$\boldsymbol{E}$

†

* ‡

E $M(x\ ) \wedge \ldots \wedge M(x_n)$

E

E σ
E

E

E

E

E

E

E

$M(x)$ σ Σ
$x\ , \ldots, x_n$ $(M(x\ ) \wedge \ldots \wedge M(x_n))\sigma$ Σ

Σ
* X $T_\Sigma(X)$
† Σ T_Σ $s \approx t$
‡ $\{t\ /x\ , \ldots, t_n/x_n\}$ $\rightleftharpoons$

σ

$\{t /x ,\ldots,t_n/x_n\}$ $(\sigma) \rightleftharpoons \{x ,\ldots,x_n\}$ $\{a/x ,b/x ,g(h^n(a))/y\}$ $n \geq$

$(\sigma) \rightleftharpoons \{t ,\ldots,t_n\}$ E E

σ $E\sigma$

σ E E

x_i t_i φ

$x ,\ldots,x_n$ $n =$ E

φ $\forall\varphi$ $\forall x \ldots\forall x_n\varphi$ E

$\exists\varphi$

$s ,t ,\ldots,s_n,t_n,s,t$ $s \approx$

$t ,\ldots,s_n \approx t_n \vdash s \approx t$ E

$\forall(s \approx t \wedge\ldots\wedge s_n\approx t_n \supset s\approx t)$

E

s t

$\{s \approx t ,\ldots,s_n\approx t_n\}$

$E \vdash_\forall s\approx t$ E

$\{s \approx t ,\ldots,s_n\approx t_n\} \vdash_\forall s\approx t$ $\leq$

σ $s \sigma \approx t \sigma,\ldots,s_n\sigma \approx t_n\sigma \vdash$

$s\sigma \approx t\sigma$ E

S

n

S n

E E

E E_i

$x\approx a \vdash_\forall$

$g(x)\approx x$ $\{x\approx a\} \vdash_\forall g(x)\approx x$

$\forall x h(x)\approx a \wedge$ S

$\forall y([h(a)\approx y \wedge y \not\approx g(h(y))] \vee [h(b)\approx f(y) \wedge y \not\approx g(f(y))])$ Σ

$\forall x$ σ S $(\sigma) =$ (S) $(\sigma) \subseteq$

$h(x)\approx a$ T_Σ

E

$\geq$

Σ_n $n \geq$

$$\left[\begin{array}{c} h(x)\approx a \\ h(x)\approx a \\ \left[\begin{array}{c} h(a)\approx y \\ y \not\approx g(h(y)) \end{array}\right] \bigvee \left[\begin{array}{c} h(b)\approx f(y) \\ y \not\approx g(f(y)) \end{array}\right] \end{array}\right]$$

n

E

Σ

E

Σ

$h(x)\approx a, h(x)\approx a, h(a)\approx y \vdash_\forall y\approx g(h(y))$ Σ_n $n \geq$

$h(x)\approx a, h(x)\approx a, h(b)\approx f(y) \vdash_\forall y\approx g(f(y))$

E

E

E

Σ

$>$

E

Σ $>$

σ S

$x\sigma$

S x S

S S

$E\sigma$

C

x $, \ldots, x_n$

Σ

L L^+ L^-

Σ $x_i^+ \rightleftharpoons x_i \approx$ $x_i^- \rightleftharpoons x_i \approx$

$(\neg x_i)^+ \rightleftharpoons x_i \approx$ $(\neg x_i)^- \rightleftharpoons x_i \approx$

