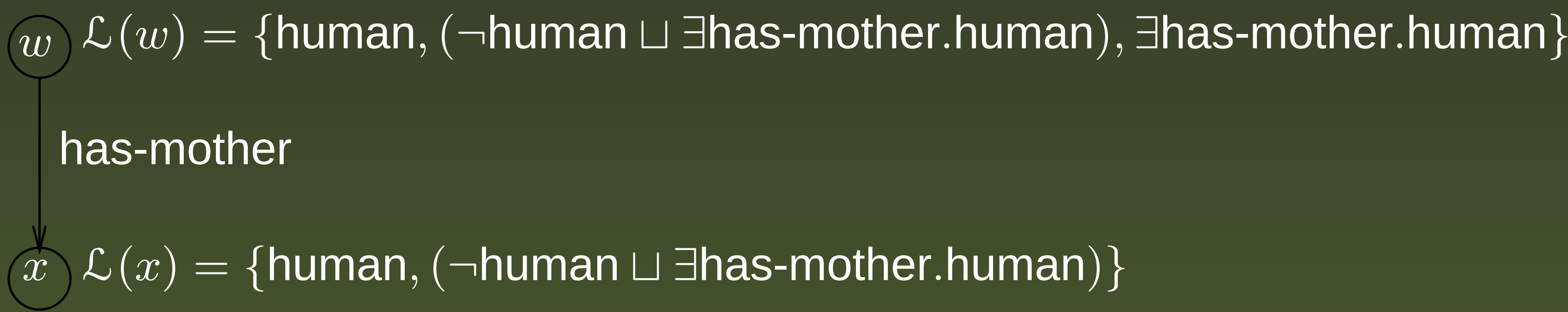

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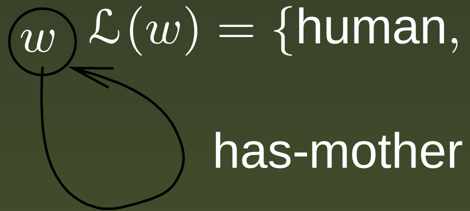

block represents **cyclical** model

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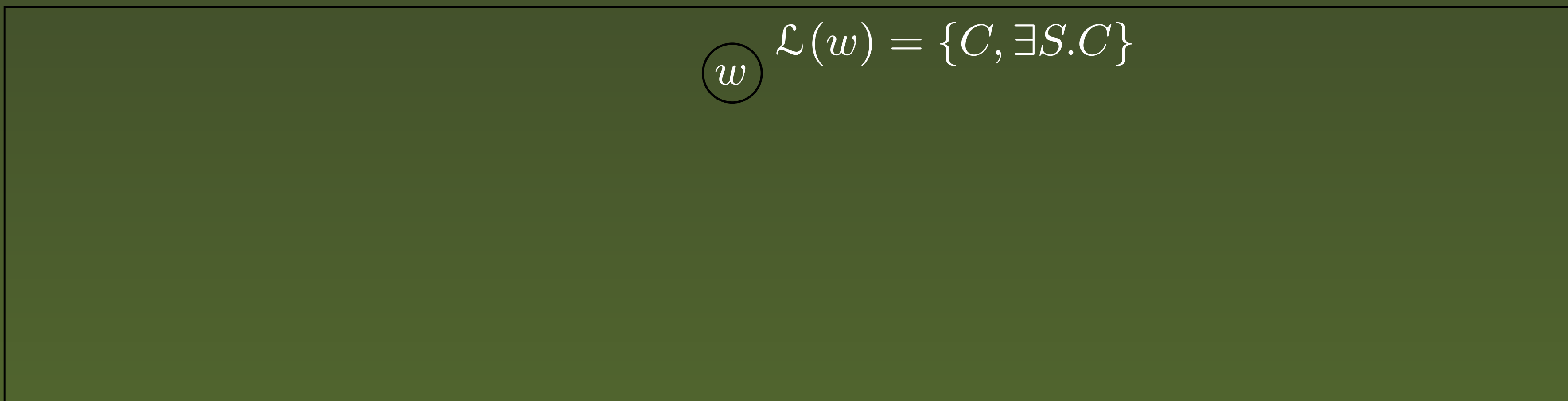

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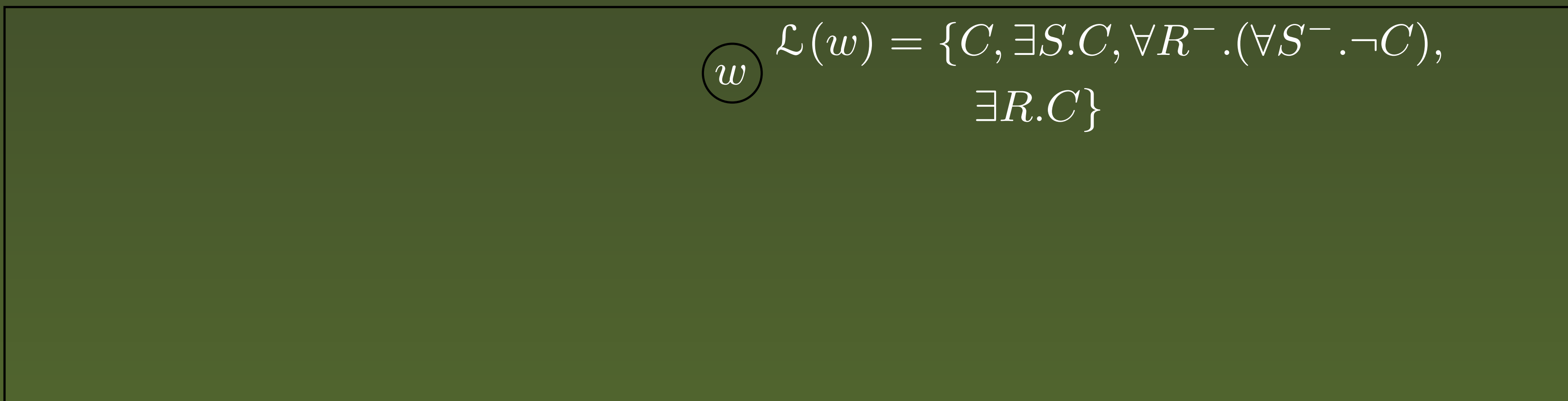

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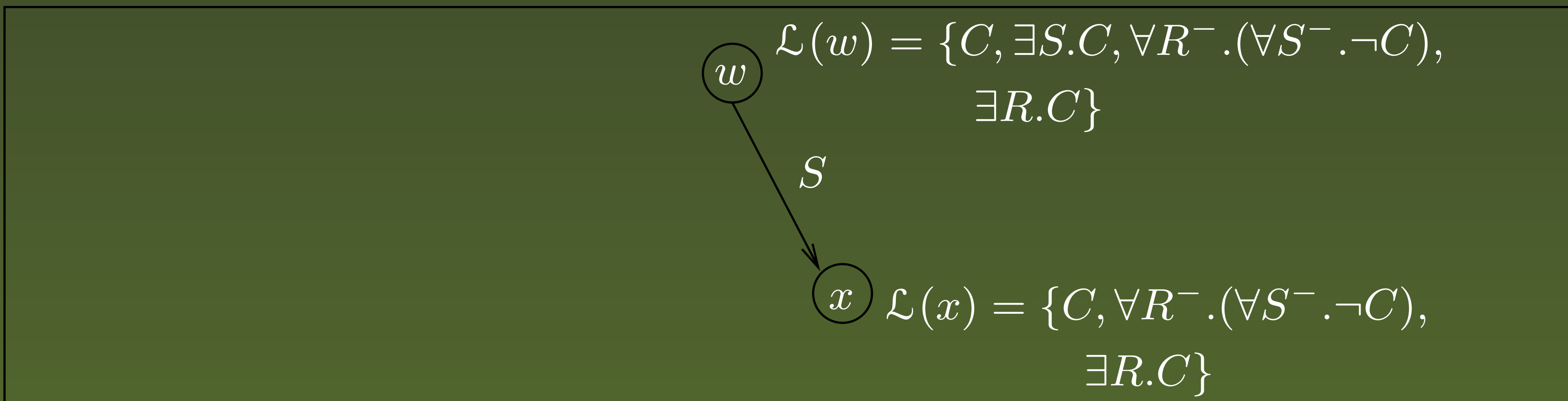

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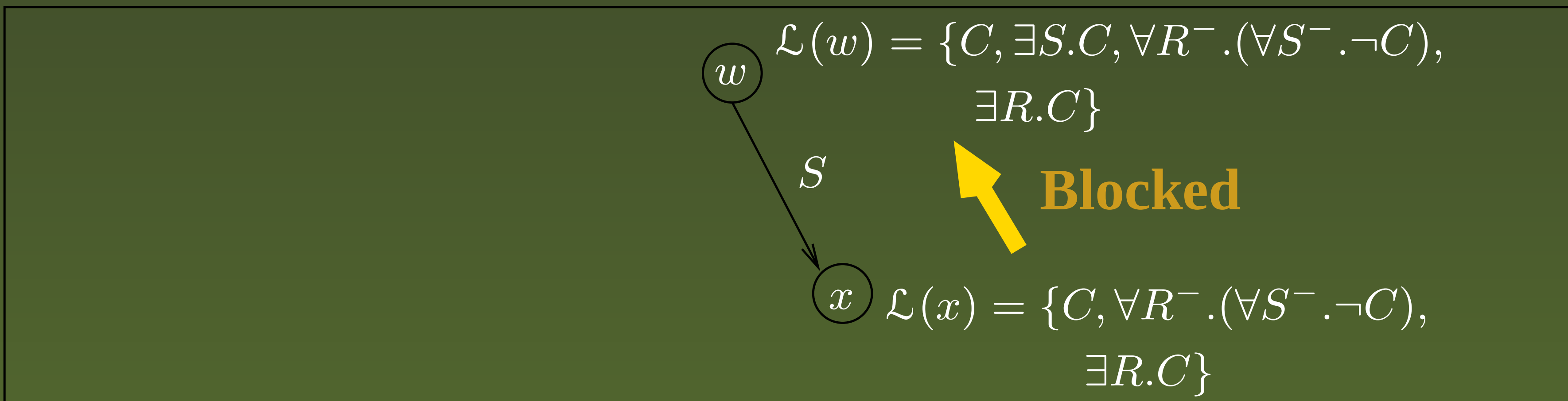

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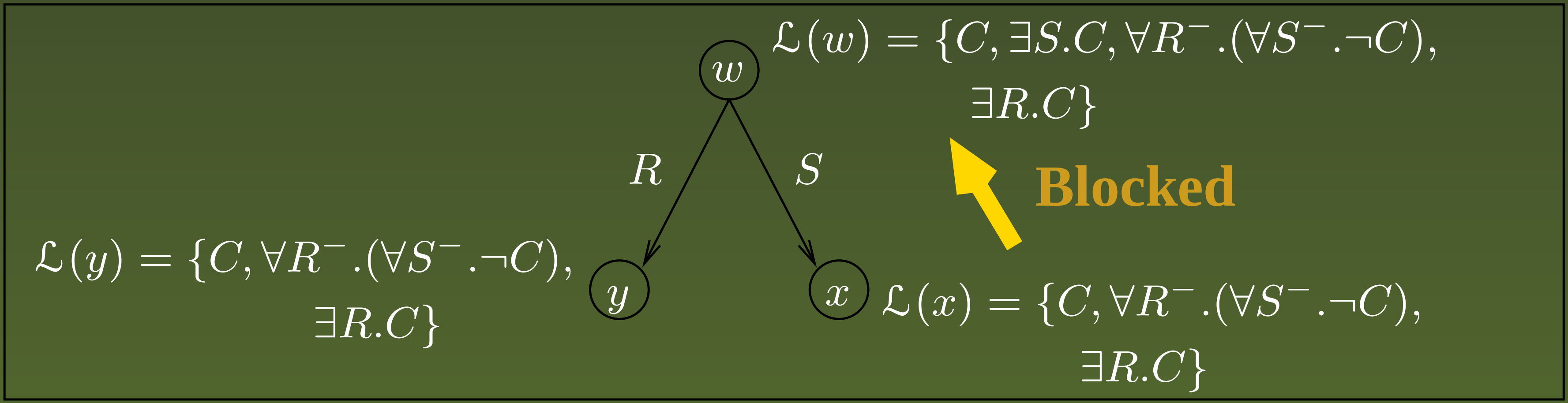

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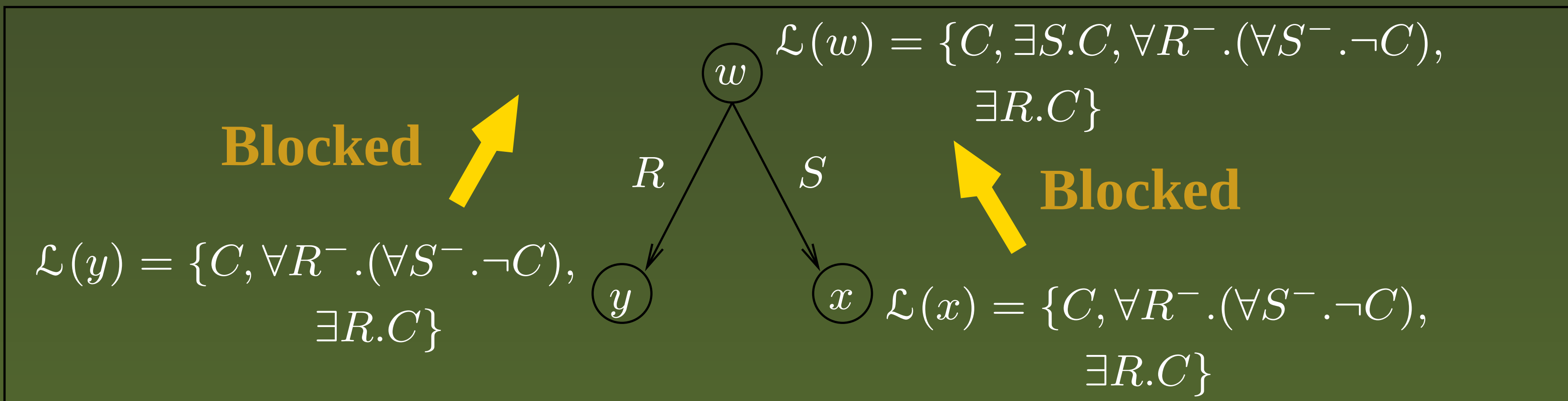

