



CS3191 Section 5

Game Models

Andrea Schalk

School of Computer Science, University of Manchester

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We will look at variants of this game before we consider its **repeated** version.

Lastly we explore the many strategies available when playing **repeated Prisoner's Dilemma**.



The Prisoner's Dilemma Revisited

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Hence game theory's recommendation for this game is to defect.

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Even if Joe knew in advance that Fred was going to **defect**, the best he could do is to **defect** in turn.

Hence game theory's recommendation for this game is to defect.

However, both parties would much prefer the outcome they get from both cooperating.



Generalizing the game

Generalization

There is nothing specific to the numbers that appear in our Prisoner's Dilemma game matrix. So let's assume we define a more general game.

Generalization

Assume that the game we are talking about is **symmetric**, and that both sides have the possibility to **defect** or to **cooperate**.

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We name the possible pay-offs as follows.

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Symmetry means that the pay-off for Player 2 can be described by the transposed matrix

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We would therefore normally give the pay-off matrix as

		Player 2	
		defect	cooperate
Player 1	defect	(P, P)	(T, S)
	cooperate	(S, T)	(R, R)

but information in second version can be derived from the first.

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Assume that $T > R > P > S$.

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Assume that $T > R > P > S$.

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- the **defect** strategy dominates the **cooperate** one;
- the **only equilibrium point** is $(1, 1)$;
- both players would prefer the outcome of $(2, 2)$.



