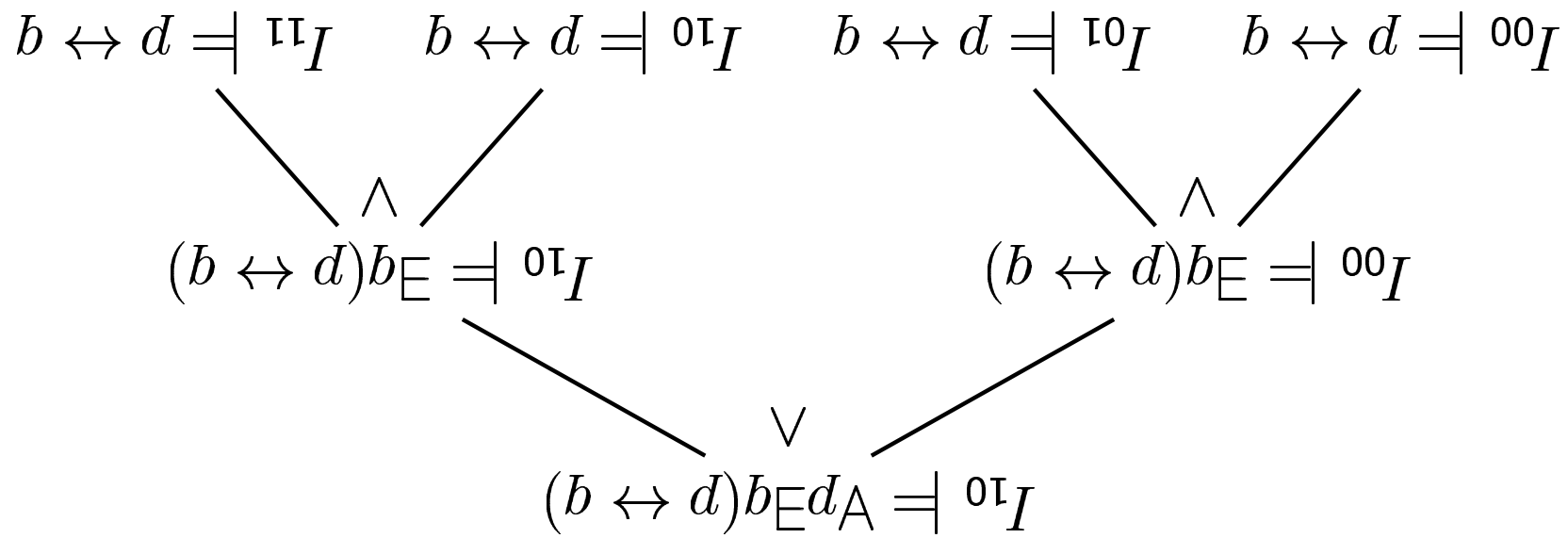


Evaluating a formula



Evaluating a formula

Truth and satisfiability

Lemma. Let for all free variables p of A we have $I_1(p) = I_2(p)$. Then $I_1 \models A$ if and only if $I_2 \models A$.

Lemma. For every interpretation I and closed formula A the following propositions are equivalent: (i) $I \models A$; (ii) A is satisfiable; and (iii) A is valid.

Lemma. Let A be a formula with free variables p_1, \dots, p_n . Then A is satisfiable (respectively, valid) if and only if the formula $\exists p_1 \dots \exists p_n A$ (respectively, $\forall p_1 \dots \forall p_n A$) is satisfiable.

Monotonic and equivalent replacement

Theorem. Let A_1 be a subformula of a formula B_1 and $A_1 \rightarrow A_2$ be valid. Let B_2 be obtained from B_1 by replacement of one or more positive (respectively, negative) occurrences of A_1 by A_2 . Then $B_1 \rightarrow B_2$ (respectively $B_2 \rightarrow B_1$).

Theorem. Let A_1 be a subformula of a formula B_1 and $A_1 \equiv A_2$. Let B_2 be obtained from B_1 by replacement of one or more occurrences of A_1 by A_2 . Then $B_1 \equiv B_2$.

Renaming bound variables

Definition. We say that a formula A' is obtained from a formula A by **renaming bound variables** if A' can be obtained from A by a sequence of the following steps. Let $\exists p B_1$ be a subformula of A and q be a variable not occurring in A . Define B_2 as the formula obtained from B_1 by replacement of all free occurrences of p by q . Replace $\exists p B_1$ in A by $\exists q B_2$.

Definition. A formula A is called **rectified** if (i) no variable appears both free and bound in A , and (ii) for every variable p , the formula A contains at most one occurrence of quantifiers $\forall p$ and $\exists p$ binding p .

Prenex form, CNF

Definition. Prenex formula has the form $\exists v_1 \dots \exists v_n B$, where B is a propositional formula. A formula A is a prenex form of a formula B if A is prenex and $A \equiv B$.

Definition. A quantified boolean formula A is in CNF, if it is either \perp , or \top , or has the form $\exists v_1 p_1 \dots \exists v_n p_n (C_1 \vee \dots \vee C_m)$, where C_1, \dots, C_m are clauses.