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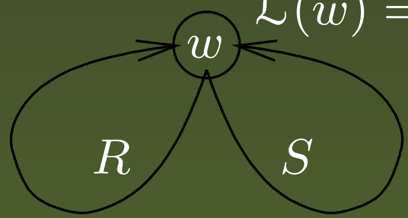

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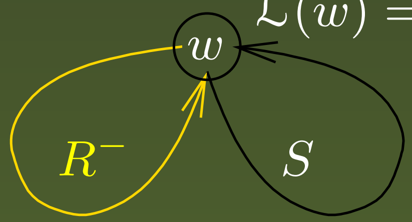

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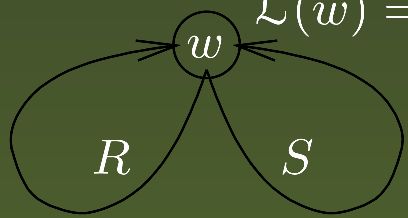

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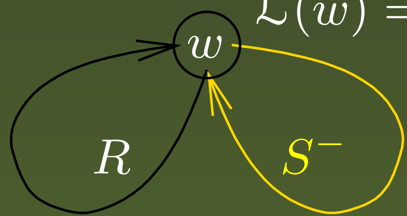

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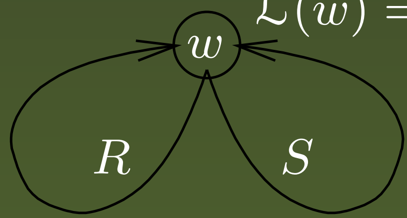

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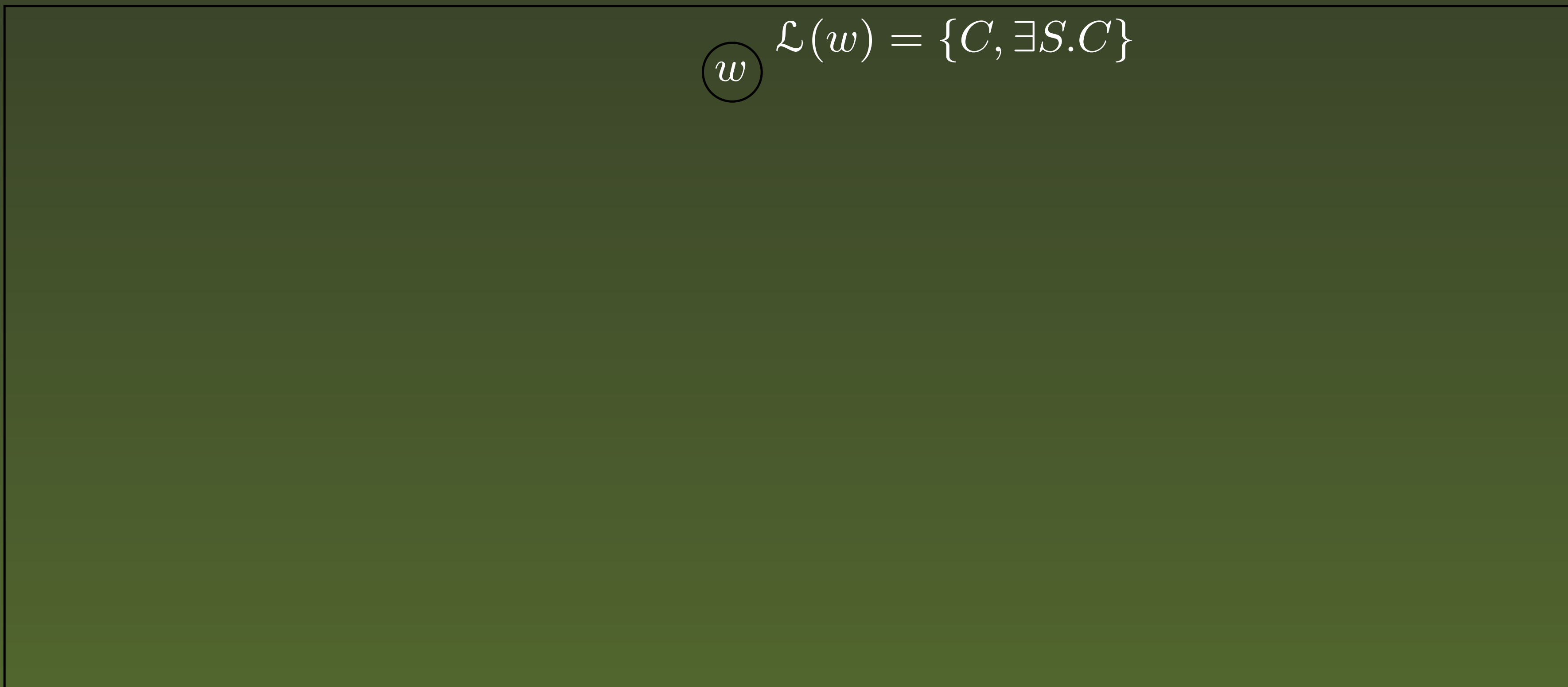

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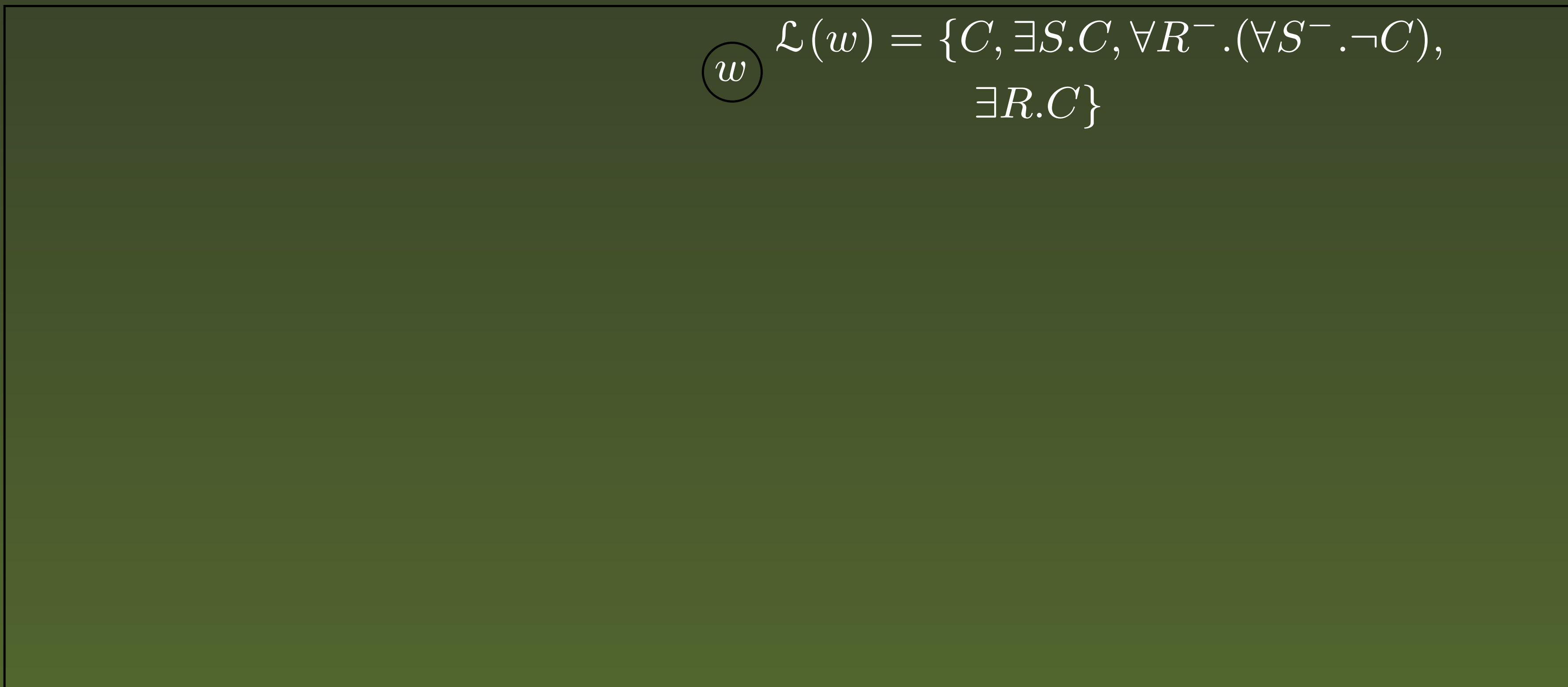

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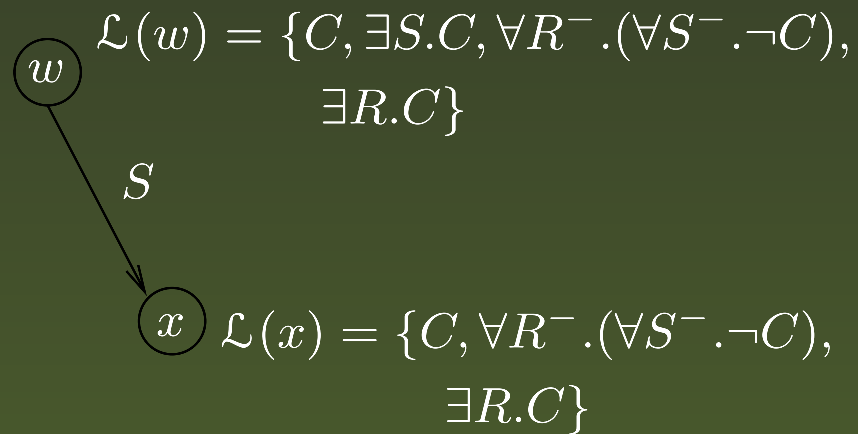

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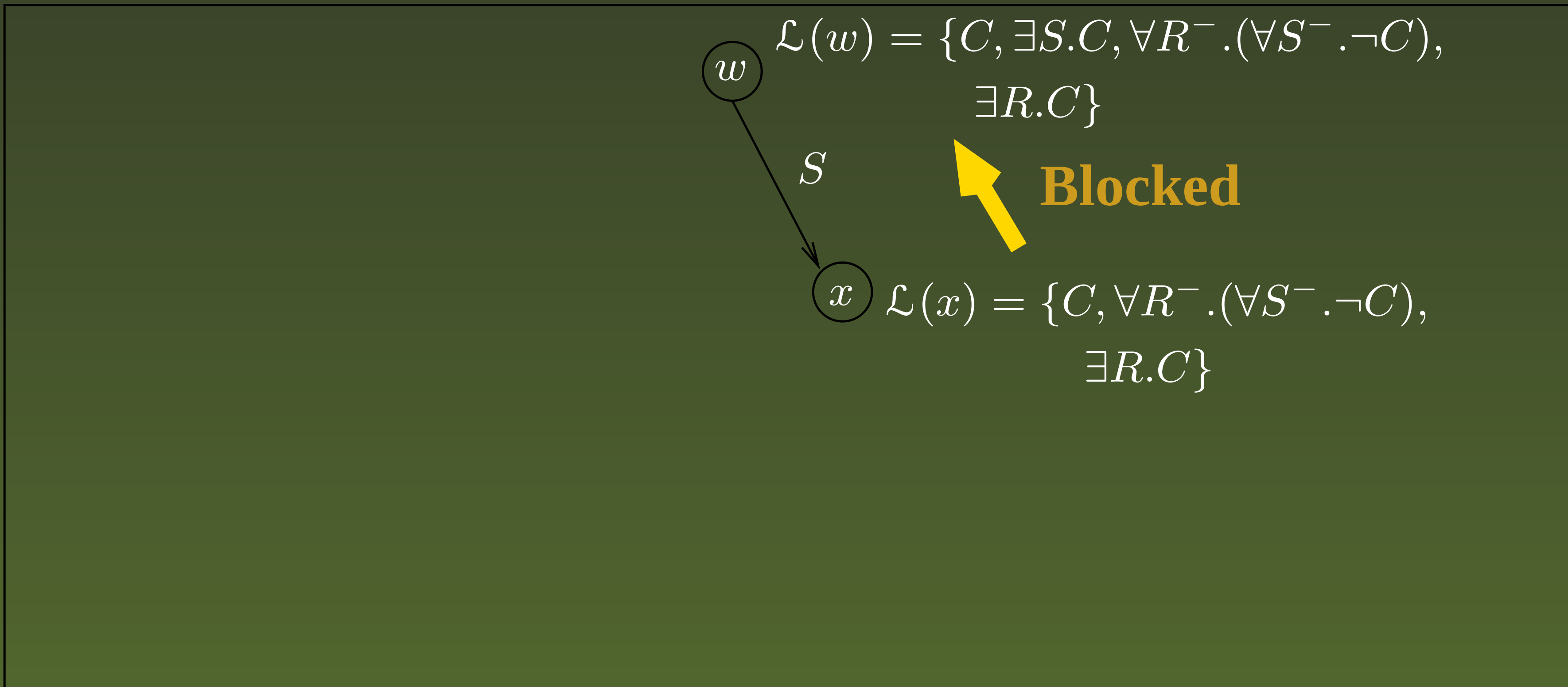