$c = (L \vee \ldots \vee L_m)$

$R(c)$ $L^-, \ldots, L^-_{m-}$ $\vdash_\forall L^+_m$

$c = (x \vee \neg x \vee \neg x)$ $R(c)$

$x \approx$ $, x \approx$ $\vdash_\forall x \approx$ σ

$\{R(c) \mid c \in C\}$ σ

C $\square$

Σ

E Σ

$$\exists x \ \ldots \exists x_n \bigwedge_{i=}^{k} A_i$$

A_i $x_m =$

$x_j + x_k$ $x_m \mid x_j$ $x_m = p$ $\mid$

p

s

Σ $\{a , a , \ldots\}$ Σ

Σ

Σ $s^n a_i$ n

E

E D

$x_m = x_j + x_k$ m, j, k

E

$G_i \rightleftharpoons (sa_i \approx a_i \vdash_\forall x_i \approx a_i)$

$i \in \{ \ , \ldots, n\}$

$P_{jkm} \rightleftharpoons (a_j \approx x_k, a_k \approx a_m \vdash_\forall$

$x_j \approx x_m)$ A_i $x_m =$

$x_j + x_k$

$D_{jm} \rightleftharpoons (a_m \approx x_m, a_j \approx a_m \vdash_\forall$

$x_j \approx a_j)$ A_i $x_m \mid x_j$

$N_m \rightleftharpoons (\vdash_\forall x_m \approx s^p a_m)$

A_i $x_m = p$

D E

θ

θ G_i $x_i\theta = s^p a_i$

p

θ G_j, G_k, G_m, P_{jkm} $x_j\theta =$

$s^p a_j, x_k\theta = s^q a_k, x_m\theta = s^{p+q} a_m$

p, q

θ G_j, G_m, D_{jm} $x_j\theta =$

$s^p a_j, x_m\theta = s^q a_m$ p, q

$q \mid p$

θ N_m $x_m\theta = s^p a_m$

$\square$

E Σ

σ

$[m \ /x \ , \ldots, m_n/x_n]$ x_i m_i

(σ) σ

$\{x \ , \ldots, x_n\}$

$s^n xa$ n x a

$s \approx t$

s, t

$s^n x_i a$ $s^{n+m_i} a$

S $s \to t$

$\to$ $\to^*$

$\to$

t $t\downarrow$

$t \to^* t\downarrow$ s $t\downarrow\to s$

D

$(t\ ,t\ ,t\ ,t\ )$ $t\ \lambda t$ λ

$=,|,<,\leq$ t_i

$+,$ D

$(\quad (x\ ,\quad),\ \ ,y+\ \ ,y)$

D

$[m\ /x\ ,\ldots,m_n/x_n]$ D

(k,l)

k,l

$i \mid j$

k $ik=j$

(j,k,l,m) $l\leq j<k\leq m$

$k-l \mid m-j$

D

$$\exists x\ \ldots\exists x_n \bigwedge_{i=}^{k} D_i$$

D_i D D

D_i

j,k,l $j\ \ =$

(k,l) $j \mid k$ $j \mid l$

d,e $kd=le+j$

D

D_i

$x_j =\quad (x_k,x_l) \Longleftrightarrow$

$x_j \mid x_k \wedge x_j \mid x_l \wedge$

$\exists y \exists z (x_k \mid y \wedge x_l \mid z \wedge y+x_j=z)$

$\square$

k,l,m $l\ <$

$k\ \leq\ m$ j

(j,k,l,m)

R

$E\vdash_\forall F$ E F

σ

R $(E,F)\subseteq$ (σ) $E\sigma\vdash$

$F\sigma$ R

$$\exists x\ \ldots\exists x_n \bigwedge_{i=}^{k} R_i$$

R_i R

$\{x\ ,\ldots,x_n\}$ σ

R_i

$[\ /x,\ /y]$

R $sxa\ \approx ya\ \vdash_\forall s\ ya \approx a$

DR

$$\exists x\ \ldots\exists x_n \bigwedge_{i=}^{k} B_i$$

B_i D R

$\{x\ ,\ldots,x_n\}$

DR

B_i

P

S P S

P

Σ S

D D

R DR

DR

D

Σ

R

E Σ

$\{x\ ,\ldots,x_n\}$ $\{a\ ,\ldots,a_q\}$

f $\{\ ,\ldots,n\}$ $\{\ ,\ldots,q\}$

E_f E

x_i $x_i a_{f(i)}$

E

$s\ x\ \approx x\ \vdash_\forall x\ \approx x$

$f(\ )=\ ,f(\ )=\ ,f(\ )=$ E_f

$s\ x\ a \approx x\ a\ \vdash_\forall x\ a \approx x\ a$

$\{\ ,\dots,n\}$
$\{\ ,\dots,q\}$ $\mathcal{M}$
P
S R

$$S \rightleftharpoons \{\exists x\ \dots \exists x_n(\bigwedge_{E\in P} E_f) \mid f \in \mathcal{M}\}$$