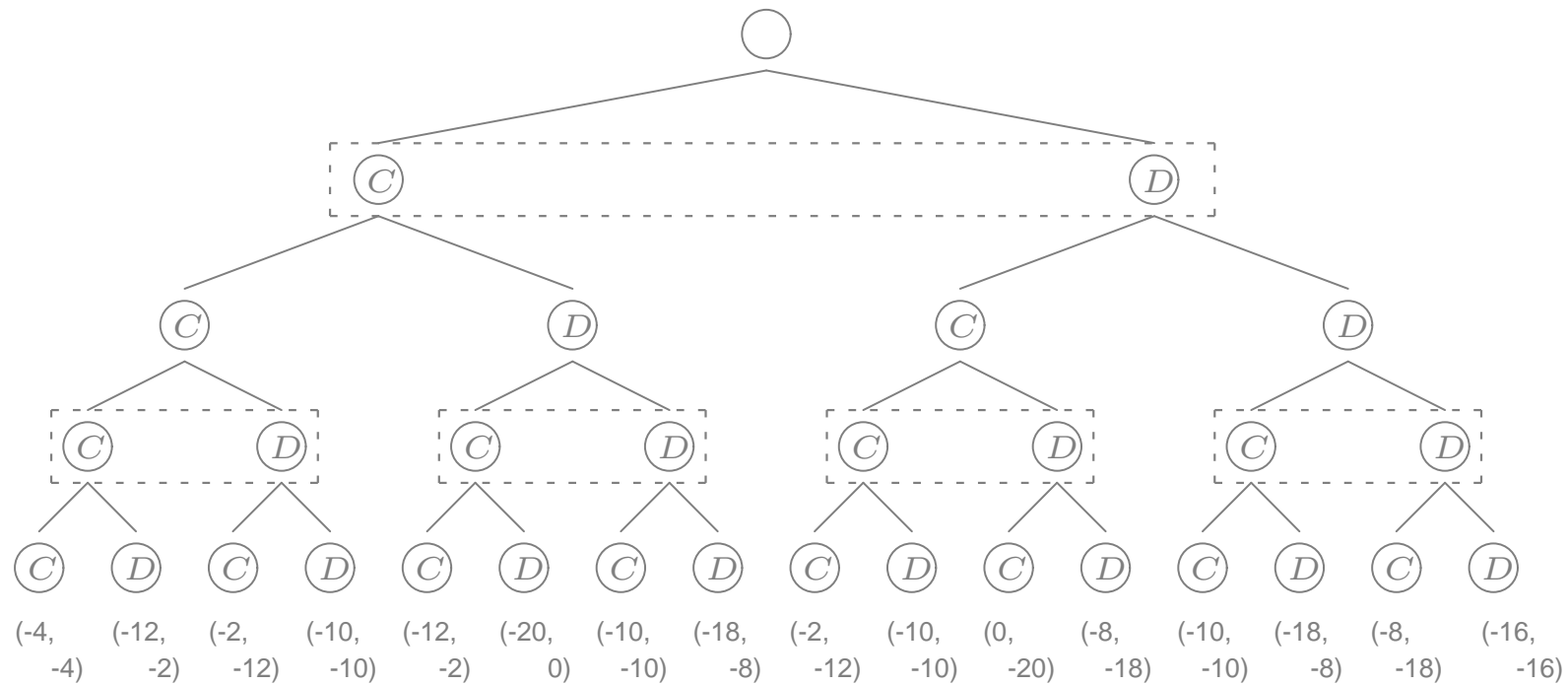
Repeated games

Repeated games

So what happens if both sides know they will play the game more than once? Does the possibility of the other side cooperating in the future (given one's own present action) make the protagonists 'play nice'?

Repeated games

Here is a tree for two round PD.



Repeated games

And here is the corresponding matrix game.

	$C(C C)$	$C(C D)$	$C(D C)$	$C(D D)$	$D(C C)$	$D(C D)$	$D(D C)$	$D(D D)$
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Note that the number of strategy increases.

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Question. How does the number of strategy increase with the number of rounds?

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The number of strategies is exponential in the number of rounds.

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If we play more rounds then there are more equilibrium points, see Exercise 22. In the five round Prisoner's Dilemma game there are at least **3 equilibrium points**, all of which lead to play where both sides **keep defecting**.

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Definition 11 *Let a **sub-game** of a given game be obtained by choosing any position in the game tree and considering it as the root, with the rest of the tree given by the part of the original tree which is below it.*

*A **sub-game equilibrium point** in a j -person game consists of a j -tuple of strategies, one for each player, such that this tuple gives an equilibrium point for each sub-game of the game whose root is reachable when playing these strategies.*

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We will see an example of an **equilibrium point** which is not a **sub-game equilibrium point** in Exercise 22.

Proposition 5.1 *The **only sub-game equilibrium point** in a game of Prisoner's Dilemma with finitely many repetitions consists of choosing the **'always defect'** strategy for each player.*

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Here is an argument (known as **backwards induction**): The last game in a series is just like ordinary PD, so both players should **defect**. But if they know that this will happen in the last round, they should also both **defect** in the last but one round—but this argument works backwards all the way until the first round is reached.

Whither now?

We have seen that even with higher stakes in the repeated PD game, it makes sense for both sides to keep **defecting**.

This is not how people behave in every-day life (compare Hofstadter's PD game with 20 players).

Whither now?

We have seen that even with higher stakes in the repeated PD game, it makes sense for both sides to keep **defecting**.

The PD game has turned out to be very useful in modelling real-world interactions, and we are not done with it yet by a long margin.

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We have seen that even with higher stakes in the repeated PD game, it makes sense for both sides to keep **defecting**.

When playing the repeated version we want to make sure that the players cannot profit from deciding to alternate **defecting** and **cooperating**, so we assume that

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There are a great number of strategies when playing the iterated PD game: For round $n + 1$ our decision is allowed to depend on the 4^n possible histories of the game!

Experiment

	defect	cooperate
defect	(1, 1)	(5, 0)
cooperate	(0, 5)	(3, 3)



A computer tournament

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He invited game theorists from a number of different academic fields such as economics, psychology, political science, mathematics and sociology to send him a computer program to play repeated Prisoner's Dilemma in a round robin tournament.

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Question. What would you have submitted? Do you think your strategy might have won?

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It is clear that given the conditions imposed by Axelrod there is no ‘best’ strategy, that is one that will win such a tournament against all comers. Instead which strategy is best depends on the **population**, that is the other strategies present and their numbers.

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TITFOR TAT scored 504 points on average over the 200 rounds.

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Note that TITFOR TAT will **never** outscore a single one of its opponents in their bout of 200 games:

- If both strategies keep cooperating, they will both score 600 points.
- If the other strategy defects first then the best TITFOR TAT can hope for is eventually to make up the disadvantage of five points caused thus.

