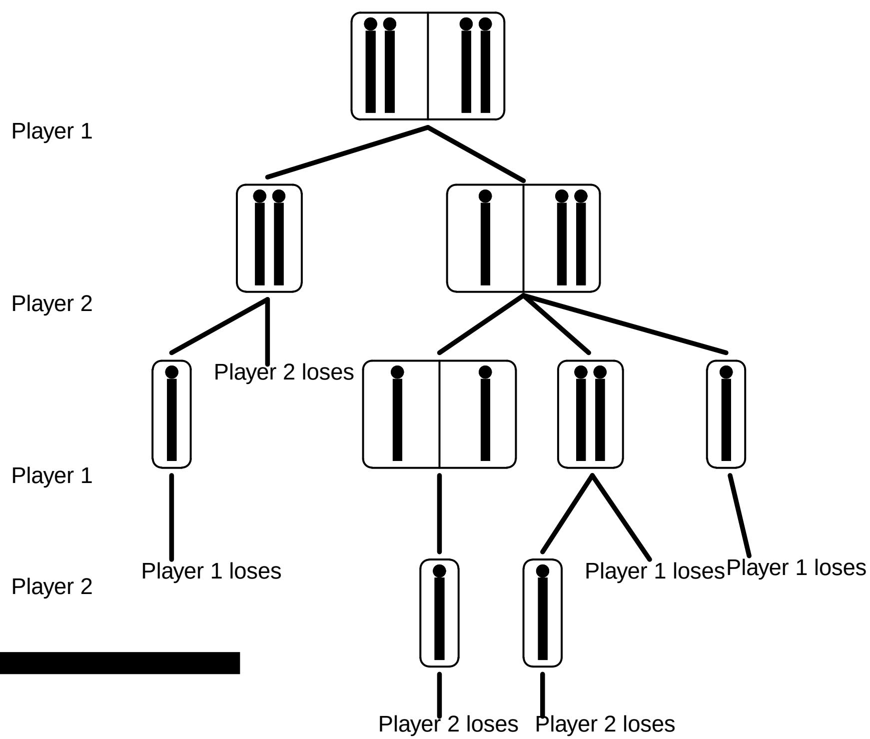
$(2, 2)$-Nim

Exercise 6 (a): $(2, 2)$-Nim as a matrix game

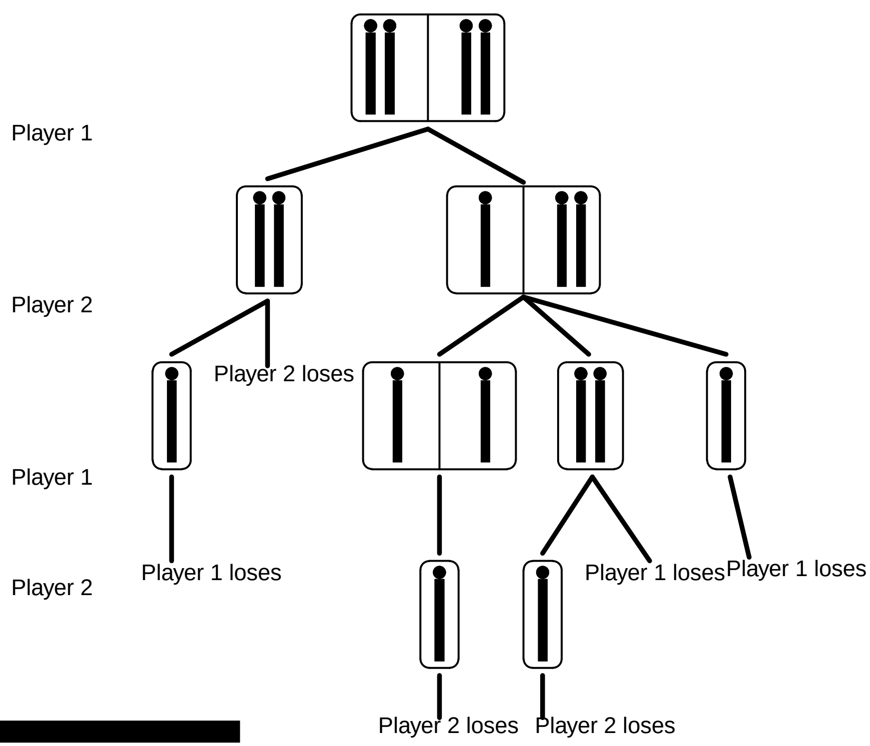
To fit all this onto the slide, we give Player 2 as the row player, and Player 1 as the column player, the entry gives the winner. In other words, we would usually give the transpose of the following matrix.