P S
P P
σ Σ
$s^m a_i$ m $\leq i \leq q$
σ $\{s^{m} a_i /x\ ,\dots,s^{m_n} a_{i_n}/x_n\}$
g $g(j) \rightleftharpoons i_j$
$[m\ /x\ ,\dots,m_n/x_n]$

$$\exists x\ \dots \exists x_n(\bigwedge_{E\in P} E_g)$$

$\square$

R E
$t\ a_i \approx t\ a_j$ E t
t DR P R
P

DR P
DR P' P

R

$s^p x_k a_i$ DR $\exists x\ \dots \exists x_n B$
B' B
$s^p x_k a_i$ $x_{n+}\ a_i$ DR
$\exists x\ \dots \exists x_{n+}\ (x_{n+}\ = x_k + p \wedge B')$
$s^p a_i$ $\square$

R E
a_i
R $s\ x\ a \approx x\ a\ ,\ s\ x\ a \approx sx\ a\ \vdash_\forall$
$x\ a \approx x\ a$ $+\ +\ +\ +\ +\ =$
DR B R
B
E DR
$j \in \{\ ,\dots,n-\ \}$
B_i $x_j < x_{j+}$

DR
D

DR
DR

DR
DR

DR P h
S
DR

B DR
$B_{x_i \leftarrow x_j}$ B
x_i x_j
DR $\exists x\ \dots \exists x_n B$
DR

$$\exists x\ \dots \exists x_n(x_i < x_j \wedge B),$$
$$\exists x\ \dots \exists x_n(x_j < x_i \wedge B),$$
$$\exists x\ \dots \exists x_n B_{x_i \leftarrow x_j}$$

h
DR
DR
$\square$

$\succ$ T_Σ
$s^n a_i \succ s^m a_j$

$i > j$

$i = j$ $n > m$

$\succ$

D
$E\ ,E$ $E\ \equiv_\forall E$
$(s \approx t) \in E$ $E\ \vdash s \approx t$
$(s' \approx t') \in E$ $E\ \vdash s' \approx t'$

$E\ ,E$
$E\ \equiv_\forall E$ E
$s \approx t$ $E, E\ \vdash s \approx t$ $E, E\ \vdash s \approx t$

j,k,l,m,n
$l < j, m \leq l, m < k$ $n =$ $(j-l, k-m) + m$
$s^j a_i \approx s^l a_i, s^k a_i \approx s^m a_i \equiv_\forall s^n a_i \approx s^m a_i$

$s^j a_i \approx s^l a_i, s^k a_i \approx s^m a_i$ $\vdash$
$s^n a_i \approx s^m a_i$
d, e $(j-$
$l)d = (k-m)e + (n-m)$ p
$(k-m)p \geq l$ $s^m a_i \rightarrow s^k a_i$
$s^n a_i\ p + e$ $s^{n+(k-m)p+(k-m)e} a_i$
$s^m a_i$ $s^m a_i \rightarrow s^k a_i\ p$
$s^l a_i \rightarrow s^j a_i\ d$
$s^{m+(k-m)p+(j-l)d} a_i$ $s^j a_i \approx s^l a_i, s^k a_i \approx s^m a_i \vdash$
$s^{n+(k-m)p+(k-m)e} a_i \approx s^n a_i$ $s^j a_i \approx s^l a_i, s^k a_i \approx s^m a_i \vdash$
$s^{n+(k-m)p+(j-l)d} a_i \approx s^m a_i$ $n+$
$(k-m)p + (k-m)e = m + (k-m)p + (j-l)d$ $\square$

k, l, m, n, p

$l < k \leq m$ (n, k, l, m)

