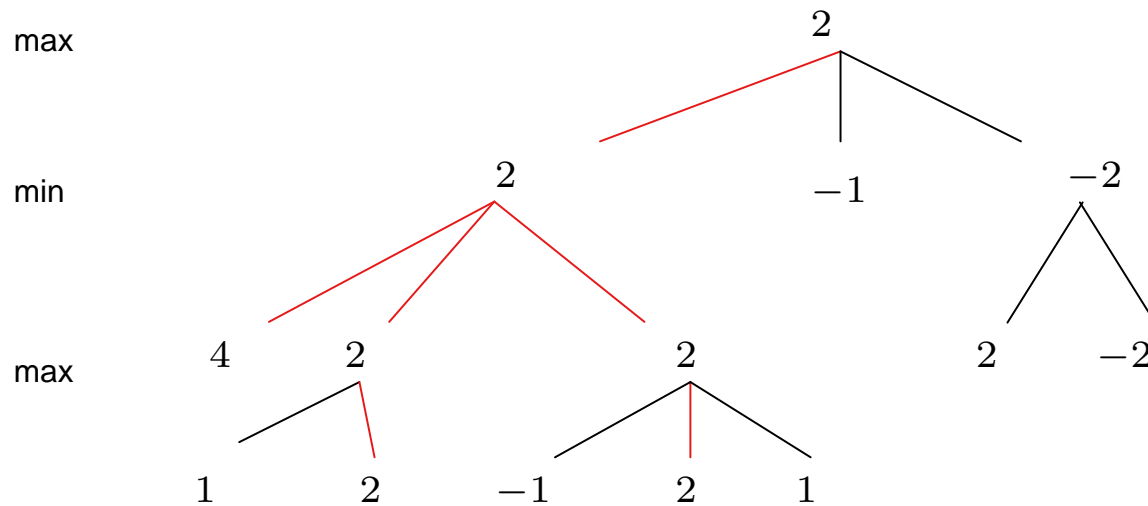


Minimax algorithm

Exercise 20 (a): Minimax algorithm.

Player 1

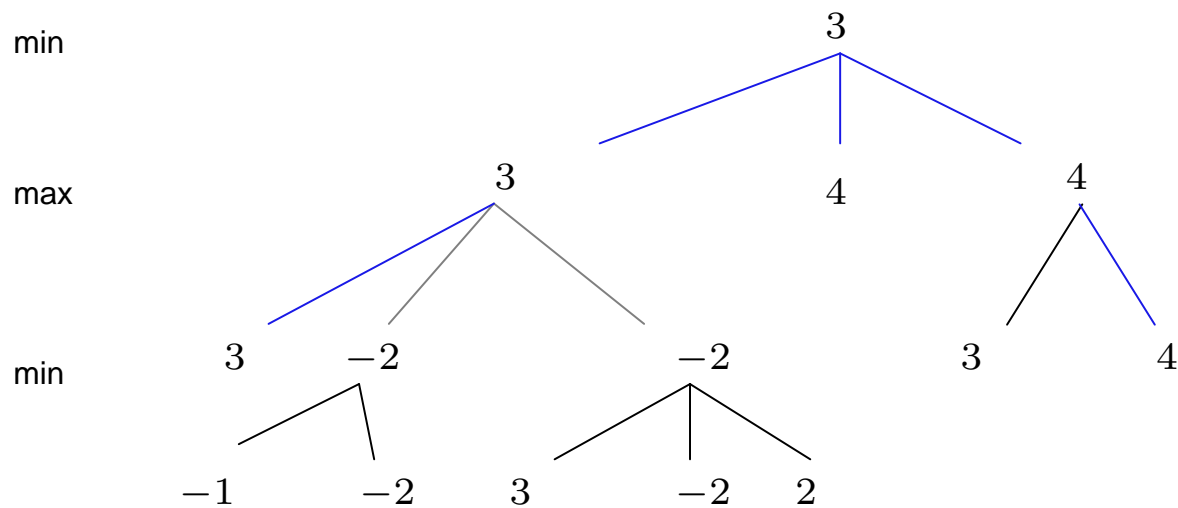


Value for Player 1: 2

Minimax algorithm

Exercise 20 (a): Minimax algorithm.

