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Abstracts

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## Relation-algebraic Computation of Fixed Points With Applications

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Computing fixed points is a central task in computer science. In most cases the task is to determine an extreme fixed point of a monotone or continuous function on a specific class of partially ordered sets. The basic fixed point theorems are the well-known Knaster-Tarski theorem dealing with the existence of extreme fixed points in the case of complete lattices and its extension to complete partial orders.

There are, however, situations where the classical fixed point theorems can't be applied since, e.g., the function under consideration is non-monotone or one has to compute all fixed points of a function. We have identified a lot of problems which can be reduced to the latter case and where, in addition, the function works on a powerset. In the talk we show how relations can be used to solve such – of course, usually computationally hard – problems. The method bases on a relation-algebraic description of sets and the modeling of a function  $F : 2^X \rightarrow 2^X$  on a powerset by a relational counterpart  $f$ . It can be applied if, in Boolean matrix terminology, the application of  $f$  to the membership-relation between  $X$  and  $2^X$  yields a relation the columns of which describe all possible results of  $F$ . Due to the use of the membership-relation the proposed procedure for computing all fixed points of  $F$  will require exponential time and space and, at a first glance, seems to be applicable only for small sets  $X$ . Nevertheless, many practical experiments with the tool RELVIEW have shown that it often works sufficiently well even in the case of sets  $X$  of medium size if relational algebra is implemented using binary decision diagrams.

# Cocktail: A tool for interactive Program Derivation

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## Abstract

At present, most tools for correct programming support functional or logical programming paradigms. However, in practice, most programs are written using procedural languages (including object-oriented languages) like Pascal, C/C++ or Java. Tools for procedural languages usually provide only syntactical support.

The essence of algorithms in procedural languages can be expressed in Dijkstra's guarded command language (GCL), for which there is a formal basis that allows semantical support. In fact, programs written in GCL can be *derived* from their specification. So far, there are no tools that support this approach to programming.

The goal of Cocktail is to provide semantical support for deriving programs from their specifications by stepwise refinement, generating proof obligations on the fly. The system is intended for programmers in an educational setting. Cocktail is mainly practice-oriented to increase its usability for the intended users. Also, the system is set up to be extensible, flexible and easy to experiment with.

The present version is powerful enough for educational purposes. The tool yields support for many sorted first order predicate logic as the specification language; a simple *While*-language as the programming language; a Hoare logic for deriving programs from their specifications and a simple tableau based automated theorem prover for verifying proof obligations.

Cocktail is written entirely in Java and provides a modern graphical user interface. Proofs are graphically represented in Fitch-style natural deduction and can be manipulated by drag-and-drop operations.

# Data Mining Procedures Based on Bit String Representation of Data

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New data mining procedures based on bit string representation of analysed data will be presented. The core of the bit string approach is representation of each category (i.e. possible value) of each attribute by a suitable string of bits. This approach was developed for implementation of the GUHA method. A system of software modules for dealing with strings of bits were prepared for the academic KDD system LISp-Miner that is developed at University of Economics, Prague by a group of teachers and students.

These modules makes possible to fast generate strings of bits representing derived Boolean attributes. They can be also used to fast computation of contingency tables of categorial attributes. These modules were used to develop several data mining procedures that can be understood as GUHA procedures.

An example is the procedure *SDKL-Miner* that mines for patterns of the form  $R \sim C/(\alpha, \beta, \gamma)$ . Intuitive meaning of this pattern is that the subsets  $\alpha$  and  $\beta$  differ in mutual relation of categorial attributes  $R$  and  $C$  when the condition  $\gamma$  is satisfied.

The input of the *SDKL-Miner* is a data matrix  $\mathcal{M}$  with columns corresponding to categorial attributes. The categorial attributes  $R$  and  $C$  are automatically chosen from defined subsets of columns of  $\mathcal{M}$ . The subsets  $\alpha$  and  $\beta$  are subsets of rows of  $\mathcal{M}$  that are represented by Boolean attributes generated in a user-defined way from the other columns of data matrix  $\mathcal{M}$ . Similarly, the condition  $\gamma$  is represented by a Boolean attribute.

The pattern  $R \sim C/(\alpha, \beta, \gamma)$  is evaluated on the basis of two contingency tables. The first one is the contingency table of categorial attributes  $R$  and  $C$  concerning the subset  $\alpha$  but only the rows from  $\alpha$  satisfying the additional condition  $\gamma$  are taken into account. The second contingency table concerns categorial attributes  $R$  and  $C$ , subset  $\beta$  and the condition  $\gamma$  in a similar way. The symbol  $\sim$  is called *SDKL-quantifier*. It defines a condition concerning these two contingency tables.