$s^k a_i \approx s^l a_i, s^m a_i \approx s^p a_j \equiv_\forall s^k a_i \approx s^l a_i, s^n a_i \approx s^p a_j$

$k - l \mid m - n$

d $(k - l)d = m -$

n $n \geq l$ $s^l a_i \to$

$s^k a_i$ d $s^n a_i$ $s^m a_i$ $s^l a_i \approx s^k a_i \vdash$

$s^n a^i \approx s^m a^i$

$\square$

k, l, m, n, p

$l < k, m < k$ $n = k - m + p$

$s^k a_i \approx s^l a_i, s^m a_i \approx s^p a_j \equiv_\forall s^l a_i \approx s^n a_j, s^m a_i \approx s^p a_j$

k, l, m, n, p, r

$l < k \leq m$ (n, k, l, m) $r = k - n + p$

$s^k a_i \approx s^l a_i, s^m a_i \approx s^p a_j \equiv_\forall s^l a_i \approx s^r a_j, s^n a_i \approx s^p a_j$

$s^k a_i \approx s^l a_i, s^m a_i \approx s^p a_j \equiv_\forall$

$s^k a_i \approx s^l a_i, s^n a_i \approx s^p a_j$ (n, k, l, m)

$n < k$ $s^k a_i \approx s^l a_i, s^n a_i \approx s^p a_j \equiv_\forall$

$s^l a_i \approx s^r a_j, s^n a_i \approx s^p a_j$ $\square$

r, l, m, n, p

$m \leq r$ $n = r - m + p$ $s^r a_i \approx s^l a_j, s^m a_i \approx s^p a_k \equiv_\forall$

$s^n a_k \approx s^l a_j, s^m a_i \approx s^p a_k$

$E \vdash_\forall F$ R

Σ E

$t \approx t$

$a_i \in \Sigma$ E

$u a_i \approx v a_j$ $j \leq i$ DR S

R

$E \vdash_\forall F$

E

$\succ$ E

t $t\downarrow$ E

D

DR P

S DR

P

P

$\exists x \ldots \exists x_n (E \vdash_\forall F \wedge \Phi)$

$E \vdash_\forall F$

P' P

$E \vdash_\forall F$

$a_i \in \Sigma$ E

$x_j a_i \approx x_l a_i$ $x_k a_i \approx x_m a_i$ $j = l$

$m = k$ $x_j a_i \approx x_l a_i$ $x_m a_i \approx x_k a_i$

$l < j, m \leq l, m <$

k E' E

$x_j a_i \approx x_l a_i, x_k a_i \approx x_m a_i$ $x_{n+} a_i \approx x_m a_i$

$\mathcal{P}'$

$\exists x \ldots \exists x_n \exists x_{n+} \exists x_{n+} \exists x_{n+}$
$(x_{n+} + x_l = x_j \wedge x_{n+} + x_m = x_k \wedge$
$x_{n+} = x_m +$ $(x_{n+}, x_{n+}) \wedge$
$E' \vdash_\forall F \wedge \Phi)$

P P'

$a_i, a_j \in \Sigma$ $i > j$ E

$x_k a_i \approx x_l a_i$ $x_m a_i \approx x_p a_j$ $k = l$

$x_k a_i \approx x_l a_i$ E

$l < k$

$k > m$ P

P'

$\exists x \ldots \exists x_n \exists x_{n+} (x_{n+} + x_m =$
$x_k + x_p \wedge E' \vdash_\forall F \wedge \Phi)$

E' E

$x_k a_i \approx x_l a_i$ $x_{n+} a_j \approx x_l a_i$

$k \leq m$ P

P'

$\exists x \ldots \exists x_n \exists x_{n+} \exists x_{n+}$
$($ $(x_{n+}, x_k, x_l, x_m) \wedge$
$x_{n+} + x_{n+} = x_k + x_p \wedge$
$E' \vdash_\forall F \wedge \Phi)$

E'