Player 2



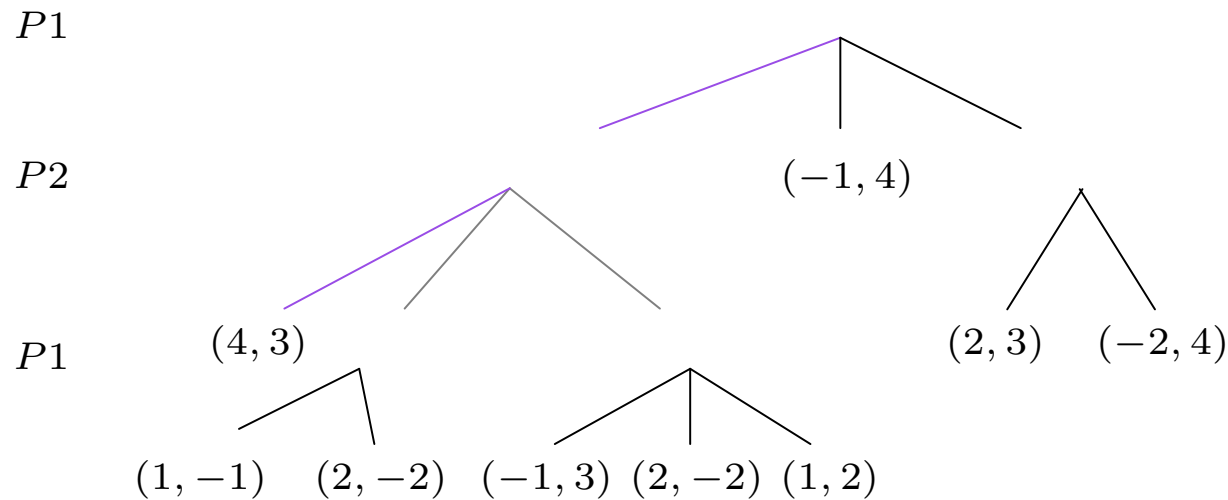
Value for Player 1: 2

Value for Player 2: 3

Minimax algorithm

Exercise 20 (a): Minimax algorithm.

