What is logic?

- Syntax and semantics;
- Proof theory and model theory;
- Reasoning.

Logic in computer science

- circuit design;
- constraint satisfaction;
- planning;
- software and hardware verification:
  - model checking;
  - Hoare’s logics;
  - higher-order logics;
- databases;
- theorem proving in mathematics.

Signature

- signature Σ — any set of symbols.
- The symbols in Σ are called relation symbols;
- Propositional formula in a signature Σ:
  - $\top$nd $\bot$ are formulas.
  - Every relation symbol is a formula, also called atomic formula, or simply atom.
  - If $A_1, \ldots, A_n$ are formulas, where $n \geq 2$, then $(A_1 \wedge \ldots \wedge A_n)$ and $(A_1 \vee \ldots \vee A_n)$ are formulas.
  - If A is a formula, then $\neg A$ is a formula.
  - If A and B are formulas, then $(A \rightarrow B)$ and $(A \leftrightarrow B)$ are formulas.

Connectives

Connective	Name	Priority
$\neg$	negation	1
$\wedge$	conjunction	2
$\vee$	disjunction	2
$\rightarrow$	implication	3
$\leftrightarrow$	equivalence	4

Semantics, Interpretation

- A boolean value, also called a truth value, is either 1 or 0.
- A (partial) interpretation for a signature Σ: a (partial) mapping from Σ to the set of boolean values $\{1, 0\}$.

Truth value

Extend I to all formulas:

1. $I(\top) = 1$ and $I(\bot) = 0$.
2. $I(A_1 \wedge \ldots \wedge A_n) = 1$ if and only if $I(A_i) = 1$ for all i.
3. $I(A_1 \vee \ldots \vee A_n) = 1$ if and only if $I(A_i) = 1$ for some i.
4. $I(\neg A) = 1$ if and only if $I(A) = 0$.
5. $I(A \rightarrow B) = 1$ if and only if $I(A) = 0$ or $I(B) = 1$.
6. $I(A \leftrightarrow B) = 1$ if and only if $I(A) = I(B)$.

Notation: $I \models A$ and $I \not\models A$.

Operation tables

$\wedge$	1	0
1	1	0
0	0	0

$\vee$	1	0
1	1	1
0	1	0

$\neg$	
1	0
0	1

$\longrightarrow$	1	0
1	1	0
0	1	1

$\longleftrightarrow$	1	0
1	1	0
0	0	1