The procedure *SDKL-Miner* and several additional procedures mining for a large variety of types of interesting patterns will be introduced and their properties will be demonstrated in details.

## Extracting Comprehensible Computational Models from Data

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Decision trees and rule based systems are a well-known and widely used methods for classification problems. To handle numerical attributes and for regression problems, traditional decision trees and rule base systems based on crisp predicates are, however, not suitable. Through the usage of a flexible language based on fuzzy predicates for different types of attributes, not only the expressive power of these models can be enhanced, but it furthermore enables us to create models for numerical attributes in a very natural manner, too. Extending the underlying logical language with ordering-based fuzzy predicates enables us to generate not only more compact, but also more accurate, models.

We will present a logical foundation and mechanisms for inductive learning of fuzzy decision trees and fuzzy rule bases from data. We will further show, how well known fuzzy logical inference methods can be applied with these models to provide continuous output for complex real-world problems.

## Modal Probability, Belief, and Actions

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We investigate a modal logic of belief and action, for which we give a regression algorithm in the style of Reiter. We add to this framework a unary modal operator of probability expressing that a proposition is more probable than its negation. Such an operator is not closed under conjunction, and its modal logic is therefore non-normal. We extend the regression algorithm to the resulting logic.

# Automated Theorem Proving in First-Order Temporal Logic

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JOINT WORK WITH B. KONEV, A. RIAZANOV, AND A. VORONKOV

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First-Order Temporal Logic (FOTL), is an extension of classical first-order logic by temporal operators for a discrete linear model of time (isomorphic to the natural numbers). Formulae of this logic are interpreted over structures that associate with each element of the natural numbers, representing a moment in time, a first-order structure with its own non-empty domain. The set of valid formulae of this logic is not recursively enumerable. However, the set of valid monodic formulae is known to be finitely axiomatisable, as recently shown by Hodkinson, Wolter, and Zakharyashev (2000). This breakthrough has led to considerable research activity examining the monodic fragment, in terms of decidable classes, extensions, applications and mechanisation, and promises important advances for the future of formal methods for reactive systems.

A formula in FOTL without equality and function symbols (constants are allowed) is called monodic if any subformula whose principal operator is a temporal operator contains at most one free variable.

In order to effectively utilise monodic temporal logics, we require tools mechanising their decision procedures. Concerning the mechanisation of monodic temporal logics, general tableau and resolution calculi have already been defined. However, neither of these is particularly practical: given a candidate formula to be tested for satisfiability, the tableau method requires representation of all possible first-order models of first-order subformulae of the candidate formula, while the resolution method involves all possible combinations of temporal clauses in the clause normal form of the candidate formula. Thus, improved tools are required.

For monodic first-order temporal logic with expanding domains we have recently been able to develop a simplified clausal resolution calculus, termed the fine-grained resolution calculus, which is more amenable to efficient implementation. We have also developed an implementation of this calculus in the form of the TeMP system. A particularly interesting aspect of this implementation is that the deduction rules of fine-grained step resolution are close enough to classical first-order resolution to allow us to use a first-order resolution prover as the core of our implementation.

# Techniques for Mechanizing Set-Based Software Development

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Successful software engineering methods like B or Z are based on a relational calculus that is derived from a set-theoretic semantics. But automated reasoning with sets and relations, which is essential for mechanizing such methods, remains a challenge. We have recently developed techniques for improving this situation. Here, we present techniques for operational reasoning with sets: a novel algebraic calculus and associated proof-search procedures. First, we show that atomic distributive lattices capture a relevant fragment of set theory. It includes element-wise reasoning, but avoids the ontological commitment to a universal set. We outline a calculus for this class that yields abstract and concise proofs from few elementary principles. Second, we introduce focused proof-search procedures for this class with simple deduction and powerful reduction and simplification rules that are guided by rewriting techniques. These procedures automatically specialize to decision procedures for interesting subclasses. Finally, we briefly discuss a novel algebraic formal method that uses these techniques in combination with relation-style reasoning based on modal Kleene algebra. It supports cross-theory reasoning about software, includes powerful decision procedures and provides small structured proof data-bases.

# **SQEMA: a new algorithm for computing correspondences of modal formulae**

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Propositional modal formulae define on Kripke frames universal monadic second-order properties. Some of them, however, are first-order definable. Such are, for instance, all Sahlqvist formulae, recently extended by us to so called inductive and complex formulae. Their first-order equivalents can be computed by applying to so called 'substitution method'. Generally stronger tools for that purpose are provided by the two implemented algorithms for elimination of second-order quantifiers over predicate variables: SCAN, based on clausification and constraint resolution, and DLS, based on suitable pre-processing and application of Ackermann's lemma, which is, in a way, dual to the substitution method. Both algorithms use skolemization and then attempt to unskolemize the result of the predicate elimination.