Playing these strategies



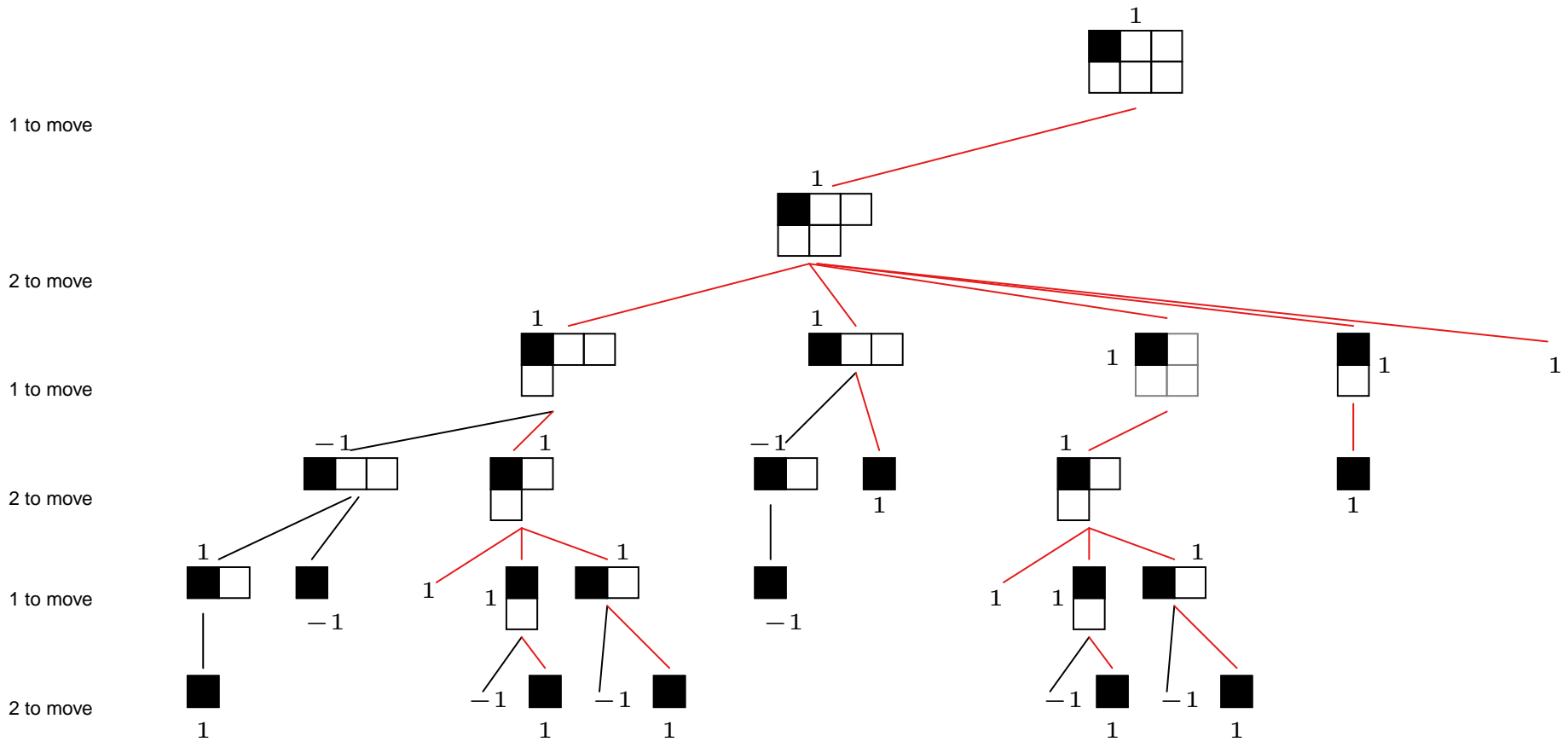
Value for Player 1: 2

Value for Player 2: 3

(2×3) -Chomp

Exercise 21 (a): Minimax algorithm.

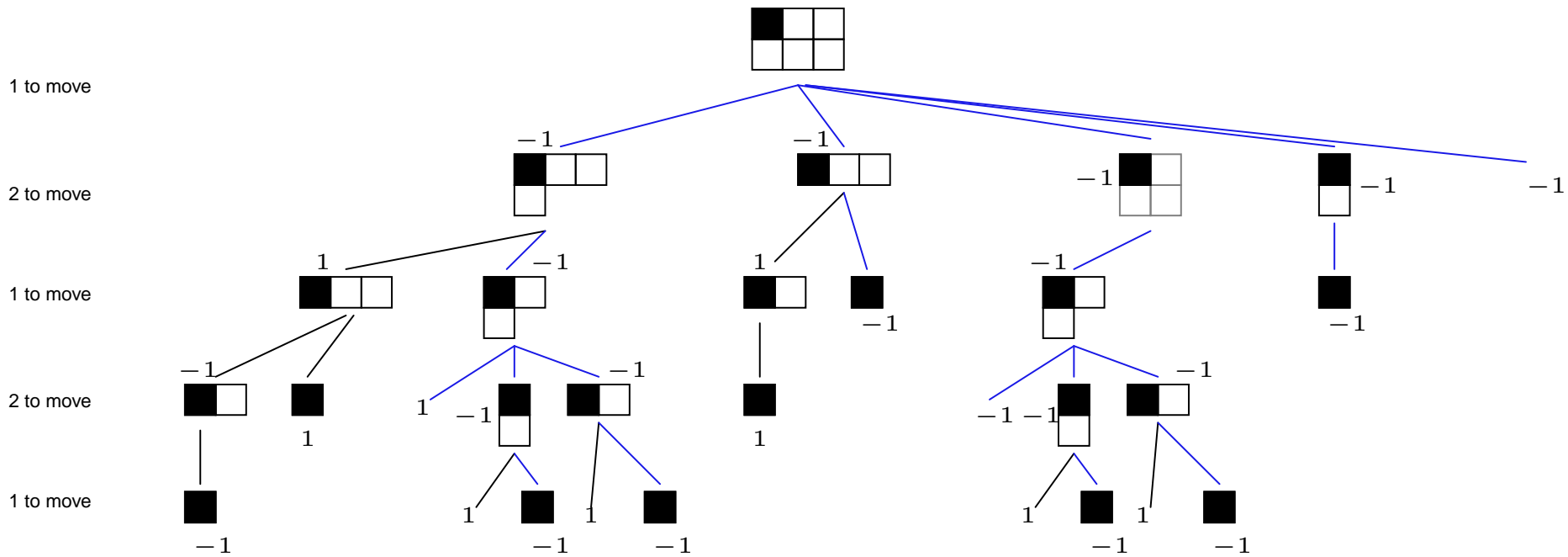
Alpha-beta pruning for Player 1 (Player 1 has winning strategy)



(2×3) -Chomp

Exercise 21 (a): Minimax algorithm.