JOSS

`Joss` (named after its creator) behaves just the same as `TITFOR TAT` but every so often it throws in a random defection and afterwards it goes back to cooperating.

	TITFOR TAT	JOSS

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TITFOR TAT	JOSS	TITFOR TAT	JOSS
<i>C</i>	<i>C</i>	3	3
\vdots	\vdots	\vdots	\vdots
<i>C</i>	<i>C</i>	3	3
<i>C</i>	<i>D</i>	0	5

When TITFOR TAT and Joss play against each other the following effect occurs: They both **cooperate**, racking up points, until Joss throws in one of its random **defections**.

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⋮	⋮	⋮	⋮
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<i>D</i>	<i>C</i>	5	0
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They are locked in a defection/cooperation loop until eventually, **Joss** will sneak in another **defection**.

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TITFORTAT	JOSS	TITFORTAT	JOSS
<i>C</i>	<i>C</i>	3	3
⋮	⋮	⋮	⋮
<i>C</i>	<i>C</i>	3	3
<i>C</i>	<i>D</i>	0	5
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They are locked in a defection/cooperation loop until eventually, JOSS will sneak in another **defection**.

At this point, both programs will **defect** forever, and their pay-off goes down to just one point per game.

JOSS

TITFOR TAT and JOSS only scored 225/230 points against each other, and JOSS had similar results against many other strategies. Ultimately it was hurt by its deviousness!

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These two probabilities are then adjusted in the course of play, starting with $1/2$. If the two probabilities are similar then **DOWNING** will tend to **defect**, having decided that it makes no difference to the other side's behaviour. If the other player instead tends to retaliate after a defection, but cooperates when **DOWNING** cooperates, it will decide that it is more prudent to **cooperate**.

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Due to the way the two probabilities are set at the start (when **DOWNING** has no real information about its playing partner) it typically starts with **defecting** twice. In the tournament that led to other programs punishing it accordingly. As a consequence the program only came 10th in the tournament, gaining only 390 points on average against other entries.

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The others presumably thought they were clever enough to produce something better than that!

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Strategies that did well were **forgiving**, that is they might punish the other side for defecting but they would not hold a grudge forever.

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All in all many participants did not do very well when judging what strategies their programs might be up against.



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This time he attracted 62 entrants from six countries. Rapoport once again submitted `TITFOR TAT`, while the British biologist John Maynard Smith put in `TITFOR TWO TATS`. `RDOWNING` was present as well, and Axelrod entered `RANDOM` to the field.

Lessons

Once again, the winner was TITFOR TAT. Surprisingly, TITFORTWOTATS was only 24th, and RDOWNING came 38th and 40th (it was submitted by two participants). This shows the importance of the **environment** when it comes to determining what does well.

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All but one of the top fifteen entries were **nice**. All but one of the bottom fifteen were **not nice**. Many people had learned this lesson, since almost two-thirds of all entries were nice.

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TITFOR TAT has all three properties identified as advantageous: It is nice (it never defects first), forgiving (it is prepared to try cooperation again if the other side shows willingness) and retaliatory (it will copy defections made by the other side in the next round).

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It did very well against strategies which forgave too easily, or needed more provocation to retaliate, such as **TITFORTWOTATS**. It also exploited strategies similar to **DOWNING**: Because **TESTER** cooperates just over half of the time, these strategies calculate that it pays off to keep cooperating—and are exploited for that. However there were not enough suckers for it to do well in the overall tournament: It only came 46th.

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The other testing strategy Axelrod dubbed 'Tranquilizer', short **TRANQ**. It works by first establishing a relation of mutual cooperation (assuming the other player allows that), and then cautiously tests whether it can get away with some defections. It is careful, however, never to defect twice in a row, nor more than at a quarter of all moves. It also defects in reply to the other side defecting.

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This fairly sophisticated program, based on reasonable sounding arguments, did poorly overall—it only came twenty-seventh (so it at least did considerably better than **TESTER**).