$(2,2)$-Nim

Exercise 6 (a): $(2,2)$-Nim as a matrix game

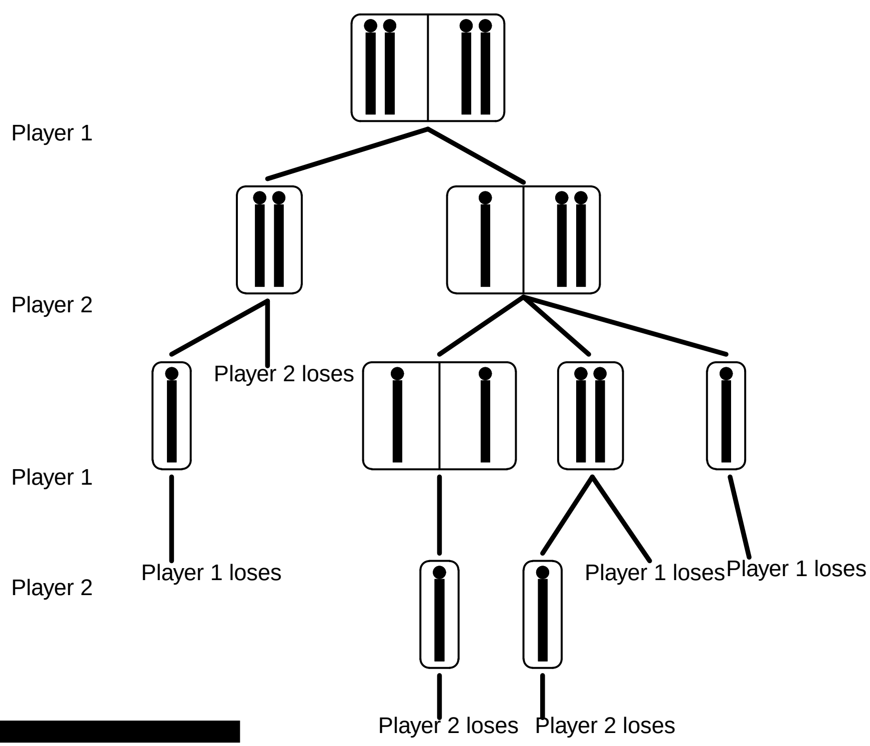
To fit all this onto the slide, we give Player 2 as the row player, and Player 1 as the column player, the entry gives the winner.



$(2, 2)$-Nim

Exercise 6 (a): $(2, 2)$-Nim as a matrix game

To fit all this onto the slide, we give Player 2 as the row player, and Player 1 as the column player, the entry gives the winner.

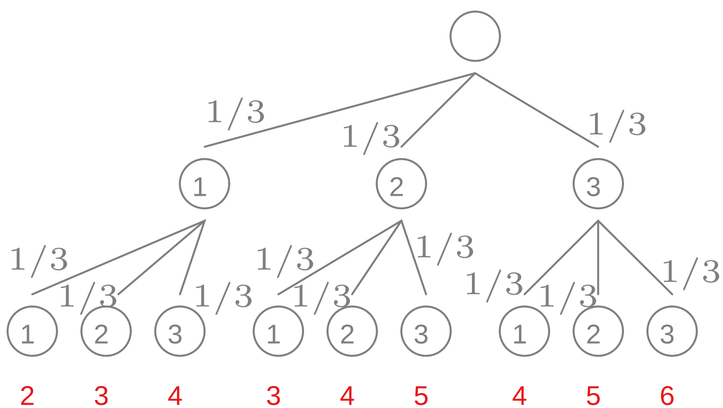


Player 2 \ Player 1	2	$(1,1)$	$(1,2)$
$(1\|1(1))$	2	1	2
$(1\|1(2))$	2	1	1
$(1\|2(2))$	2	2	2
$(2\|1(1))$	1	1	2
$(2\|1(2))$	1	1	1
$(2\|2(2))$	1	2	2

Note that $(1|2(2))$ is a winning strategy for Player 2: if she plays according to it, she will always win.