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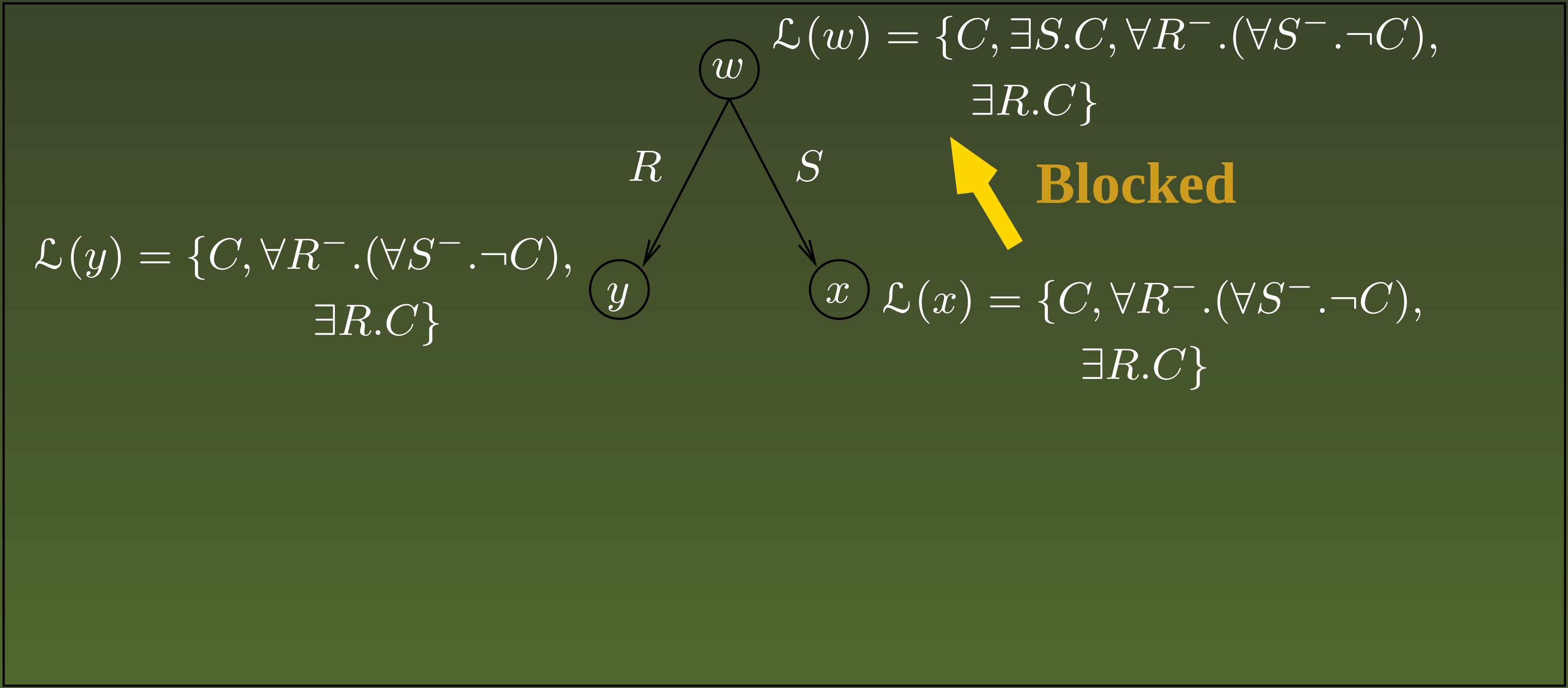

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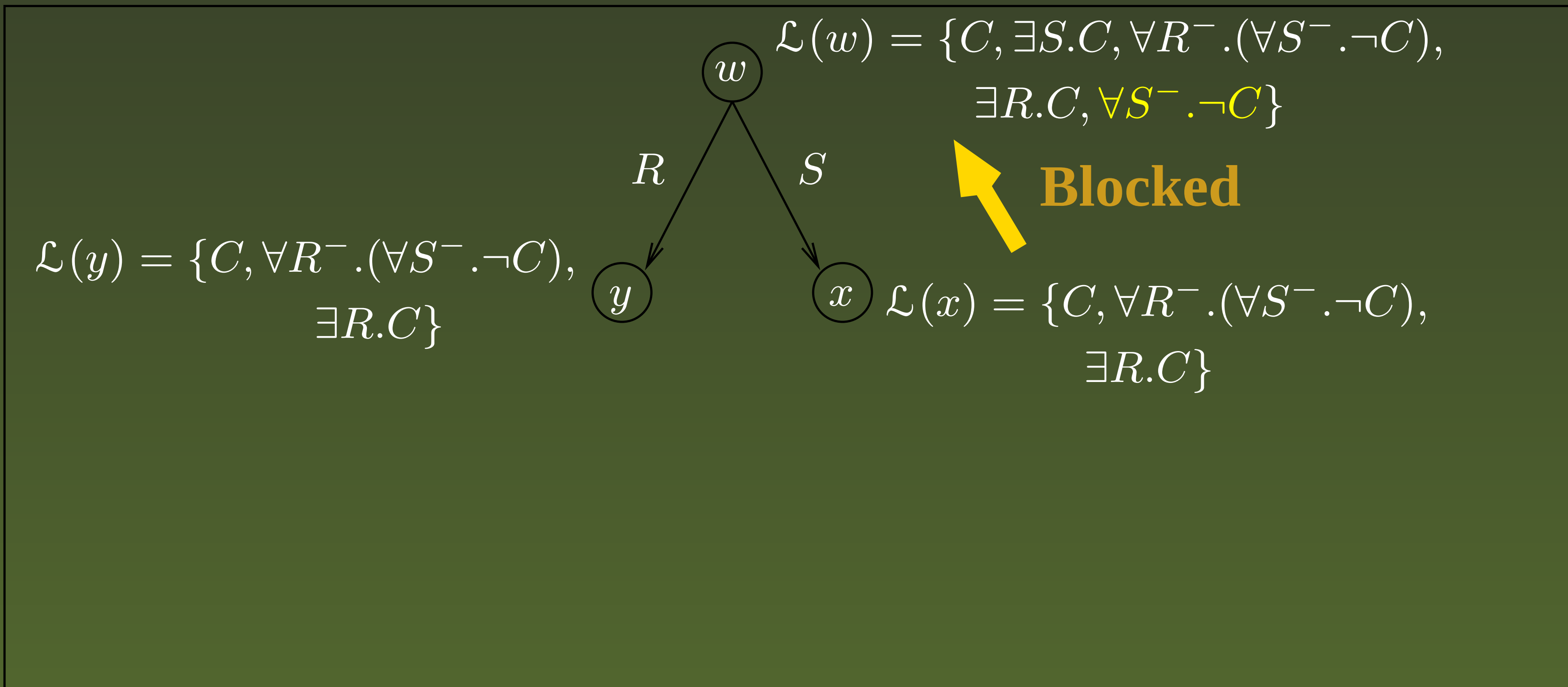

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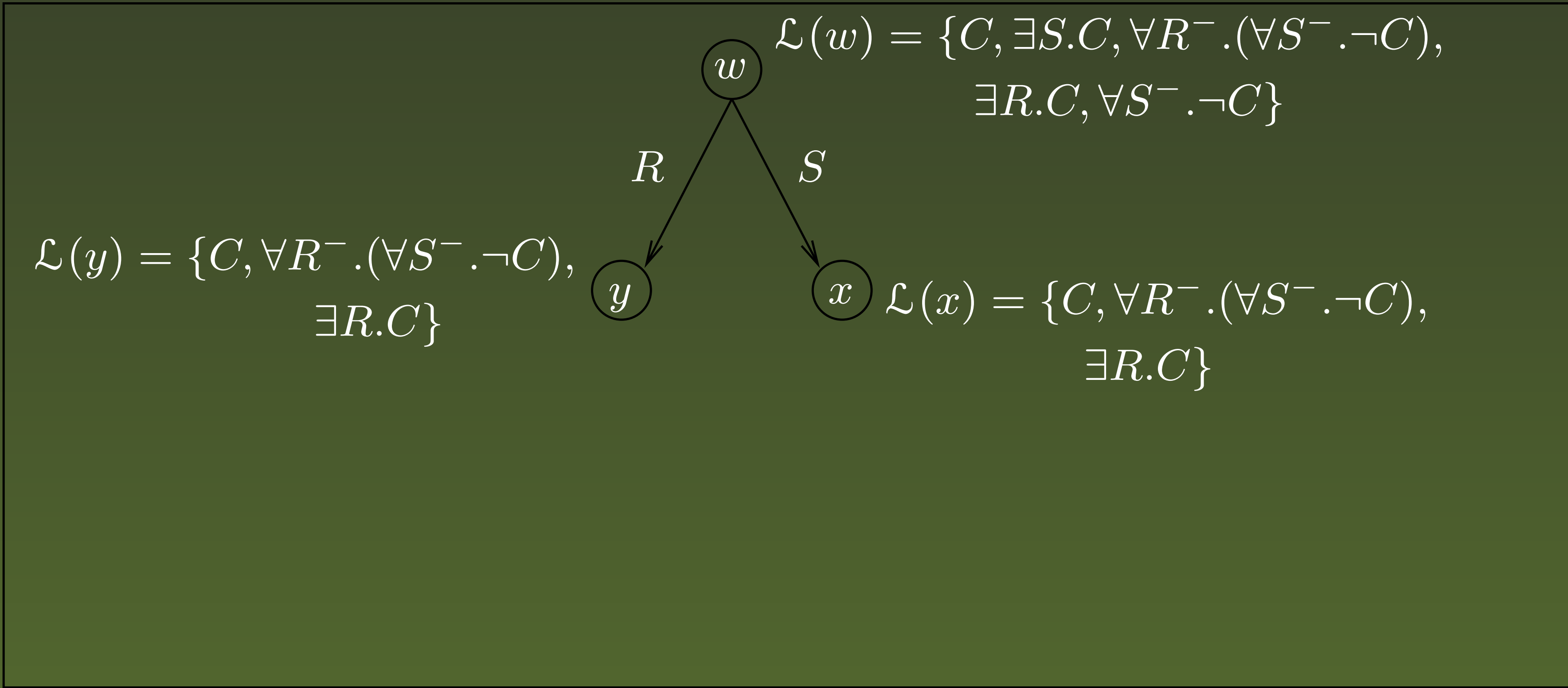

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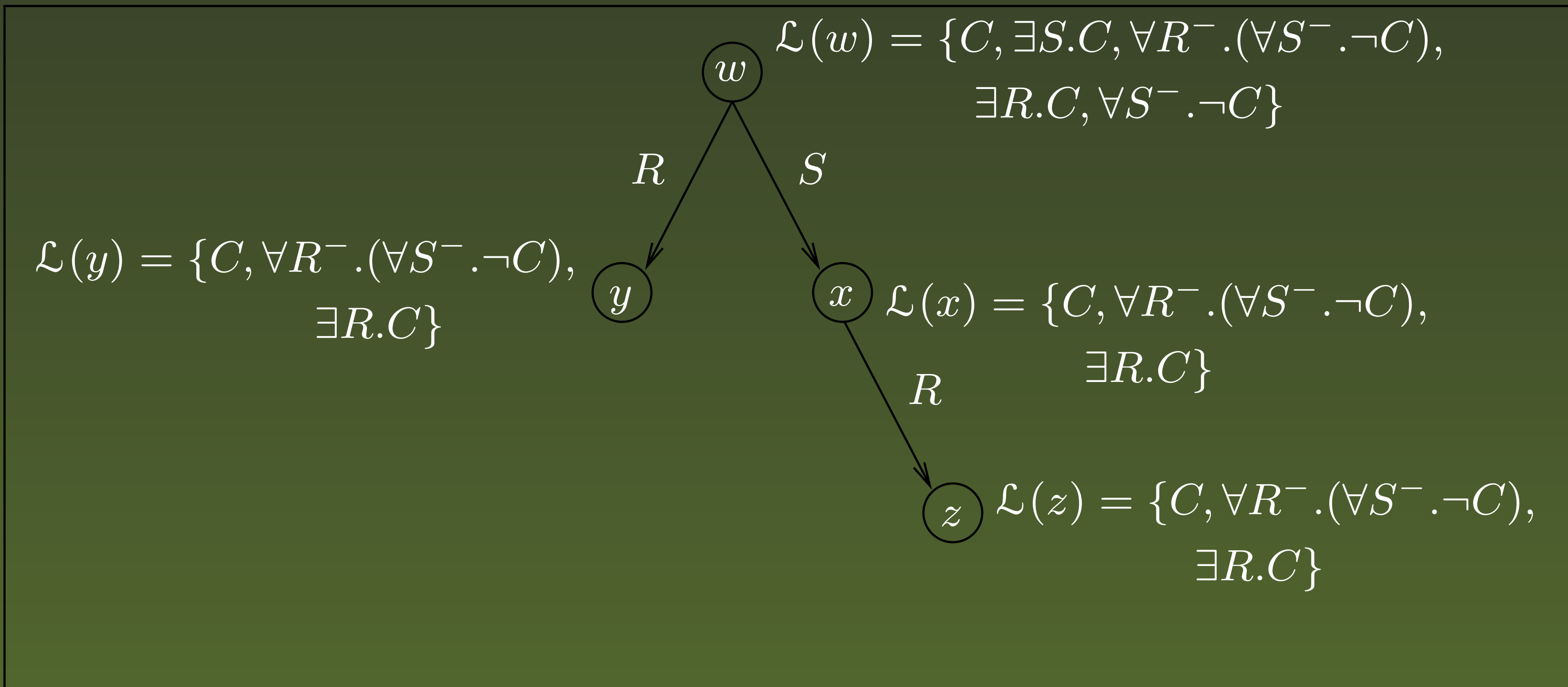