Player 1 plays different opening move (then Player 2 can force a win)



Repeated Prisoner's Dilemma

Exercise 22 (a): Repeated Prisoner's Dilemma.

The hint says, for Player 2, to consider the strategy which

- defects five times in a row and
- in the last round
 - ▶ cooperates if the other side cooperated five times,
 - ▶ defects otherwise.

As an abbreviation, we call this strategy AAD, for 'almost always defect'.

Repeated Prisoner's Dilemma

Exercise 22 (a): Repeated Prisoner's Dilemma.

The hint says, for Player 2, to consider the strategy AAD, for 'almost always defect'.

Many plays against AAD leads to **mutual defection for all six rounds**, giving a pay-off of $6 \times (-8) = -48$ for both players.

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In order to do better the other strategy has to achieve a different outcome, and the only chance of that is to **cooperate five times in a row** and then **defect**. This leads to a pay-off for that strategy of $5 \times (-10) + 0 = -50$, which gives no improvement.

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This is almost enough to show that the following are equilibrium points:

(ALWAYS D, AAD)

(AAD, AAD).

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So let's now show that these are indeed equilibrium points.

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So let's now show that these are indeed equilibrium points.

If Player 1 changes his mind, then he will be worse off:

- We know that Player 2 will **defect** on the first five moves.
 - ▶ We know that **cooperating** five times to get him to **cooperate** in round 6 doesn't give a better pay-off.

Hence our best answer leads to him **defecting** 6 times.

- The best play against **defecting** 6 times is also to **defect** 6 times. Both the given strategies do just that.

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- The best play against **defecting** 6 times is also to **defect** 6 times. Both the given strategies do just that.

The argument for Player 2 is almost identical.

Repeated Prisoner's Dilemma

Exercise 22 (a): Repeated Prisoner's Dilemma.

This is almost enough to show that the following are equilibrium points:

$(\text{ALWAYS D}, \text{AAD})$ (AAD, AAD) .

$(\text{ALWAYS D}, \text{AAD})$ and (AAD, AAD) are not **subgame equilibrium points** since there are subgames where they would **cooperate** (on the sixth round), but that is known not to be an equilibrium point strategy.

ALWAYS D is collectively stable

Exercise 23 (a): ALWAYS D is collectively stable

Show: No strategy can get higher pay-off against ALWAYS D than ALWAYS D against itself.

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When **ALWAYS D** plays against itself we get a game where both parties get pay-off P for each round.

ALWAYS D is collectively stable

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When ALWAYS D plays against itself we get a game where both parties get pay-off P for each round.

But any strategy playing against ALWAYS D in any round can

- defect and get pay-off P or
- cooperate and get pay-off S .

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But $P \geq S$, and so (as we have known all along) the best answer against somebody who is known always to defect is to defect in turn.

In other words, there is nothing the other strategy can do which would allow it to achieve a higher pay-off against ALWAYS D than ALWAYS D does against itself.

The Hawk-Dove Game

Exercise 24 (a): The Hawk-Dove Game

The matrix for the game is

$$\begin{vmatrix} -40 & 40 \\ 0 & 10 \end{vmatrix}$$

where the pay-offs are given for the row player and the symmetry of the game means that they we can read off pay-offs for both, Player 1 and Player 2 from this.

The Hawk-Dove Game

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where the pay-offs are given for the row player.

A population with a proportion p of **HAWKS** and $1 - p$ **DOVES** is stable iff the pay-off per round for a **HAWK** is the same as that for a **DOVE**.

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The former is

$$-40p + 40(1 - p) = 40 - 80p,$$

while the latter is

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The two are equal iff

$$40 - 80p = 10 - 10p, \quad \text{that is iff} \quad p = 3/7.$$