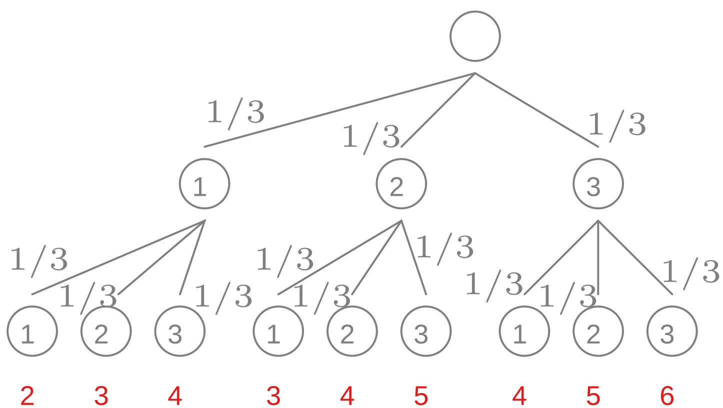
Throwing two 3-faced dice

Exercise 7 (a): Throwing two 3-faced dice



Throwing two 3-faced dice

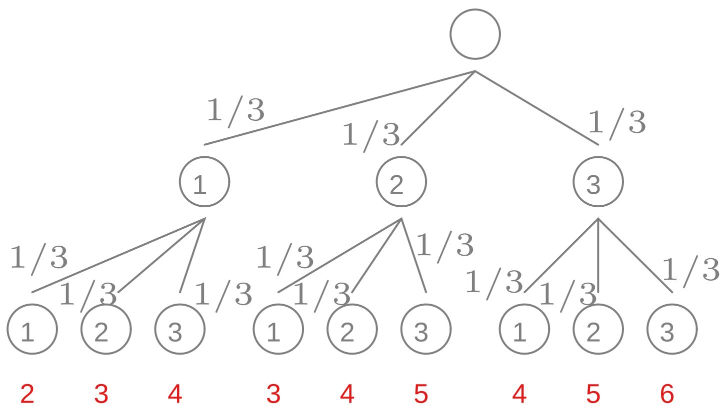
Exercise 7 (a): Throwing two 3-faced dice



The probabilities for the various outcomes (the sum of the faces of the two thrown dice) is given in the following table.

Throwing two 3-faced dice

Exercise 7 (a): Throwing two 3-faced dice

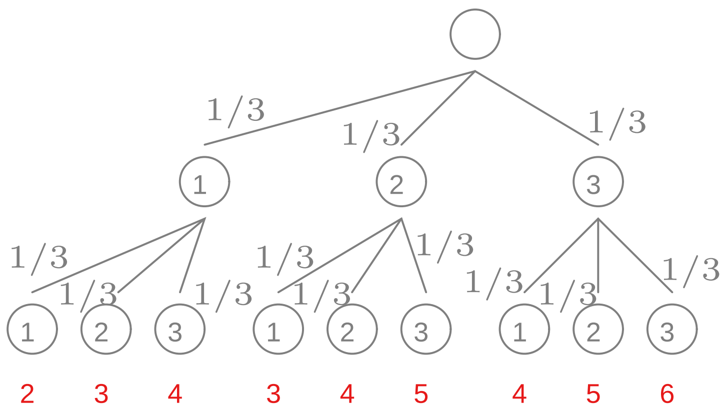


The probabilities for the various outcomes (the sum of the faces of the two thrown dice) is given in the following table.

2	3	4	5	6
$1/9$	$2/9$	$3/9$	$2/9$	$1/9$

Throwing two 3-faced dice

Exercise 7 (a): Throwing two 3-faced dice



The probabilities for the various outcomes (the sum of the faces of the two thrown dice) is given in the following table.

