Satisfiability, validity

- If a formula A is true in I we say that A is satisfiable in I and valid in I. We also say that I satisfies A and that I is a model of A, denoted by $I \models A$.
- A is satisfiable (valid) if it is true in some (every) interpretation.
- Two formulas A and B are called equivalent, denoted $A \equiv B$ if they have the same models.

Example equivalences

For all formulas A and B, the following equivalences hold.

$$
\begin{aligned}
A \rightarrow \bot &\equiv \neg A; && (1)\\
\top \rightarrow A &\equiv A; && (2)\\
A \rightarrow B &\equiv \neg(A \wedge \neg B); && (3)\\
A \wedge B &\equiv \neg(\neg A \vee \neg B); && (4)\\
A \vee B &\equiv \neg A \rightarrow B. && (5)
\end{aligned}
$$

Connections between these notions

1. A formula A is valid if and only if $\neg A$ is unsatisfiable.
2. A formula A is satisfiable if and only if $\neg A$ is not valid.
3. A formula A is valid if and only if A is equivalent to $\top$.
4. Formulas A and B are equivalent if and only if the formula $A \leftrightarrow B$ is valid.

Equivalent replacement

Lemma Let I be an interpretation, a formula A_1 be a subformula of a formula B_1 and $I \models A_1 \leftrightarrow A_2$. Let the formula B_2 be obtained from B_1 by replacement of one or more occurrences of A_1 by A_2. Then $I \models B_1 \leftrightarrow B_2$.

Theorem (Equivalent Replacement) Let A_1 be a subformula of a formula B_1 and $A_1 \equiv A_2$. Let the formula B_2 be obtained from B_1 by replacement of one or more occurrences of A_1 by A_2. Then $B_1 \equiv B_2$.

Evaluating a formula

	formula	value
1	$(p \to q) \land (p \land q \to r) \to (p \to r)$	1
2	$p \to r$	1
3	$(p \to q) \land (p \land q \to r)$	0
4	$p \land q \to r$	1
5	$p \to q$	0
6	$p \land q$	0
7	p $\quad$ p $\quad$ p	1
8	q $\quad$ q	0
9	r $\quad$ r	1