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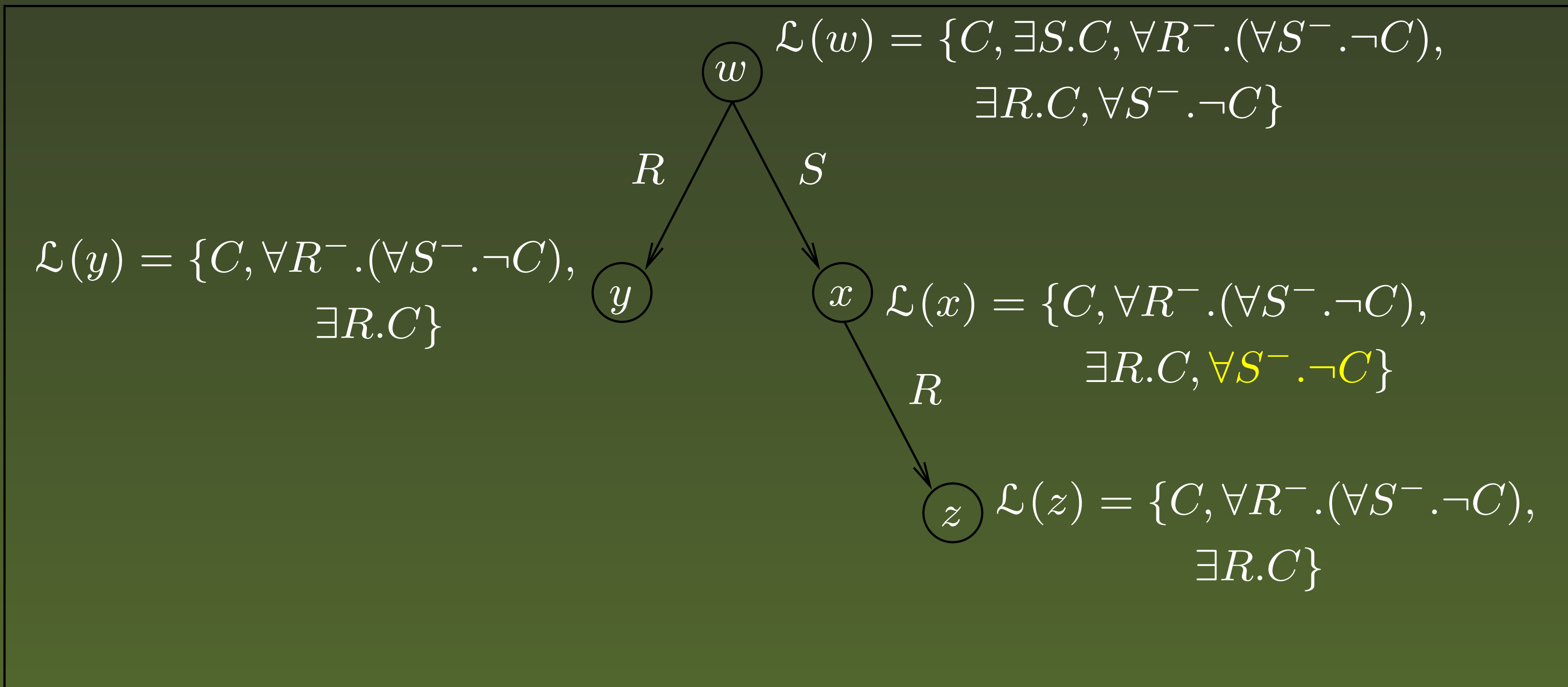

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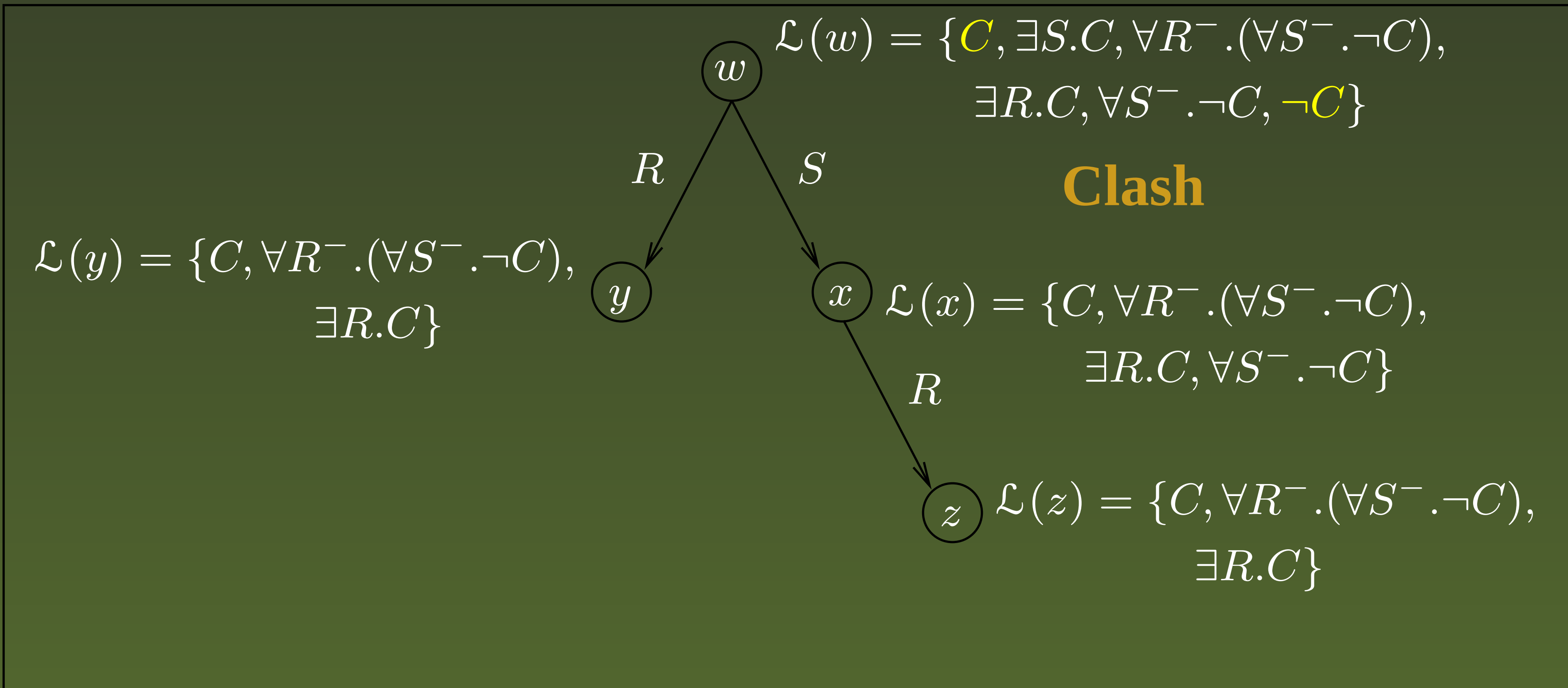

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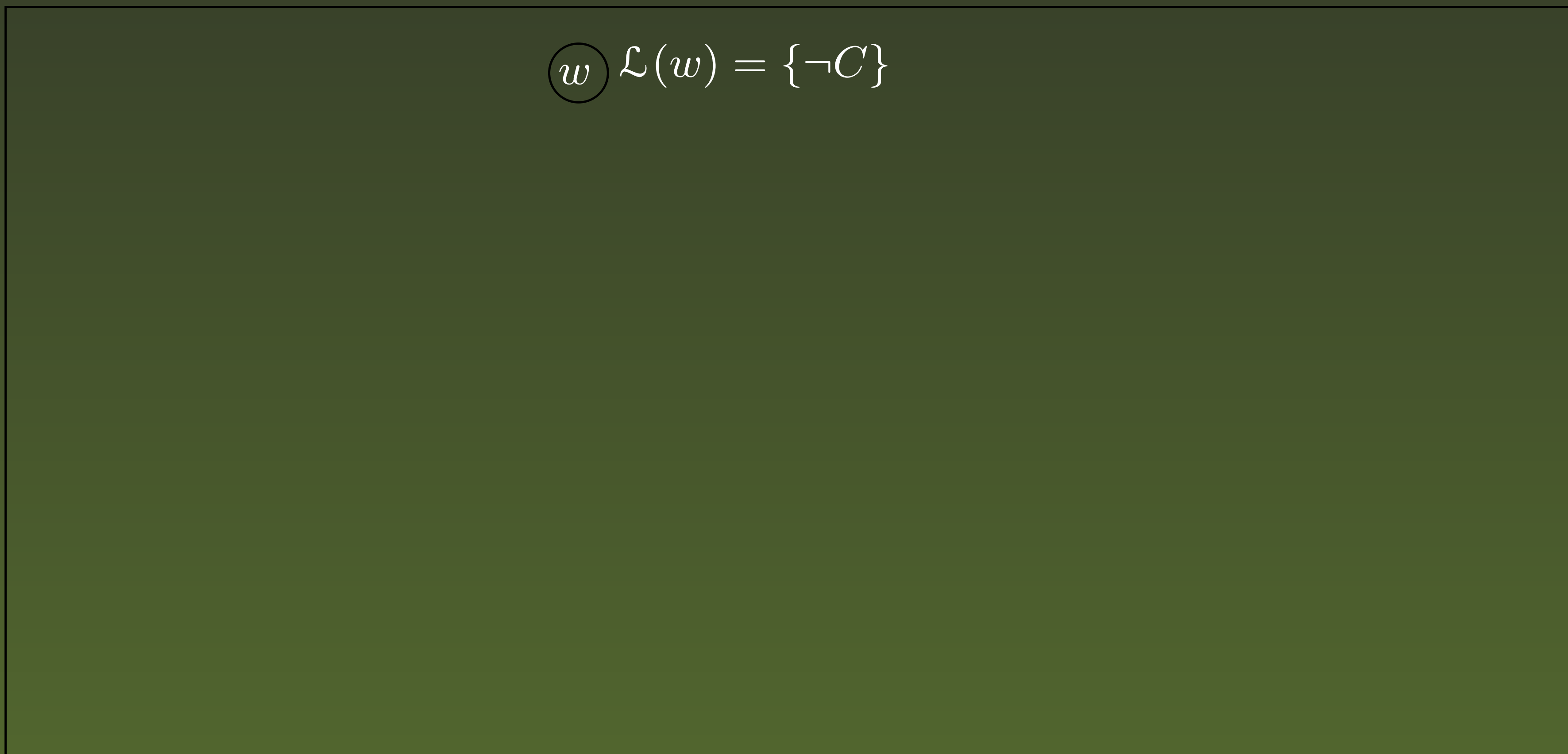

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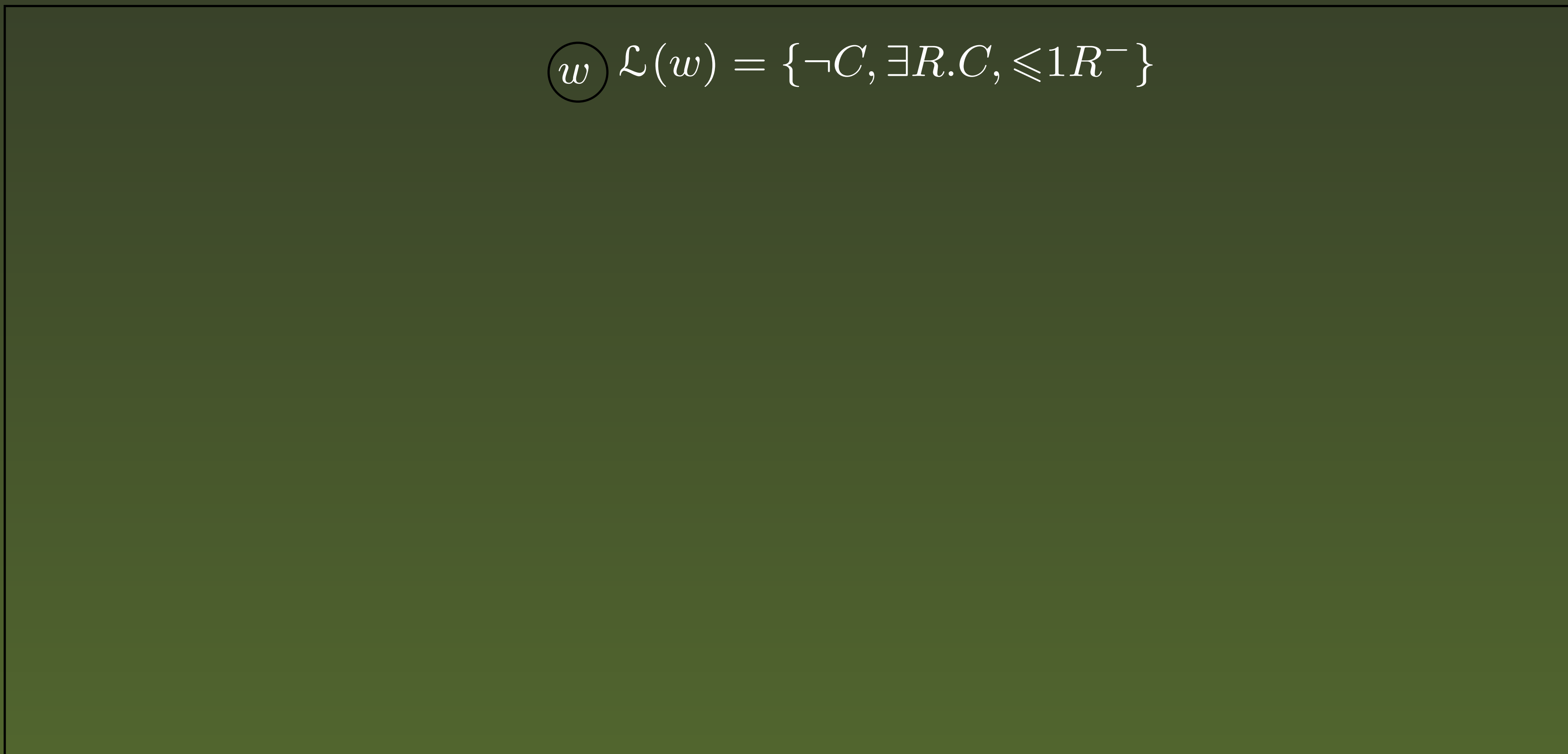