We have recently developed a new algorithm, SQEMA, for computing first-order equivalents of modal formulae, based on formal transformation rules enabling application of Ackermann's lemma, and combining features of both DLS and SCAN. Unlike these, SQEMA works directly on modal formulae, and does not use skolem functions.

In this talk I will introduce briefly the core of SQEMA and will demonstrate it with several examples. Time permitting, will also discuss issues such as correctness, completeness for important classes of formulae, and canonicity of the successfully computed formulae.

# Connections Relations in Mereotopology

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Achille C. Varzi

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We provide a model-theoretic framework for investigating and comparing a variety of mereotopological theories. In the first part we consider different ways of characterizing a mereotopology with respect to (i) the intended interpretation of their connection primitives, and (ii) the composition of their intended domains (e.g., whether or not they allow for boundary elements). This second part extends this study by considering a second, orthogonal dimension along which varieties of topological connection can be classified: the strength of the connection.

## S4u as a unifying spatial formalism

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In qualitative spatial representation and reasoning, a number of formalisms (such as RCC-8, BRCC-8, 9-intersections, etc.) have been introduced with the aim of specifying and reasoning about topological relations between spatial entities. On the other hand, it has been known from Tarski's classical results of the 1930s that propositional modal logic S4 and some of its extensions can be regarded as logics of topological spaces. Bennett and others have established some connections between the two kinds of formalisms. In this talk we analyse these connections in depth. In particular, we identify some natural fragments of the modal logic S4u which correspond to the spatial languages mentioned above, as well as some new, more expressive ones. We investigate the expressive power and the computational complexity of the introduced fragments.

# Relational Representation of Metric Spaces

Oliver Kutz

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Logics of metric (or weaker distance) spaces were conceived as knowledge representation formalisms aimed at bringing a numerical, quantitative concept of distance into the conventional qualitative representation and reasoning, where the notion of 'distance' can be interpreted in a variety of ways, amongst them, various notions of spatial distance, as well as similarity measures.

In the talk, I will briefly introduce a number of (modal) languages for talking about the intended models, metric spaces or weakenings thereof, which typically contain modal operators such as 'everywhere in the circle of radius  $x$ ', where ' $x$ ' is some rational number.

In order to provide complete axiomatisations and to establish or disprove logical properties like interpolation, we show how, depending on the expressivity of the language at hand, adequate relational representations of metric spaces can be given and techniques and results from modal logic can be employed.

In particular, we discuss a 'finitary' and elementary relational representation of metric spaces that captures theoremhood in metric spaces for a modal (hybrid) language that is expressively complete over metric spaces for the (undecidable) two-variable fragment of first-order logic with binary predicates interpreting the metric.

# Logics of metric spaces with tessellation operator

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A tessellation operator is a binary operator on sets of points of a metric space such that for any two sets it gives the set of all points which are closer to the first set than to the other. We consider extensions of metric logics by the tessellation operator. We prove that the universal modality and the operator of interior of a set in the topology induced by metric are expressible via the tessellation operator. More precisely, we prove that with respect to metric models, the language with the tessellation operator is equivalent to a language with some sort of restricted quantification over distances. Furthermore, we prove the tree-model property for the introduced logics and define an appropriate notion of bisimulation of metric models for which satisfiability of modal formulae of the language with the tessellation operator is an invariant.

## Propositional modal logics related to RCC

Philippe Balbiani, Tinko Tinchev, Dimiter Vakarelov

We consider a propositional modal language suitable for spatial reasoning in terms of RCC, including both continuous (topological) and discrete (in the sense of Galton) semantics. We axiomatize and prove completeness theorems for several RCC-like systems. One of them is exactly the propositional version of RCC. For some of the considered systems we establish the decidability of provability and the corresponding complexity.

# Enhanced visual tracking of moving objects by reasoning about spatio-temporal relationships

Brandon Bennett, Derek R. Magee, Anthony G. Cohn, David  
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We present a framework for annotating dynamic scenes involving occlusion and other uncertainties. Our system comprises an object tracker, an object classifier and an algorithm for reasoning about spatio-temporal continuity. The principle behind the object tracking and classifier modules is to reduce error by increasing ambiguity (by merging objects in close proximity and presenting multiple hypotheses). The reasoning engine resolves error, ambiguity and occlusion to produce a most likely hypothesis, which is consistent with global spatio-temporal continuity constraints. The system results in improved annotation over frame-by-frame methods. It has been implemented and applied to the analysis of a team sports video.