CS3191 Section 4

Large Games

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Section 4 covers how computer programs for games such as Chess, Go, Othello, Checkers and similar games work.
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There are three problems which have to be solved to write such a program which we will discuss in some detail. Finally we will have a look at how Chess-playing programs developed, since Chess is the game for which the most effort has been made when it comes to writing programs.
The three problems

In order to write a game-playing program, the following problems have to be solved.
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**Board representation and move generation.** Clearly we have to think about how the board (and the pieces) are represented internally, and how the moves are to be generated. Typically, once this has been solved it can be left alone.
The three problems

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**Board representation and move generation.**

**Alpha-beta search.** Despite the fact that we cannot hope to employ the minimax algorithm with alpha-beta pruning, this technique still plays a vital role in game-playing programs. There are some variants that might be implemented, and typically some effort is spent on cataloguing search results.
The three problems

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Board representation and move generation.

Alpha-beta search.

Evaluation function. Since alpha-beta search cannot be carried out until a leaf is reached, the search stops instead at a pre-defined depth. To obtain a value for a position at this depth, a function has to be created which assigns one based entirely on the state of the board at the time. This is known as the ‘evaluation function’.
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The faster the program, the higher the depth to which it can carry out alpha-beta search (before it has to ‘guess’ a value for a position), and the better it will play. Hence speed is of the essence when writing such programs, and is a concern for all the components mentioned above.
Representing the board and related issues
Representing the board–array

In order to illustrate our thoughts, we often use Chess as an example. However, there’s no need to be familiar with the game beyond the rudiments.
Representing the board–array

Obvious representation of a Chess board: $8 \times 8$ array. Each field holds information about the piece that occupies the corresponding field on the board (if any).
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- if piece is a rook, bishop, pawn or queen, check whether the way to target is empty;
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Need:
- loop over all fields (to pick piece);
- loop over all possible target positions;
- loop to check for obstructions along the way.

Complicated, not fast.
Board representation – $0x88$

Faster: Assign a number to each square on the board given by one byte, four high bits: row; four low bits: column.
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<table>
<thead>
<tr>
<th>high bits</th>
<th>0000</th>
<th>0001</th>
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This is much faster than the first version. To check whether a number \( i \) is a valid position on the board, check whether it satisfies \( i \& 0x88 == 0 \) (\&: bitwise).
Board representation – $0x88$

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</table>

To move one field to the left or right, just subtract or add one.
To move up a row, add 16, to move down a row, subtract 16.
Board representation—$0x88$

<table>
<thead>
<tr>
<th>high bits</th>
<th>1</th>
<th>0000</th>
<th>0010</th>
<th>0100</th>
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<th>1100</th>
<th>1110</th>
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<th>low bits</th>
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Board: represented as an array with 128 entries, only 64 of which correspond to actual fields.
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Board representation–bitboards

Idea: for each colour and piece, use a ‘bitboard’.

The white pawns:

\[
\begin{array}{cccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Need: one 64-bit word for each piece. Operations: bit-wise—this is really fast!

Only disadvantage: the code becomes more complicated; turning a bitboard of possible moves into a list of possible moves, for example.

Advantages: fast; bitboards required more than once only have to be computed once; several moves can be generated at the same time.
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0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0
1 1 1 0 0 0 0 1
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
Board representation–bitboards

Idea: for each colour and piece, use a ‘bitboard’.

The white pawns:

![Chess board with white pawns]

0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 1 0 0
0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0
1 1 1 0 0 0 0 1
0 0 0 0 0 0 0 0

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```
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 1 0 0
0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0
1 1 1 0 0 0 0 1
0 0 0 0 0 0 0 0
```

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Example: bitboard for all black pieces: bit-wise ‘or’ of all bitboards for black pieces.
Board representation–bitboards

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The white pawns:

0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 1 0 0

Need: one 64-bit word for each piece. Operations: bit-wise—this is really fast!

Move of a piece by a row: shift the bitboard by 8.
Idea: for each colour and piece, use a ‘bitboard’.

The white pawns:

0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 1 0 0
0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0
1 1 1 0 0 0 0 1
0 0 0 0 0 0 0 0

Need: one 64-bit word for each piece. Operations: bit-wise—this is really fast!

Empty fields: bitboard for all pieces negated.
Board representation–bitboards

Idea: for each colour and piece, use a ‘bitboard’.

The white pawns:

All legal moves of pawns by one field can be stored in a bitboard (similarly for all legal moves of pawns by two fields). Constant bitboards can be prepared at compile time to be available in a library.
Board representation–bitboards

Idea: for each colour and piece, use a ‘bitboard’.

The white pawns:

