

## Integrating a node in a dag

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procedure *integrate*( $n_1, p, n_2, D$ )

input: dag  $D$ , nodes  $n_1, n_2$  in  $D$ , atom  $p$ .

begin

if  $D$  contains a node  $n$  labelled by  $p$  such that  $n_1 = \text{left}(n)$   
and  $n_2 = \text{right}(n)$

then return  $n$

else

add to  $D$  a node  $n$  labelled by  $p$  and arcs from  $n$  to  $n_1$  and  $n_2$   
labelled by 0 and 1 respectively

return  $n$

end

## Building OBDDs

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procedure *obdd*( $A$ )

input: propositional formula  $A$

parameters: a dag  $D$  containing  $N_0$  and  $N_1$

output: a node  $n$  in (modified)  $D$  which represents  $A$

begin

$A := \text{simplify}(A)$

if  $A = \perp$  then return  $N_0$

if  $A = \top$  then return  $N_1$

$p := \text{max\_atom}(A)$

$n_1 := \text{obdd}(A_p^\perp)$

$n_2 := \text{obdd}(A_p^\top)$

if  $n_1 = n_2$  then return  $n_1$

return  $\text{integrate}(n_1, p, n_2, D)$

end

## *Composing OBDDs*

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procedure compose( $A, q_1, \dots, q_m, n_1, \dots, n_m$ )
parameters: a dag  $D$  containing  $n_1, \dots, n_m$  and  $N_0, N_1$  as nodes
input: propositional formula  $A(q_1, \dots, q_m)$  of atoms  $q_1, \dots, q_m$ 
         nodes  $n_1, \dots, n_m$  representing  $A_1, \dots, A_m$  in  $D$ 
output: a node  $n$  representing  $A(A_1, \dots, A_m)$  in (modified)  $D$ 
begin
   $A := \text{simplify}(A)$ 
  if  $A = \perp$  then return  $N_0$ 
  if  $A = \top$  then return  $N_1$ 
  if some  $n_i$  is a leaf labelled 0 then
    return  $\text{compose}(A_{q_i}^\perp, q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_m, n_1, \dots, n_{i-1}, n_{i+1}, \dots, n_m)$ 
  if some  $n_i$  is a leaf labelled 1 then
    return  $\text{compose}(A_{q_i}^\top, q_1, \dots, q_{i-1}, q_{i+1}, \dots, q_m, n_1, \dots, n_{i-1}, n_{i+1}, \dots, n_m)$ 
   $p := \text{max\_atom}(n_1, \dots, n_m)$ 
  forall  $i = 1 \dots m$ 
    if  $n_i$  is labelled by  $p$ 
      then  $(l_i, r_i) := (\text{left}(n_i), \text{right}(n_i))$ 
      else  $(l_i, r_i) := (n_i, n_i)$ 
   $k_1 := \text{compose}(A, q_1, \dots, q_m, l_1, \dots, l_m)$ 
   $k_2 := \text{compose}(A, q_1, \dots, q_m, r_1, \dots, r_m)$ 
  if  $k_1 = k_2$  then return  $k_1$ 
  return  $\text{integrate}(k_1, p, k_2, D)$ 

```

end

*Special case: disjunction*

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procedure *disjunction*( $n_1, \dots, n_m$ )

parameters: a dag  $D$  containing  $n_1, \dots, n_m$  and  $N_0, N_1$  as nodes

input: nodes  $n_1, \dots, n_m$  representing  $A_1, \dots, A_m$  in  $D$

output: a node  $n$  representing  $A_1 \vee \dots \vee A_m$  in (modified)  $D$

begin

if  $m = 1$  then return  $n_1$

if some  $n_i$  is a leaf labelled 0 then

return *disjunction*( $n_1, \dots, n_{i-1}, n_{i+1}, \dots, n_m$ )

if some  $n_i$  is a leaf labelled 1 then return  $N_1$

$p := \text{max\_atom}(n_1, \dots, n_m)$

forall  $i = 1 \dots m$

if  $n_i$  is labelled by  $p$

then  $(l_i, r_i) := (\text{left}(n_i), \text{right}(n_i))$

else  $(l_i, r_i) := (n_i, n_i)$

$k_1 := \text{disjunction}(l_1, \dots, l_m)$

$k_2 := \text{disjunction}(r_1, \dots, r_m)$

if  $k_1 = k_2$  then return  $k_1$

return *integrate*( $k_1, p, k_2, D$ )

end