2	3	4	5	6
$1/9$	$2/9$	$3/9$	$2/9$	$1/9$

The expected value is

$$2/9 + 6/9 + 12/9 + 10/9 + 6/9 = 36/9 = 4.$$

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

$$\begin{vmatrix} 4 & 3 & 1 & 1 \\ 3 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 \\ 3 & 3 & 1 & 2 \end{vmatrix}$$

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

$$\left|\begin{array}{cccc} 4 & 3 & \color{red}{1} & 1 \\ 3 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 \\ 3 & 3 & 1 & 2 \end{array}\right| \quad \color{red}{1}$$

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

4	3	1	1	1
3	2	2	2	2
4	4	2	2	
3	3	1	2	

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

4	3	1	1	1
3	2	2	2	2
4	4	2	2	2
3	3	1	2	

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

4	3	1	1	1
3	2	2	2	2
4	4	2	2	2
3	3	1	2	1

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

4	3	1	1	1
3	2	2	2	2
4	4	2	2	2
3	3	1	2	1
				2

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

4	3	1	1	1
3	2	2	2	2
4	4	2	2	2
3	3	1	2	1
				2

So $\max_{1 \leq i \leq 4} \min_{1 \leq j \leq 4} a_{i,j} = 2$.

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

4	3	1	1	1
3	2	2	2	2
4	4	2	2	2
3	3	1	2	1
4				2

So $\max_{1\leq i\leq 4} \min_{1\leq j\leq 4} a_{i,j} = 2$.

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

4	3	1	1	1
3	2	2	2	2
4	4	2	2	2
3	3	1	2	1
4	4			2

So $\max_{1\leq i\leq 4}\min_{1\leq j\leq 4} a_{i,j} = 2$.

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

4	3	1	1	1
3	2	2	2	2
4	4	2	2	2
3	3	1	2	1
4	4	2		2

So $\max_{1\leq i\leq 4}\min_{1\leq j\leq 4} a_{i,j} = 2$.

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

4	3	1	1	1
3	2	2	2	2
4	4	2	2	2
3	3	1	2	1
4	4	2	2	2

So $\max_{1 \leq i \leq 4} \min_{1 \leq j \leq 4} a_{i,j} = 2$.

So $\min_{1 \leq j \leq 4} \max_{1 \leq i \leq 4} a_{i,j} = 2$.

Maxmin and minmax

Exercise 8 (a): Maxmin and minmax

4	3	1	1	1
3	2	2	2	2
4	4	2	2	2
3	3	1	2	1
4	4	2	2	2\2

So $\max_{1 \leq i \leq 4} \min_{1 \leq j \leq 4} a_{i,j} = 2$.

So $\min_{1 \leq j \leq 4} \max_{1 \leq i \leq 4} a_{i,j} = 2$.

Maxmin and minmax don't agree

Exercise 9 (a) Such a matrix is given in Exercise 8(b), or on page 33 of the notes.

Finding equilibria

Exercise 10 (a): Finding equilibria

$$\begin{vmatrix} 4 & 3 & 1 & 1 \\ 3 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 \\ 3 & 3 & 1 & 2 \end{vmatrix}$$

We are looking for values which are minimal in their row and maximal in their column.

We check that no value in the first row satisfies this criterion. In the second row, we are finally successful.

Finding equilibria

Exercise 10 (a): Finding equilibria

We are looking for values which are minimal in their row and maximal in their column.

$$\begin{vmatrix} 4 & 3 & 1 & 1 \\ 3 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 \\ 3 & 3 & 1 & 2 \end{vmatrix}$$

Since the corresponding 2 is minimal in its row and maximal in its column, $(2,3)$ is an equilibrium point.

Finding equilibria

Exercise 10 (a): Finding equilibria

$$\begin{vmatrix} 4 & 3 & 1 & 1 \\ 3 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 \\ 3 & 3 & 1 & 2 \end{vmatrix}$$

We are looking for values which are minimal in their row and maximal in their column.

We now know that the value of the game is 2 and it is sufficient to check all the remaining entries of that value.

Since the corresponding 2 is minimal in its row and maximal in its column, $(2,4)$ is an equilibrium point.