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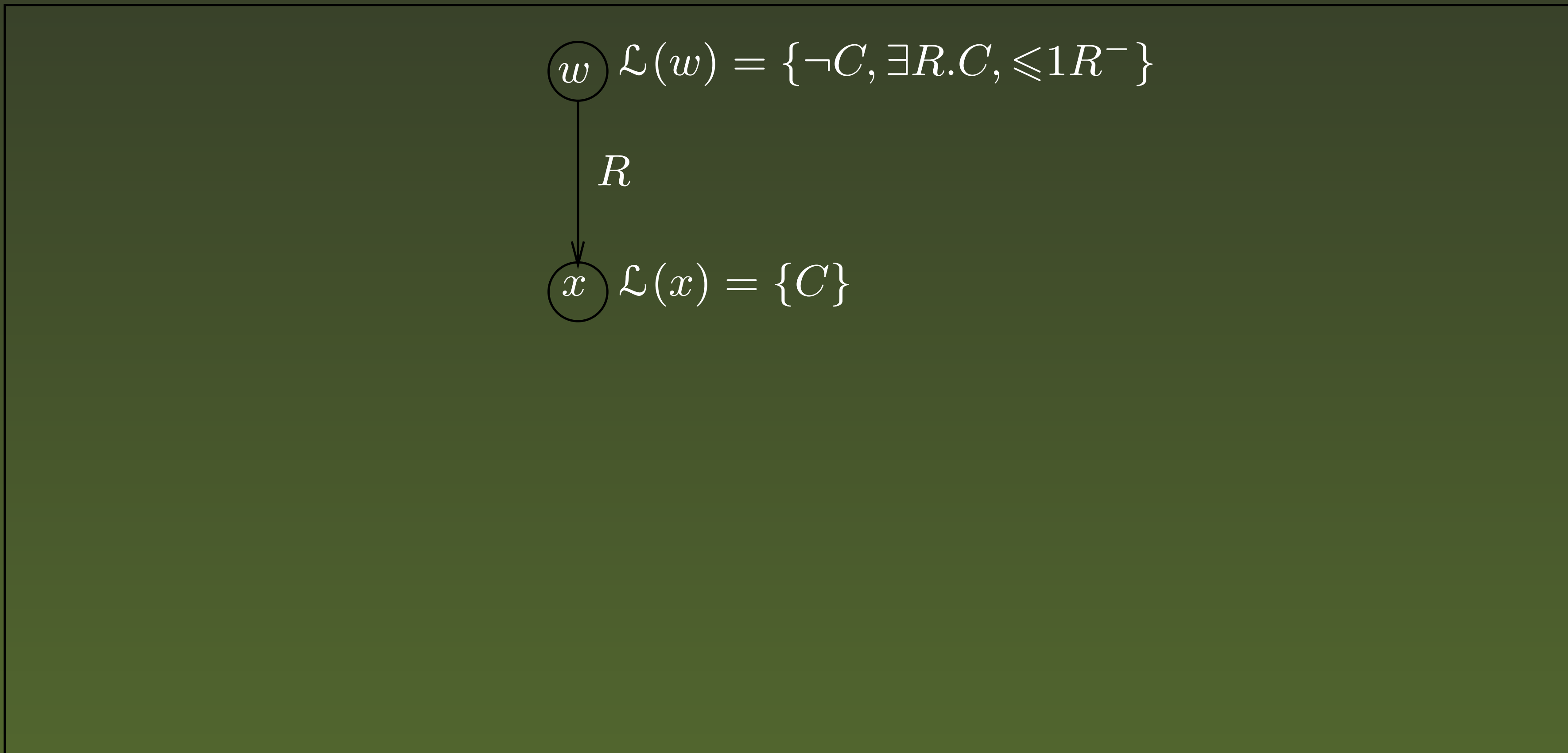

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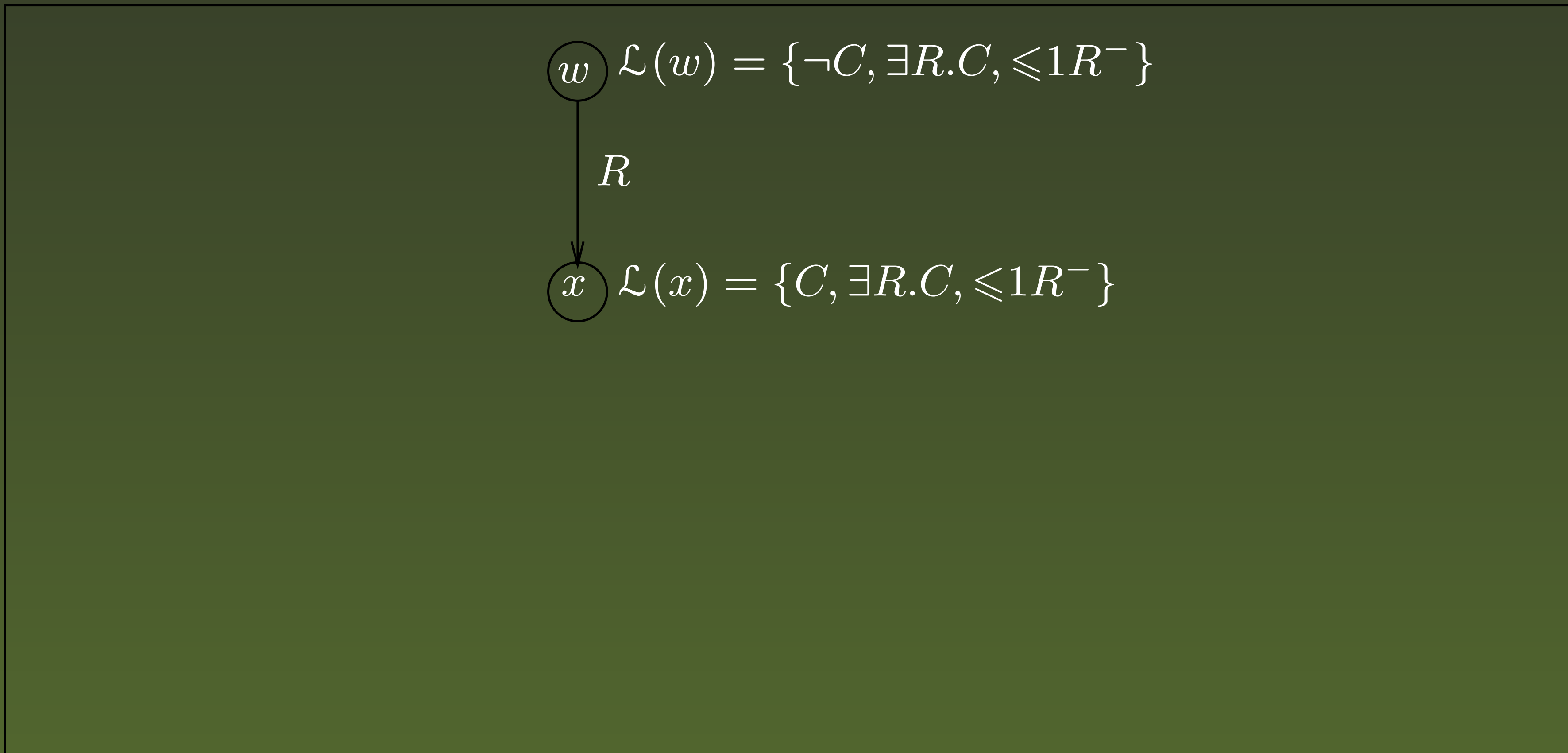

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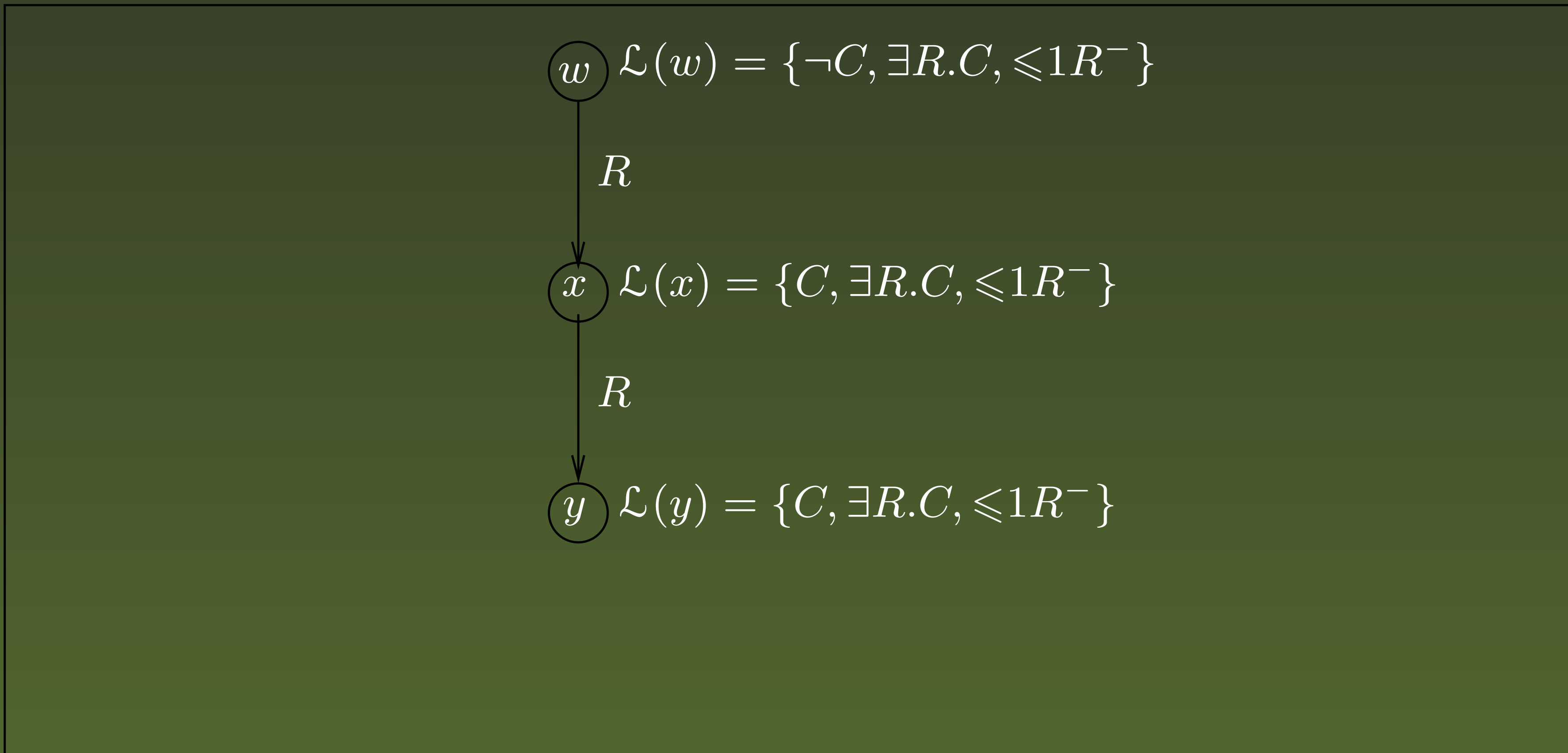

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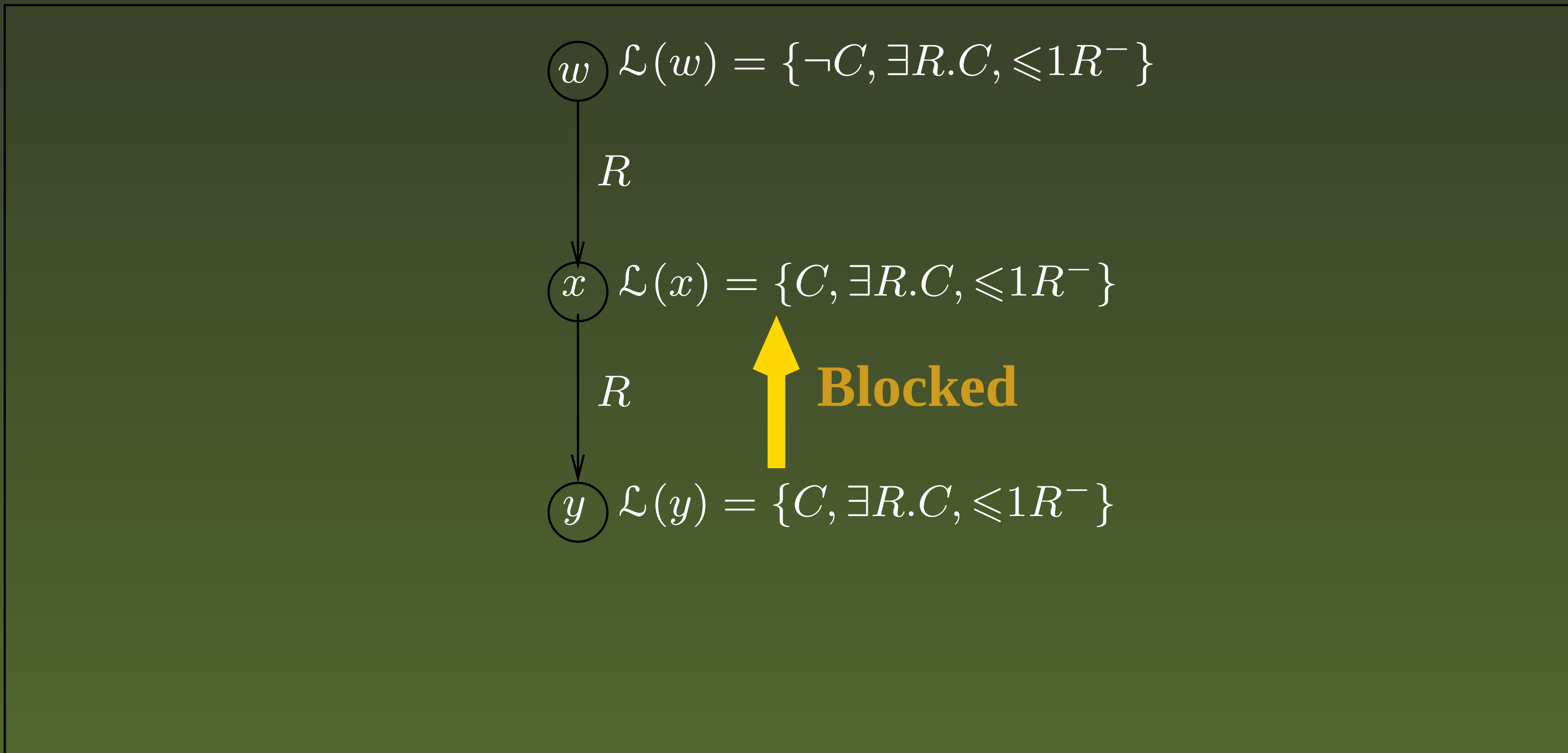

