Complexity of Some Problems in Modal and Superintuitionistic Logics

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A logic is called *tabular* if it can be characterized by finitely many finite models; and *pretabular* if it is maximal among non-tabular logics. A logic L is said to have *Craig's interpolation property (CIP)*, if for every formula $(A \to B) \in L$ there exists a formula C such that (i) both $A \to C$ and $C \to B$ belong to L, and (ii) every variable of C occurs in both A and B. A logic L is said to have *interpolation property (IPD)*, if $A \vdash_L B$ implies that exists a formula C such that (i) $A \vdash_L C$, $C \vdash_L B$, and (ii) every variable of C occurs in both A and B.

Denote by $L + A$ the extension of a logic L by an extra axiom scheme A. By the *tabularity (pretabularity etc.) problem over* L we mean the problem of determining for arbitrary A, whether $L + A$ is tabular (pretabular, etc.), and consider its complexity with respect to the size of A over intuitionistic logic Int and the modal S4.

Theorem 1 *(i) The tabularity problems over both* Int *and* S4 *are NP-complete. (ii) The pretabularity problems over both* Int *and* S4 *are in* Δ_2^p *and coNP-hard.*

Theorem 2 *(i) The interpolation problem over* Int *is PSPACE-complete. (ii) The problem of determining whether* $\mathrm{Int} + A$ *is a tabular logic with CIP is NP-complete.*
(iii) Both CIP and IPD problems over S4 *are in coNEXP and PSPACE-hard. (iv) The problems of determining whether* $\mathrm{S4} + A$ *is a tabular (pretabular or locally tabular) logic with CIP (or IPD) are in* Δ_2^p *and coNP-hard.*

By rewriting these results in the algebraic language, we get

Theorem 3 *(i) The amalgamability problem for finitely based varieties of Heyting algebras is PSPACE-complete. (ii) The amalgamability and super-amalgamability problems for finitely based varieties of closure algebras are in coNEXP and PSPACE-hard.*

References

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