Literal, clause

- Literal: either an atom p (positive literal) or its negation $\neg p$ (negative literal).
- The complementary literal to L:

$$\tilde{L} \rightleftharpoons \begin{cases} \neg L, & \text{if } L \text{ is positive;} \\ p, & \text{if } L \text{ has the form } \neg p. \end{cases}$$

- Clause: a disjunction $L_1 \vee \ldots \vee L_n$, $n \geq 0$ of literals.
- Empty clause, denoted by $\square$: $n = 0$.
- Unit clause: $n = 1$.

CNF transformation

$$
\begin{aligned}
A \leftrightarrow B &\;\Rightarrow\; (\neg A \vee B) \wedge (\neg B \vee A), \\
A \rightarrow B &\;\Rightarrow\; \neg A \vee B, \\
\neg(A \wedge B) &\;\Rightarrow\; \neg A \vee \neg B, \\
\neg(A \vee B) &\;\Rightarrow\; \neg A \wedge \neg B, \\
\neg\neg A &\;\Rightarrow\; A, \\
(A_1 \wedge \ldots \wedge A_m) \vee B_1 \vee \ldots \vee B_n &\;\Rightarrow\; (A_1 \vee B_1 \vee \ldots \vee B_n) \quad \wedge \\
&\qquad\qquad \ldots \qquad\qquad\quad \wedge \\
&\qquad (A_m \vee B_1 \vee \ldots \vee B_n).
\end{aligned}
$$

Structure-Preserving Clausal Form Transformation

Simple formula: if all of its immediate subformulas are literals. Short definition: if it has the form $p \rightarrow A$ or $A \rightarrow p$, where A is simple.

1. If S is a set of clauses, then terminate. Otherwise, take a formula A in S which is not a clause, remove it from S and perform the following transformations:
   (a) If A is a conjunction $A_1 \wedge \ldots \wedge A_n$, add to S the set of formulas $A_1, \ldots, A_n$.
   (b) Otherwise, if A is a simple formula or a short definition, then transform it into a set of clauses using the standard CNF transformation.
   (c) Otherwise, select in A a simple subformula B and introduce a definition of B in S.