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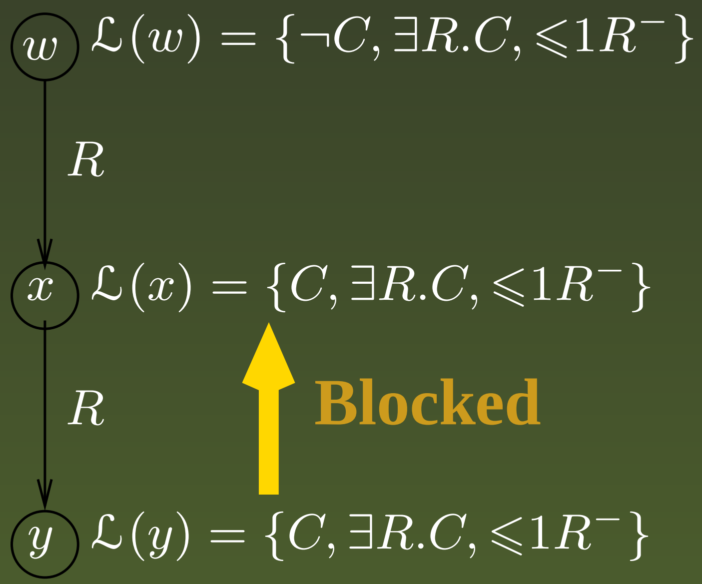

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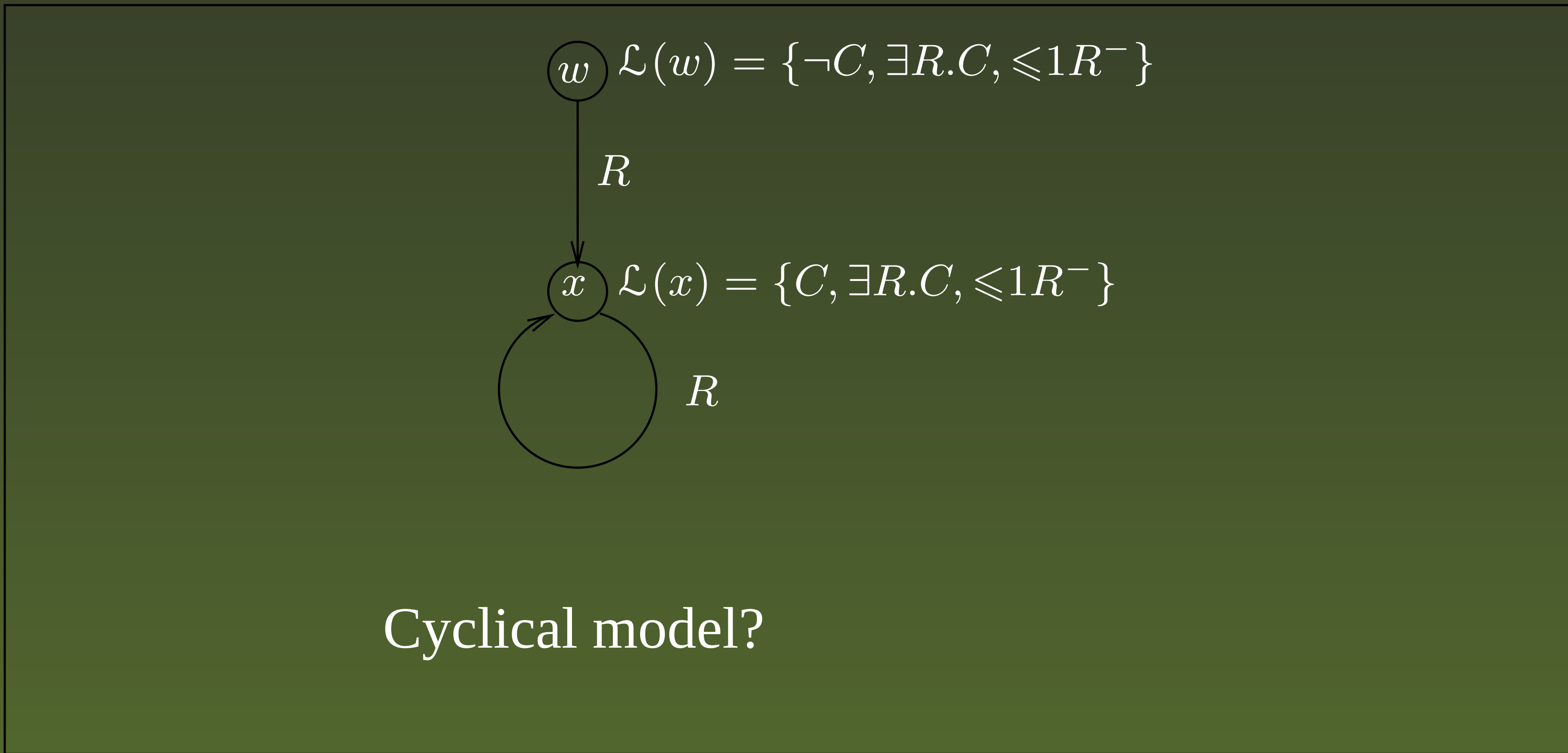

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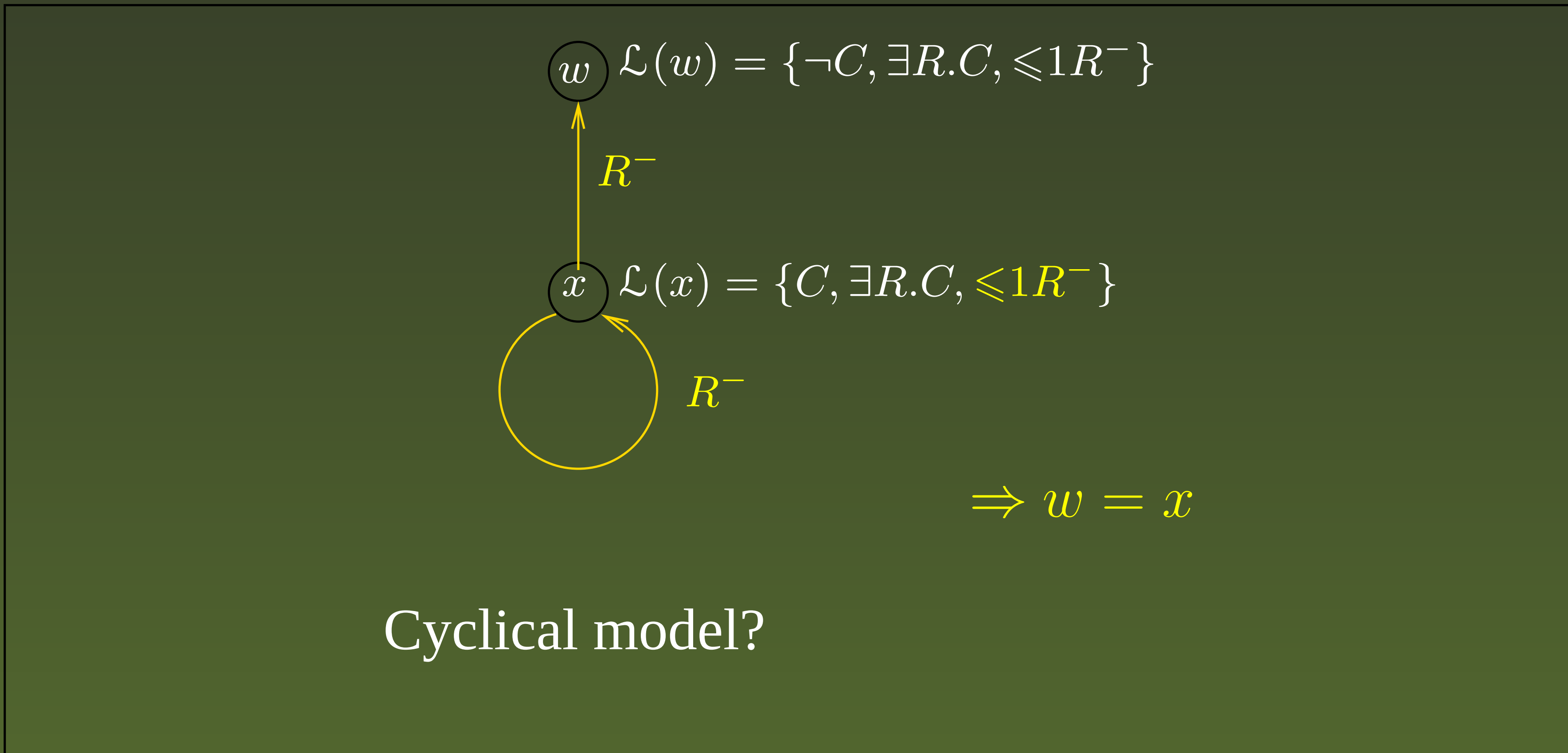

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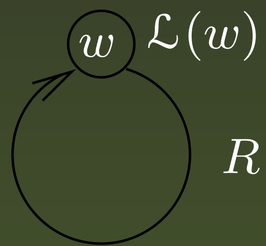

Clash

Cyclical model?

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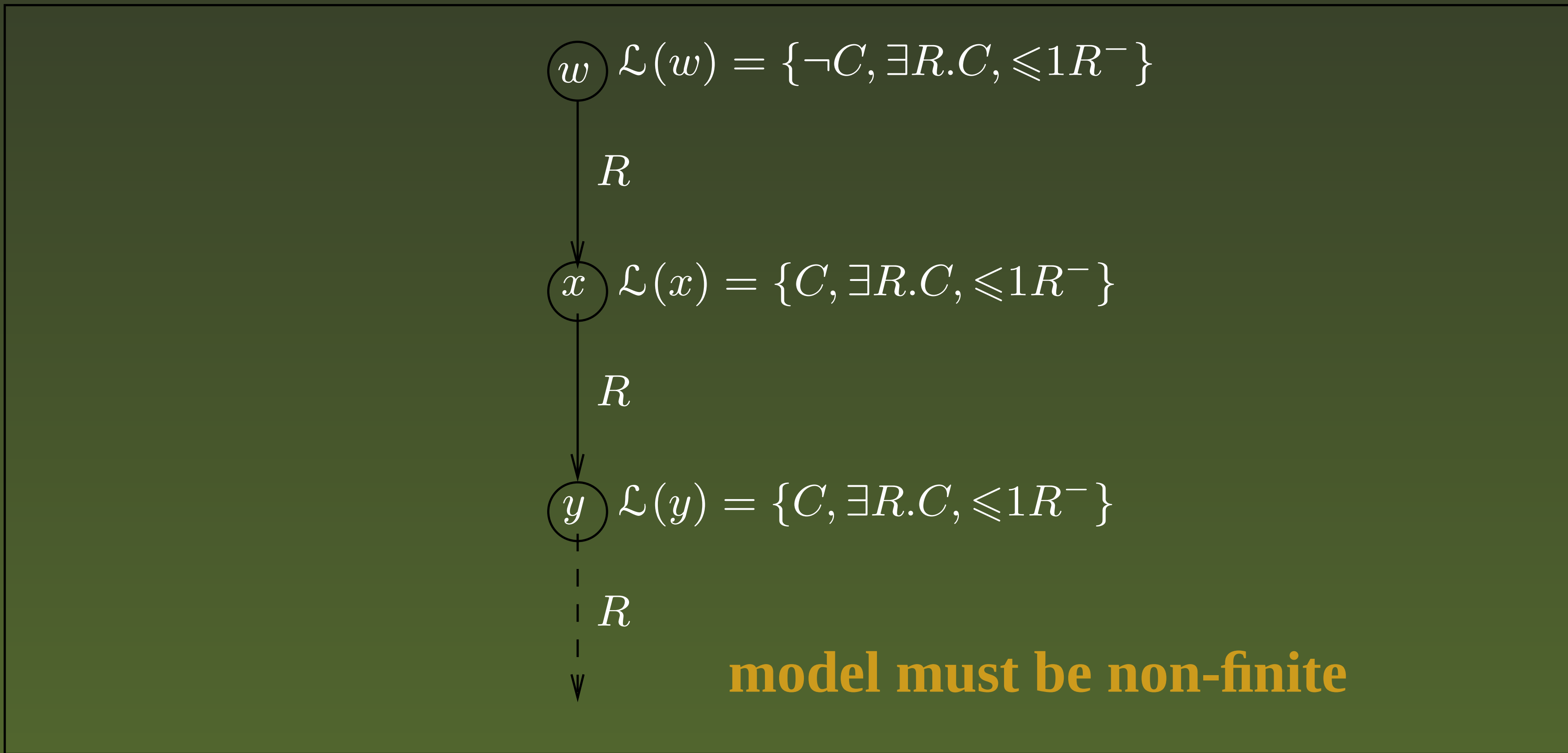

