Hyperresolution (cont’d)

- There are many variants of resolution. (Refer to Bachmair and Ganzinger (2001), “Resolution Theorem Proving”, for further reading.)
- One well-known example is hyperresolution (Robinson 1965):
  - Assume that several negative literals are selected in a clause $D$.
  - If we perform an inference with $D$, then one of the selected literals is eliminated.
  - Suppose that the remaining selected literals of $D$ are again selected in the conclusion.
  - Then we will eliminate the remaining selected literals one by one by further resolution steps.

Craig-Interpolation (cont’d)

Proof of Property [17]
Transform $F$ and $\neg G$ into CNF.
Let $N$ and $M$, resp., denote the resulting clause sets.
Choose any atom ordering $\succ$ for which the prop. variables that occur in $F$ but not in $G$ are maximal.
Saturate $N$ wrt. $\text{Res}_S^{-}$ (with empty selection function $S$) to get $N^{*}$. Let
$N' = N^{*} \setminus \{C \mid C \text{ contains a symbol in } F \text{ but not in } G\}$.
I.e. $C \in N'$ iff $C \in N^{*}$ and $C$ contains only symbols in $G$.
Let $H = \land N'$. Then, clearly $F \models H$. (Why?)
To see that $H \models G$, take $N^{*} \cup M$ and saturate wrt. $\text{Res}_S^{-}$.
This derives $\bot$, but no inferences are performed on clauses in $N^{*} \setminus N'$.
This implies $N' \cup M \models \bot$ and therefore $H \models G$.

Hyperresolution (cont’d)

- Hyperresolution replaces these successive steps by a single inference.
- As for $\text{Res}_S^{-}$, the calculus is parameterised by an atom ordering $\succ$ and a selection function $S$.
- But $S$ is the ‘maximal’ selection function, i.e. selects all negative literals in a clause.

Hyperresolution

- Hyperresolution calculus $\text{HRes}$

\[
\begin{array}{c}
C_1 \lor A_1 \ldots \lor C_n \lor A_n \lor \neg B_1 \lor \ldots \lor \neg B_n \lor D \\
\hline
(C_1 \lor \ldots \lor C_n \lor D)\sigma
\end{array}
\]

provided $\sigma$ is the mgu s.t. $A_1\sigma = B_1\sigma$, $\ldots$, $A_n\sigma = B_n\sigma$, and
(i) $A_i\sigma$ strictly maximal in $C_i\sigma$, $1 \leq i \leq n$;
(ii) nothing is selected in $C_i$ (i.e. $C_i$ is positive);
(iii) the indicated $\neg B_i$ are exactly the ones selected by $S$, and $D$ is positive.

- Similarly as for resolution, hyperresolution has to be complemented by a factoring rule. I.e. the ordered positive factoring rule from before.