```
0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 1 0 0
0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0
1 1 1 0 0 0 0 1
0 0 0 0 0 0 0 0
```

Pawn captures: shifting the bitboard by 7 or 9 and bit-wise ‘and’ it with the bitboard for pieces of the opposite colour.
Idea: for each colour and piece, use a ‘bitboard’.

The white pawns:

0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0
0 0 0 0 0 1 0 0
0 0 0 0 0 1 0 0
0 0 0 0 1 0 0 0
0 0 0 0 1 0 0 0
0 0 0 0 1 0 0 0
1 1 1 1 0 0 0 0
0 0 0 0 0 0 0 0

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 0 0 0 1 0 0 0 0
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Only **disadvantage**: the code becomes more complicated; turning a bitboard of possible moves into a list of possible moves, for example.

**Advantages**: fast; bitboards required more than once only have to be computed once; several moves can be generated at the same time.
Beyond board representation

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Chess programs typically use a large hash table to keep track of positions that have occurred in play.
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When we carry out an alpha-beta search from a given position, we will search to a given depth. When it is our turn again, we will repeat that from the now current position—but we have searched this position before, only to a depth of two less than we now require!
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A hash function frequently used consists of assigning to each pair, consisting of a piece and a field on the board, a large random number. The idea is that this number encodes the fact that the corresponding piece occupies the corresponding field. Then one sums up the appropriate numbers for the given position to obtain the hash key. A checksum process can be applied to make sure later that ‘the right’ position is looked up.
Doing and undoing moves

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This is best done by keeping a stack of moves with sufficient information to undo them. This is typically much cheaper than keeping a list of positions through which one has gone.
Evaluation function
The task

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This is entirely different for the evaluation function. In order to turn a game position into a meaningful number, the programmer must have considerable knowledge about the game.
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There are no hard and fast rules for what makes a good evaluation function; they are mostly based on heuristics.
When writing a game-playing program, speed is always an issue. Hence it pays to calculate the desired evaluation function in such a way to make the process as fast as possible.

When calculating the evaluation function for two successive positions, the value often does not change very much, and, in fact, the actual calculations are very similar. We can make this work in our favour if we can express the evaluation function in terms of the contributions made by the different pieces on their various fields.

Let $p$ be the current position, and $e$ the evaluation function. Then if $e(p) = e_{s_1}(s_1's place in p) + \ldots + e_{s_n}(s_n's place in p)$; where $s_1, \ldots, s_n$ are the pieces involved, the value of a new position resulting from one piece $s$ being moved is $s_{core}(move) = e_{s}(s's new field) - e_{s}(s's old field)$.

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It is important that an evaluation function judge any position from both players’ point of view. Having many pieces on the board does not give White any advantage if Black is about to checkmate him!
Relevant constituent parts

Material. The number and kind of pieces on the board. Chess: Each piece has a value; Go: count number of pieces on board, Othello: same.
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Not equally useful for all games: Othello: not number of pieces is important, but their locations (corners). Player with fewer pieces might have better position. There are other games where the number of pieces may be irrelevant.
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Chess: count number of fields threatened by one player; Othello: count number of pieces which cannot be taken by the opponent. Calculate size, possible with weights for very important fields.
Relevant constituent parts

**Material.** The number and kind of pieces on the board.

**Space.** Influence.

**Mobility.** Ability to move. Having many different available moves: advantageous, *e.g.* in Othello. Chess: not clear this is useful.
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**Tempo.** Initiative. Go: one player has the *initiative*, that is, he acts, other player reacts to his moves.
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Other games: try ‘parity argument’: often find positions where player who moves next wins/loses, can be simple to evaluate (see Nim, Connect-4).
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**Threats.** Can one of the players capture (or threaten to capture) a piece? Connect-4, Go-Moku: can a player win in the next move? Othello: is a player threatening to take a corner?