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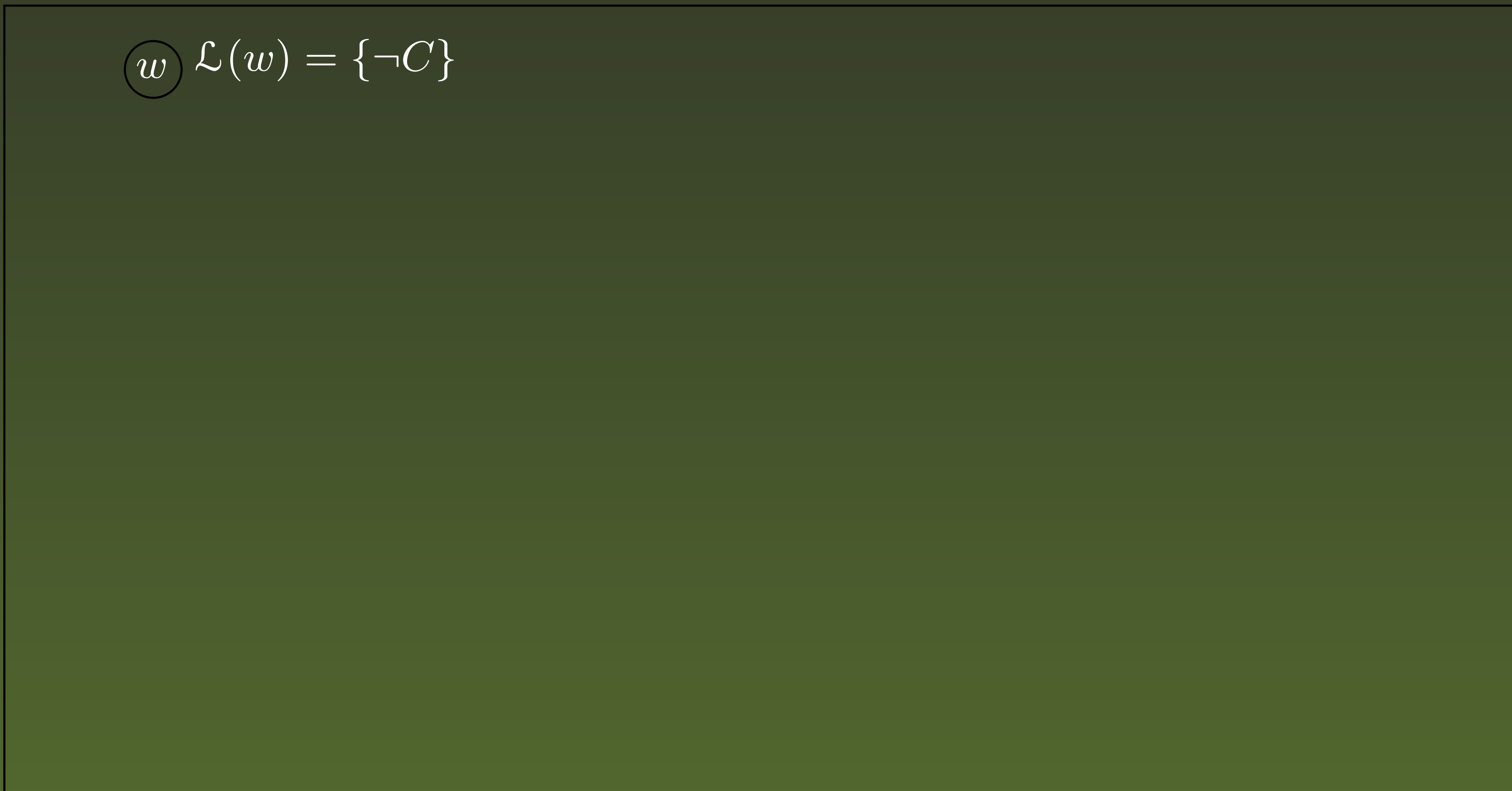

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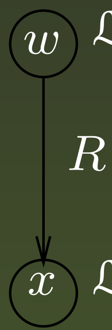

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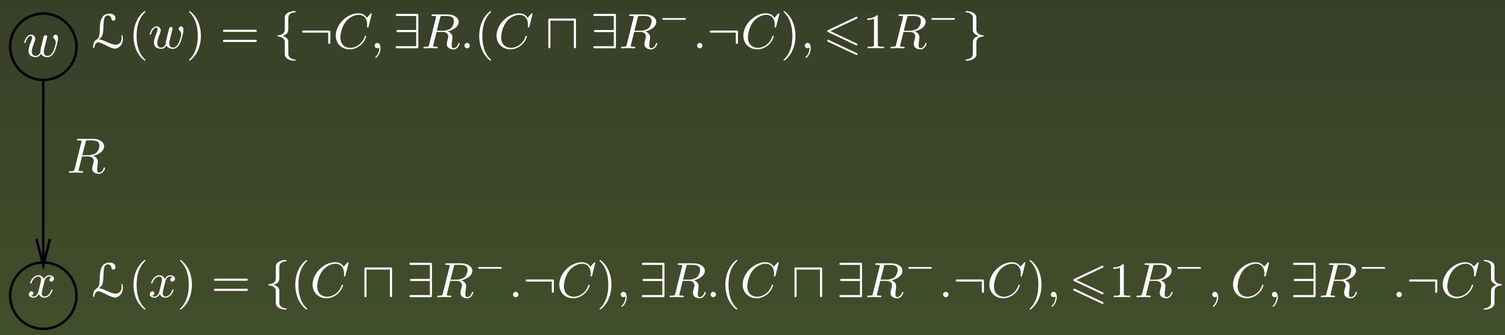

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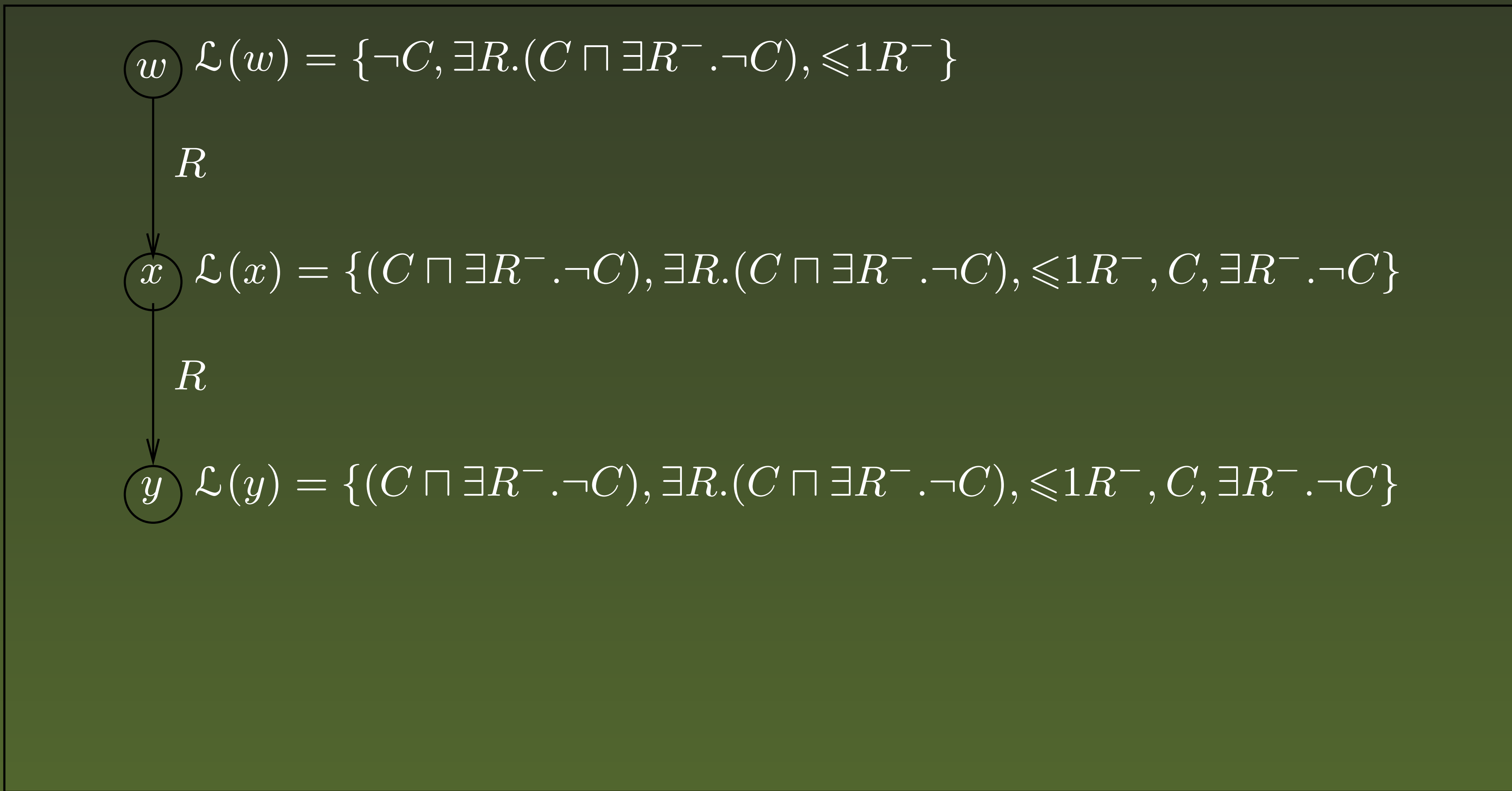

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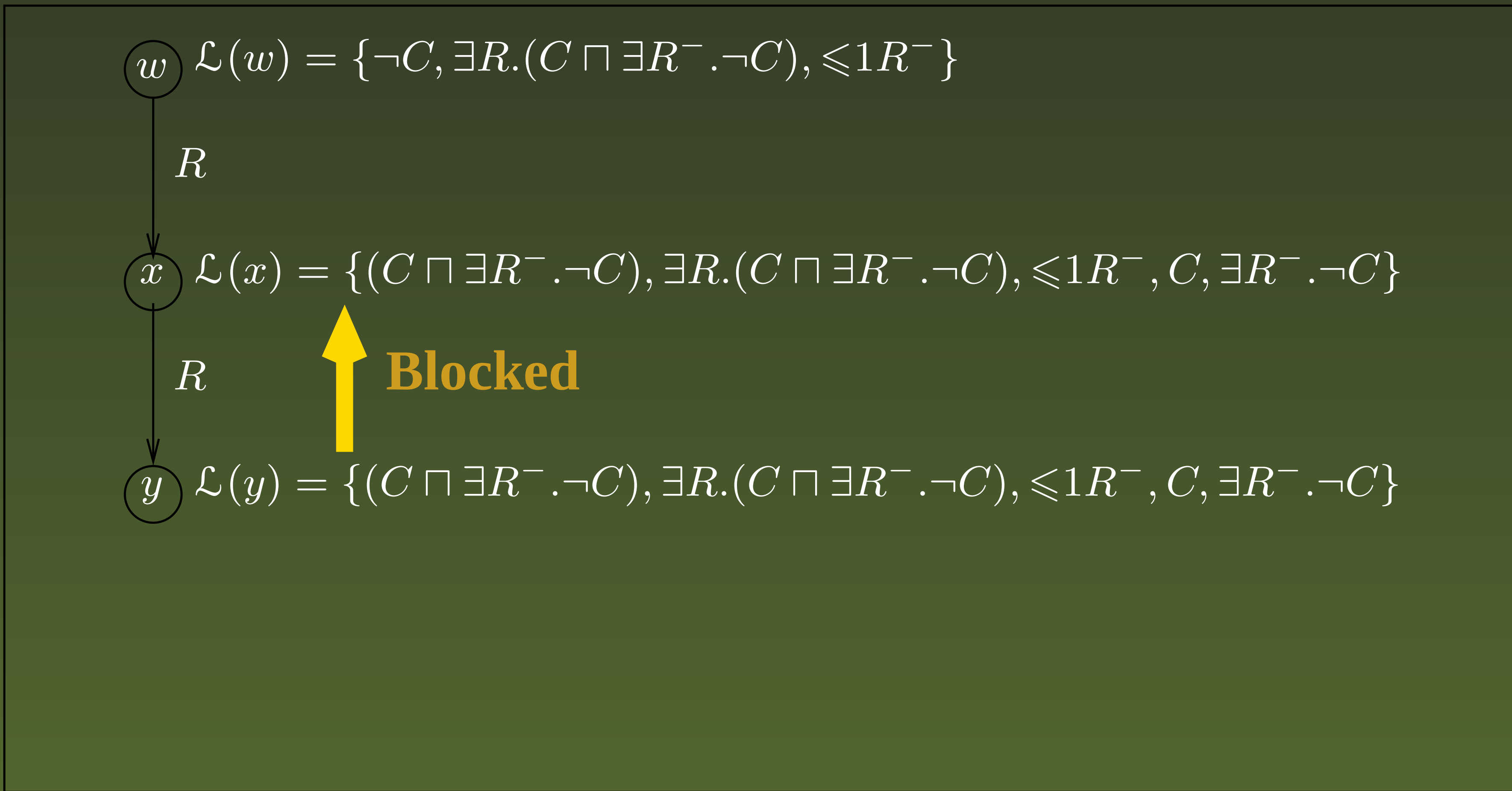

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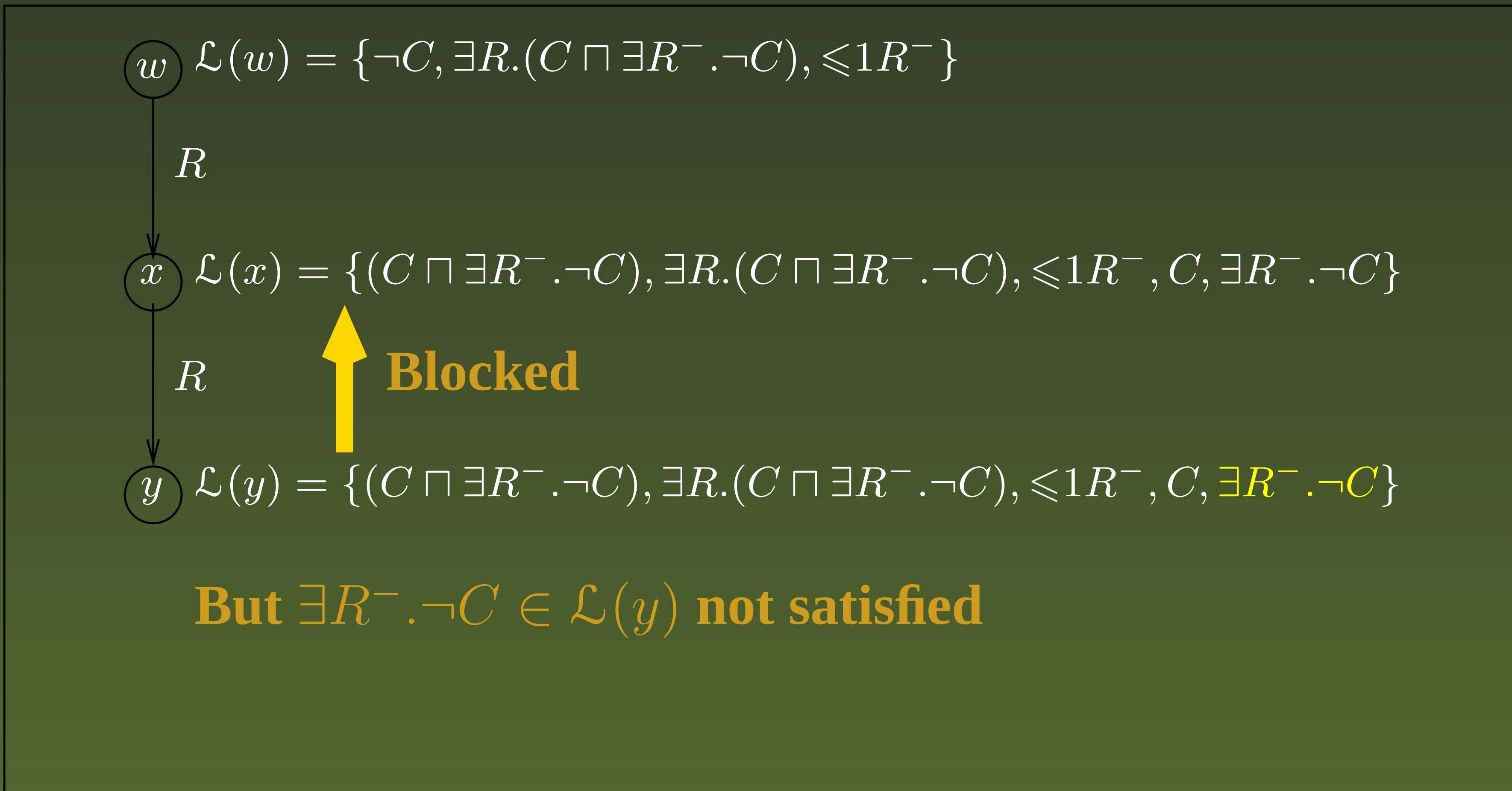