The equilibrium points are: $(2,3)$

Finding equilibria

Exercise 10 (a): Finding equilibria

We are looking for values which are minimal in their row and maximal in their column.

$$\begin{vmatrix} 4 & 3 & 1 & 1 \\ 3 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 \\ 3 & 3 & 1 & 2 \end{vmatrix}$$

We now know that the value of the game is 2 and it is sufficient to check all the remaining entries of that value.

Since the corresponding 2 is minimal in its row and maximal in its column, $(3, 3)$ is an equilibrium point.

The equilibrium points are: $(2, 3)$, $(2, 4)$.

Finding equilibria

Exercise 10 (a): Finding equilibria

We are looking for values which are minimal in their row and maximal in their column.

$$\begin{vmatrix} 4 & 3 & 1 & 1 \\ 3 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 \\ 3 & 3 & 1 & 2 \end{vmatrix}$$

We now know that the value of the game is 2 and it is sufficient to check all the remaining entries of that value.

Since the corresponding 2 is minimal in its row and maximal in its column, $(3, 4)$ is an equilibrium point.

The equilibrium points are: $(2, 3)$, $(2, 4)$, $(3, 3)$.

Finding equilibria

Exercise 10 (a): Finding equilibria

We are looking for values which are minimal in their row and maximal in their column.

$$\begin{vmatrix} 4 & 3 & 1 & 1 \\ 3 & 2 & 2 & 2 \\ 4 & 4 & 2 & 2 \\ 3 & 3 & 1 & 2 \end{vmatrix}$$

We now know that the value of the game is 2 and it is sufficient to check all the remaining entries of that value.

Since the corresponding 2 is not minimal in its row, $(4,4)$ is not an equilibrium point.

The equilibrium points are: $(2,3)$, $(2,4)$, $(3,3)$ and $(3,4)$.

Finding equilibria

Exercise 10 (a): Finding equilibria

$$\begin{vmatrix} 4 & 3 & 1 & 1 \\ 3 & 2 & \color{orange}{2} & \color{orange}{2} \\ 4 & 4 & \color{orange}{2} & \color{orange}{2} \\ 3 & 3 & 1 & 2 \end{vmatrix}$$

We are looking for values which are minimal in their row and maximal in their column.

We now know that the value of the game is 2 and it is sufficient to check all the remaining entries of that value.

The equilibrium points are: $(2, 3)$, $(2, 4)$, $(3, 3)$ and $(3, 4)$.

Equilibria in non zero-sum games

Exercise 11 (a): Equilibria in non zero-sum games.

$$\begin{vmatrix} (-10,5) & (2,-2) \\ (1,-1) & (-1,1) \end{vmatrix}$$

If Player 1 changes his mind from strategy 1 to strategy 2 while Player 2 sticks with her strategy 1 Player 1 will be better off, so $(1,1)$ is no equilibrium point.

Equilibria in non zero-sum games

Exercise 11 (a): Equilibria in non zero-sum games.

$$\begin{vmatrix} (-10,5) & (2,-2) \\ (1,-1) & (-1,1) \end{vmatrix}$$

If Player 2 changes her mind from strategy 2 to strategy 1 while Player 1 sticks with his strategy 1 Player 2 will be better off, so $(1,2)$ is no equilibrium point.

Equilibria in non zero-sum games

Exercise 11 (a): Equilibria in non zero-sum games.

$$\begin{vmatrix} (-10,5) & (2,-2) \\ (1,-1) & (-1,1) \end{vmatrix}$$

If Player 2 changes her mind from strategy 1 to strategy 2 while Player 1 sticks with her strategy 2 Player 2 will be better off, so $(2,1)$ is no equilibrium point.

Equilibria in non zero-sum games

Exercise 11 (a): Equilibria in non zero-sum games.

$$\begin{vmatrix} (-10,5) & (2,-2) \\ (1,-1) & (-1,1) \end{vmatrix}$$

If Player 1 changes his mind from strategy 2 to strategy 1 while Player 2 sticks with her strategy 2 Player 1 will be better off, so $(2,2)$ is no equilibrium point.

Equilibria in non zero-sum games

Exercise 11 (a): Equilibria in non zero-sum games.

$$\begin{vmatrix} (-10,5) & (2,-2) \\ (1,-1) & (-1,1) \end{vmatrix}$$

Hence this game has no equilibrium points.