E $x_k a_i \approx x_l a_i, x_m a_i \approx x_p a_j$

$x_l a_i \approx x_{n+} a_j, x_{n+} a_i \approx x_p a_j$

$a_i, a_j, a_k \in \Sigma$ $i > j$ $i > k$

E $x_r a_i \approx x_l a_j$

$x_m a_i \approx x_p a_k$ $m \leq r$

P P'

$\exists x \ldots \exists x_n \exists x_{n+}$
$(x_{n+} + x_m = x_r + x_p \wedge E' \vdash_\forall F \wedge \Phi)$

E' E $x_r a_i =$

$x_l a_j$ $x_{n+} a_k = x_l a_j$

P'

P $\square$

$E \vdash_\forall F$

Σ E'

E $\succ$ $(s \to t) \in E'$

$s \approx t \in E$ $s \succ t$

Σ E'

$$s^m a_i \downarrow = \begin{cases} s^n a_i, & (n,k,l,m) \\ \quad (s^k a_i \to s^l a_i) \in E' \quad k \leq m \\ s^{m-k+l} a_j \downarrow \\ \quad (s^k a_i \to s^l a_j) \in E' \;\; i > j \quad k \leq m \\ s^m a_i \end{cases}$$

s, t $E \vdash s \approx t$ $s \downarrow =$ $t \downarrow$

DR D

DR P

$>$

DR

P P

$\exists x \ \ldots \exists x_n (E \vdash_\forall F \wedge \Phi)$

P'

P

F $t \approx t$ $P' \rightleftharpoons \exists x \ \ldots \exists x_n \Phi$

P P'

F $x_m a_i \approx x_p a_i$ $m \neq p$ E

$x_k a_i \approx x_l a_i$ $k \leq m, k \leq p$

$k > l$ P'

$$\exists x \ \ldots \exists x_n \exists x_{n+} \; (\quad (x_{n+} \ , x_k, x_l, x_m) \wedge \\ (x_{n+} \ , x_k, x_l, x_p) \wedge \Phi)$$

P P'

F $x_m a_i \approx x_p a_i$ E

$x_k a_i \approx x_l a_i$ $k \leq m, k > p$ $k > l$

P'

$$\exists x \ \ldots \exists x_n (\quad (x_p, x_k, x_l, x_m) \wedge \Phi)$$

F $x_m a_i \approx t$ E

$x_k a_i \approx x_l a_j$ $i > j$ $k \leq m$ P'

$$\exists x \ \ldots \exists x_n \exists x_{n+} \; (E \vdash_\forall x_{n+} \ a_j \approx t \wedge \\ x_{n+} \ + x_k = x_m + x_l \wedge \Phi)$$

P

D $\exists x \ (x \ < x \ )$

P'

P $\square$

P Σ

D

P DR

D $\square$

E

Σ

Σ_n

E Σ

E

Σ_n $n >$ $\Sigma_n^m \rightleftharpoons$

$\{f \ , \ldots , f_n, c \ , \ldots , c_m\}$ $f \ , \ldots , f_n$

$c \ , \ldots , c_m$

E

Σ_n $n >$

E Σ

$\Rightarrow$ $\Leftarrow$

S

Σ_n $x \ , \ldots , x_m$ $k \geq$

S

$c \ , \ldots , c_k$ f_i

Σ_n $f \ f^i$ Σ

S' S

f_i $f \ f^i$

$j \in$

$\{ \ , \ldots , m\}$

$$\{f \ f^i c \ \approx c \ \mid i \in \{ \ , \ldots , n\}\} \cup \\ \{c \ \approx c \ , \ldots , c \ \approx c_k\} \\ \vdash_\forall x_j \approx c$$

S S' $\square$

E

E

E

x_i x_j x_k E

R R

x_l

$\{f\ ,\ldots,f_n,c_l\}$ $l\in\{i,j,k\}$

$c_i\approx x_j, c_j\approx c_k \vdash_\forall x_i\approx x_k$

R

$\{Vc_i/x_i, Wc_j/x_j, VWc_k/x_k\}$

V,W $f\ ,\ldots,f_n$

$\square$

E Σ

E

E

E

E

E

E

E

E

E E

E

E

E

E

E

E

$\vec{E}$