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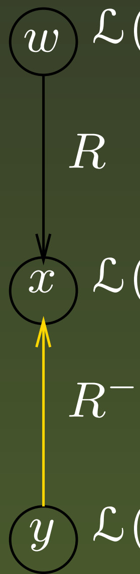

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Blocked

$\mathcal{L}(y) = \{(C \sqcap \exists R^{-}.\neg C), \exists R.(C \sqcap \exists R^{-}.\neg C), \leqslant 1 R^{-}, C, \exists R^{-}.\neg C\}$

But $\exists R^{-}.\neg C \in \mathcal{L}(y)$ not satisfied

Inconsistency due to $\leqslant 1 R^{-} \in \mathcal{L}(y)$ and $C \in \mathcal{L}(x)$

Double Blocking I

- Problem due to $\exists R^{-}.\neg C$ term **only** satisfied in **predecessor** of blocking node

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$$\downarrow R$$

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- Solution is **Double Blocking** (pairwise blocking)

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- Solution is **Double Blocking** (pairwise blocking)
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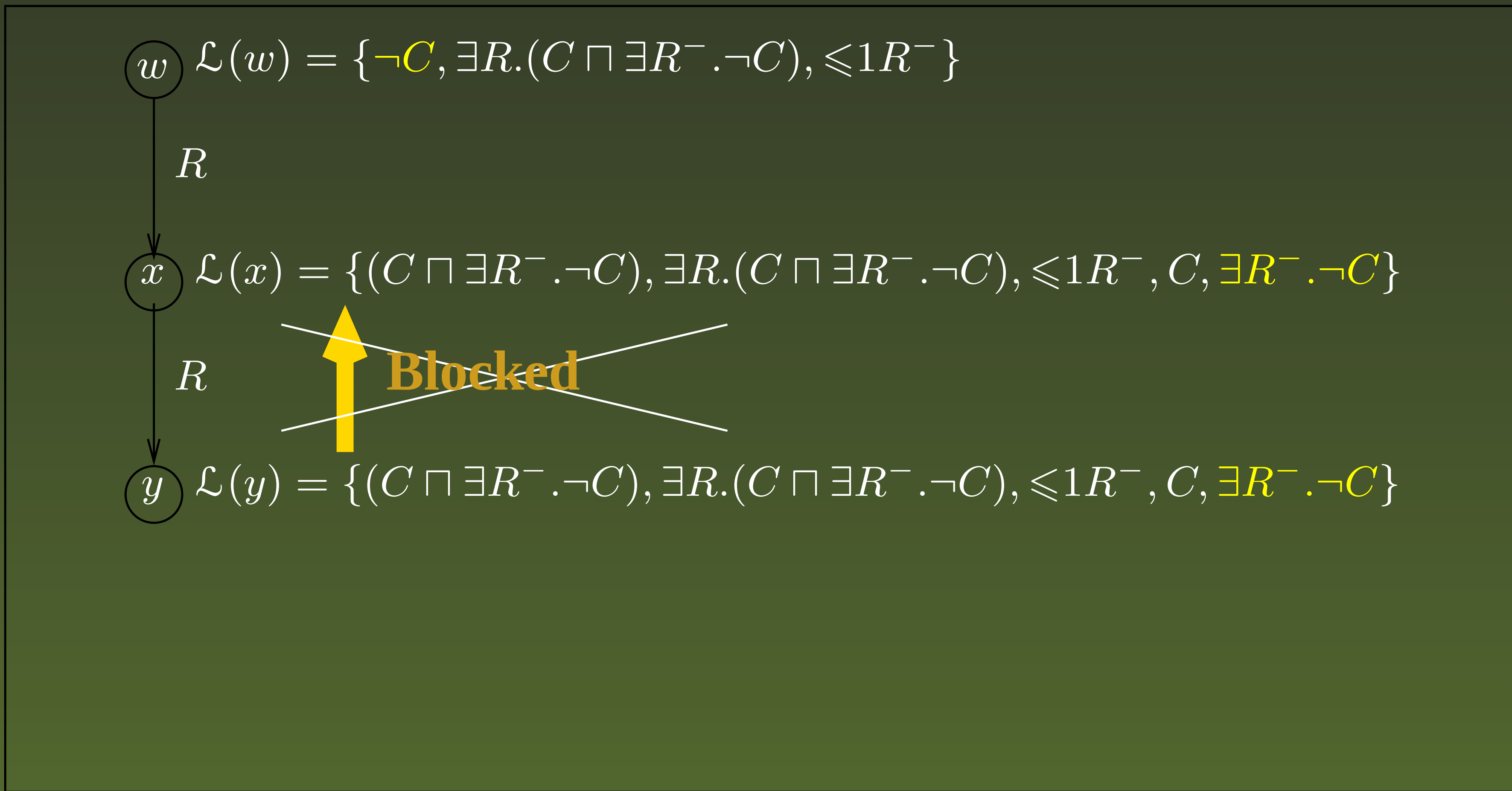
  - In particular, $\exists R.C$ terms satisfied in predecessor of blocking node must also be satisfied in predecessor of blocked node $\neg C \in \mathcal{L}(w)$

Double Blocking II

- Due to pairwise condition, block no longer holds



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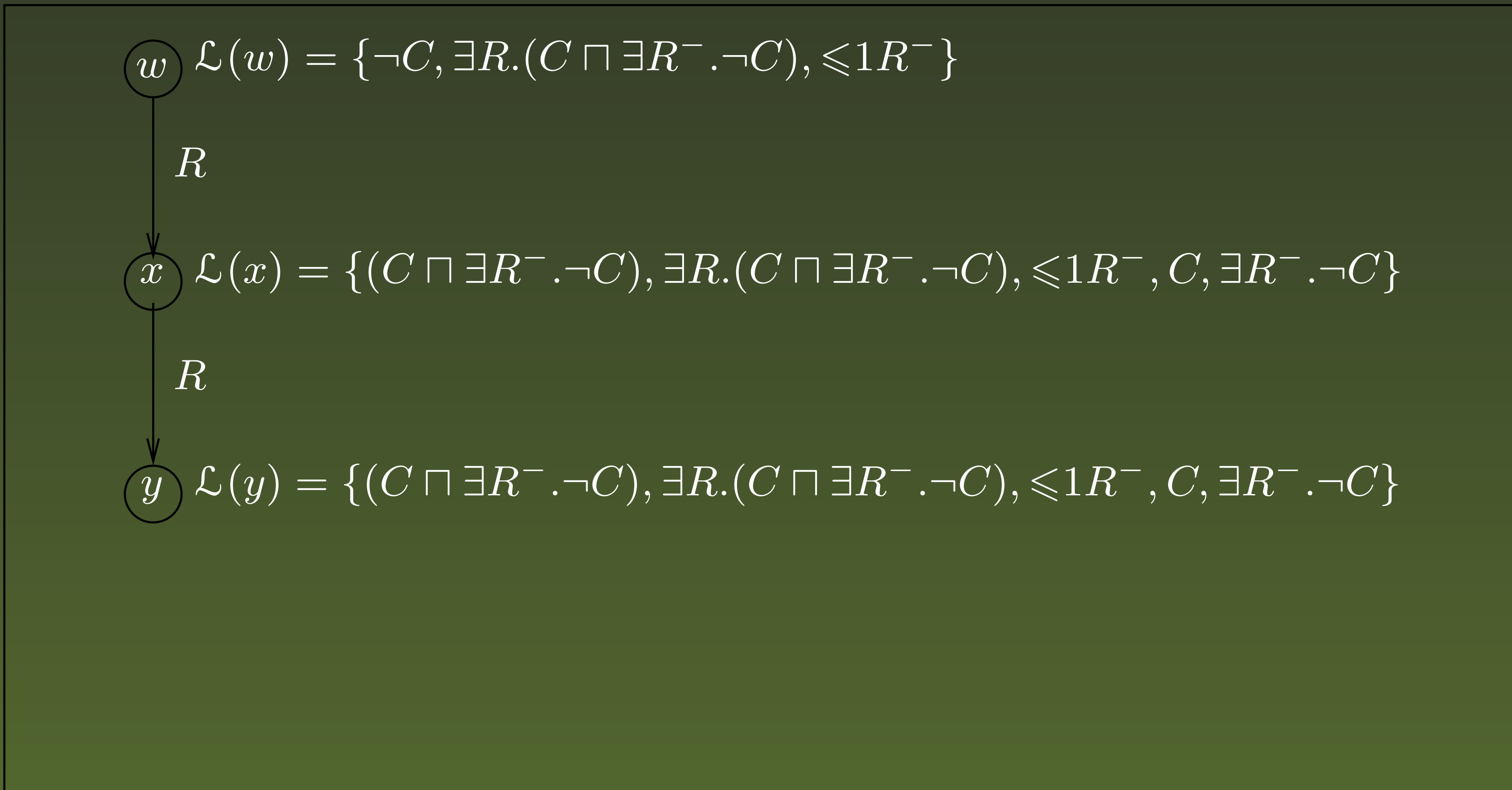

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$\downarrow R^{-}$

z: $\mathcal{L}(z) = \{\neg C\}$

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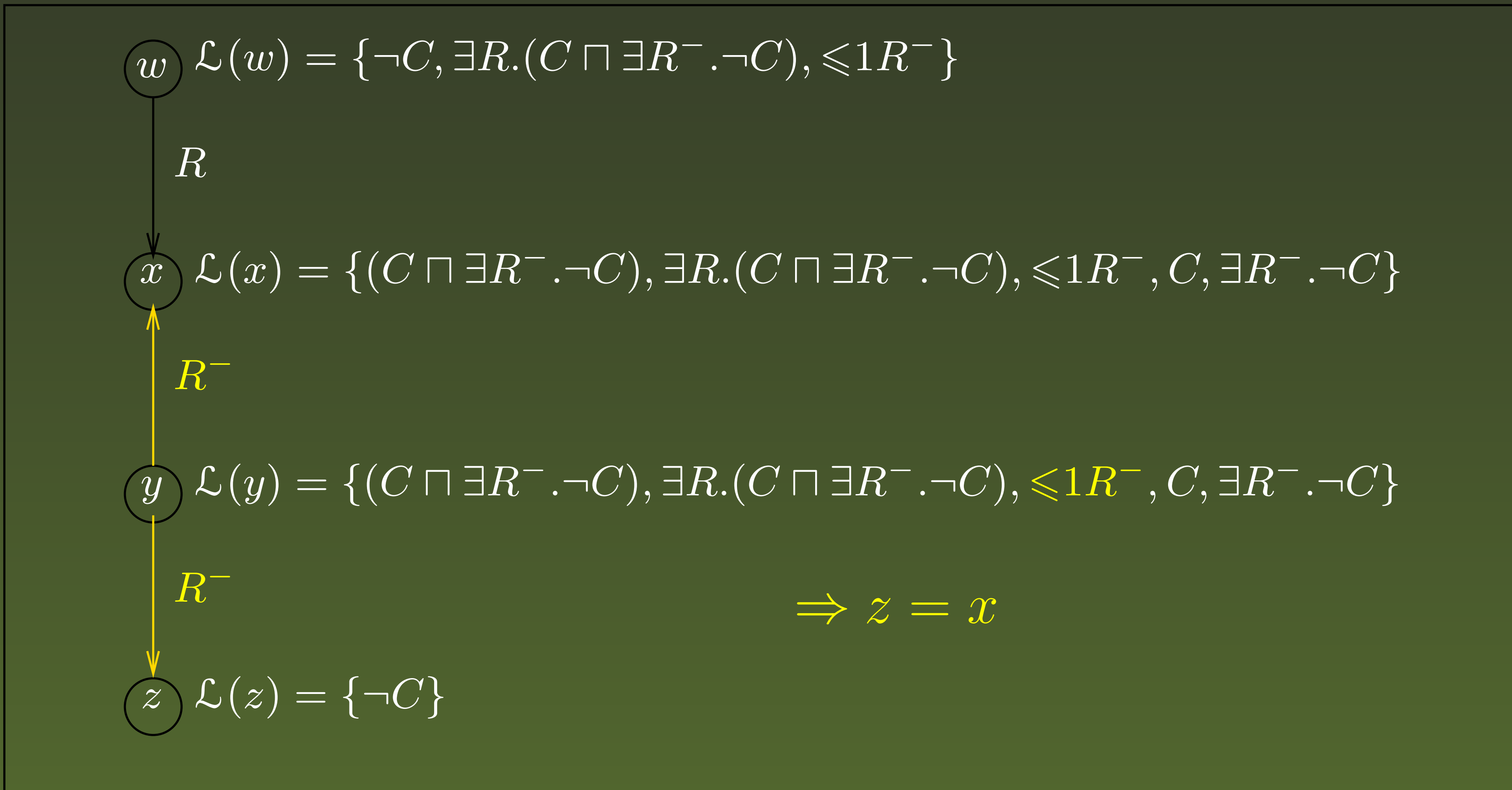

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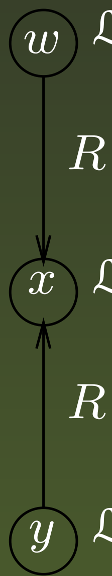

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