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In order to do well against programs like **TESTER** and **TRANQ**, a program must retaliate as soon as the other side defects without provocation and so make it clear that will not tolerate this sort of behaviour.

Representative entries

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$$120 + .202P(T, S_1) + .198P(T, S_2) + .11P(T, S_3) + .072P(T, S_4) + .086P(T, S_5).$$

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This can be used to predict the score of a strategy T in the tournament to an amazing degree of accuracy (typically a difference of fewer than 10 points out of about three to four hundred). The strategy S_4 is TESTER while S_5 is TRANQ. The strategies S_1, S_2 and S_3 are nice.

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Among these, S_2 , called REVISEDSTATETRANSITION by Axelrod, keeps a model of the other player as a one-step Markov process. Given the history of the game so far, it estimates the probability that the other player will defect based on the result of the previous round.

Hypothetical tournaments

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In any of Axelrod's tournaments, ALWAYS D would have done very poorly, with a pay-off of 200 against most other strategies. Some of the strategies that finished towards the end of the field were not clearly enough distinguishable from RANDOM, so their partners gave up on trying to cooperate with them.

Further developments

A tournament organized by D.R Hofstadter and Marek Lugowski had ALWAYS_D come almost last, but **variations of TITFOR_{TAT}** did better than the original. They switched to ALWAYS_D if the other player was too uncooperative.

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What is successful depends on the **environment**. But surely one would expect such an environment to change, as unsuccessful strategies vanish and the better ones multiply. This is the idea behind **evolutionary game theory**.

Further developments

In 2004 a competition with 223 entries celebrated the 20th anniversary of Axelrod's first tournament. A team from the University of Southampton submitted 60 programs which were designed to **recognize each other**. Then one of them would turn itself into point fodder for the other. If meeting some other program these would turn into ALWAYS D.

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The top three programs were theirs, but so were a lot of the bottom ones. The Southampton team did some further research and decided that **20 of their programs** would have led to a similar result.



Infinitely and indefinitely repeated versions

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One alternative is to try to go for infinitely many repetitions. There are two main problems with that.

- It is not particularly realistic, since nobody has an infinite amount of time.
- Calculating expected outcomes is hard—the only sensible way of doing so is to
 - ▶ insist that all strategies have a finite definition;
 - ▶ calculate the **cycle** that two strategies playing against each other will eventually run into;
 - ▶ calculate the pay-off for the strategies over the cycle and thus an **average pay-off per round**.

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The closer w is to 1 the higher the probability that there will be another game, and the higher the difference between the expected pay-off from mutual defection *versus* that from mutual cooperation. People sometimes call this '**the shadow of the future**' to capture the idea that current behaviour is likely to influence future gains.

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There is another way of reading this, which seems to be preferred by economists: Rather than read w as a probability that there will be another game they tend to view it as a **discount factor**, the idea being that a pay-off in the future is worth less than one now.

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Proposition 5.2 *For each of the following properties there is at least one sub-game equilibrium point for the indefinitely repeated Prisoner's Dilemma game satisfying it. In the play resulting from employing these strategies against each other it is the case that*

- *they both cooperate throughout;*
- *they both defect throughout;*
- *they both defect in the first round and cooperate ever thereafter;*
- *they both cooperate in every odd round and defect in every even one;*
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This shows that indefinitely repeated Prisoner's Dilemma has a huge number of sub-game equilibrium points, which means that it is very hard to predict just what will happen in this game. Also note that TITFOR TAT is **not** a sub-game equilibrium point, but it is an equilibrium point in this game!

Summary of Section 5

- The **generalized Prisoner's Dilemma** has four pay-offs, $T > R > P > S$ with $R > (S + T)/2$. Its only equilibrium point is given by the strategy which always defects (for both players), and this is also the dominant strategy.

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- In two computer tournaments playing this game, the strategy known as **TitForTat** won twice. Successful strategies were **nice**, **forgiving** and **retaliatory**.

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- More realistic versions of this game are **indefinitely repeated**, where after each round a chance experiment determines whether a further round will be played. This will occur with a fixed probability.
- Prisoner's Dilemma situation can be readily identified **in the real world**.