**Shape.** How pieces on the board relate to each other. Chess: line of pawns much stronger than other grouping. Go: shape is ‘territory to be’—stones outline territory which the player can defend when threatened. Judging shape: often very difficult. Change of shape value: incremental over time, long-term target. Evaluation function partially based on shape: can’t just simply add piece-based functions.

**Known Patterns.** Go: libraries of sequences of moves in small areas (joseki) — preserves balance between players. Chess: bishop capturing a pawn on border is often trapped; Othello: sacrifice one corners in exchange for another. Deciding when a pattern applies is hard!
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Know, e.g.: one rook less than two pawns and bishop, or two pawns and knight, but not less than one pawn and bishop/knight.

So: weight of a rook should be below weight of pawns and bishop, but above one pawn and bishop. Get fewer possibilities to try.
Deducing constraints.

Hand tweaking. Happens typically in practice. Programmers watch implementation play, judge which parameters to change and how. Perform the change and watch again. Reasonably fast but requires game-specific knowledge.
Fine-tuning

- Deducing constraints.
- Hand tweaking.
- Optimization techniques. Employ general optimization techniques.

Example: 'hillclimbing': Make small changes to parameters, keep them if they improve the performance. Need a measure to judge performance, for example the percentage of won games against some opponent. Often slow; risks being stuck when each small change makes performance worse, but big change might bring huge gains ('local optima'). Can be modified by randomly sticking with some changes which do not improve performance. 'Randomness' controlled by some probabilities (start out fairly high, become smaller as a good value is approached). Adjusted method is slower than original, but can get good values.

Learning:
- Early: Thought good Chess programs would mimic human reasoning with machine-based learning, most important aspect. That's not what happened! All world-class game-playing programs use other principles foremost.
- Examples for learning: genetic algorithms, neural networks. Both: rather slow; main advantage: do not require game-specific knowledge. Reason for slowness: number of test games required is typically very high (commercial game programmers tried about 3000 matches to allow the program to learn—the result was worse than hand tweaking). Further problem: If opponent is too good program loses all the time and never starts learning.
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Problem with playing program against versions of itself: same lines are explored over and over. To avoid this: start the program(s) from positions a few moves into a game.
Alpha-beta search
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There are some ways of fiddling with this to adjust it to the game in question. The thought is always to make it faster so that it can search deeper.
Iterative Deepening

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Obvious advantage: When time runs out we give the best move found so far, and that will at least be sensible. This is known as iterative deepening.
Modified alpha-beta search

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But if we use a hash table to keep track of results so far we can estimate a value.
Reminder: alpha-beta search

As the search reports back a value for the child of the current position we get

- successively increasing lower bounds for a max node ($\alpha$);
- successively decreasing upper bounds for a min node ($\beta$).
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If we find a value below $\alpha$: that part of the tree is irrelevant; return to the parent without adjusting $\alpha$ or $\beta$. 
Modified alpha-beta search

Get provisional value \( v \) from earlier searches. Decide that real value will be between \( \alpha \leq v \) and \( \beta \geq v \). Use this as start for our search (instead of \( -\infty \) and \( \infty \)).
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Search will report new value $w$ and the following cases may arise:

- **Value $w$ is between**...
  - If $w$ is between two values, this will be the correct value.

- **Value $w$ is larger than**...
  - If $w$ is larger than a certain value, it means our original upper bound was too low and our preliminary value was too pessimistic. We need to adjust our preliminary value to $w$ and consider allowing a larger range. This is known as "failing high".

- **Value $w$ below**...
  - If $w$ is below a certain value, it means our original lower bound was too high and our preliminary value was overly optimistic. We need to adjust our preliminary value to $w$, maybe allowing a larger range. This is known as "failing low".

This technique is known as aspiration search.
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- Value $w$ below $\alpha$. Means our original lower bound $\alpha$ was too high and preliminary value $v$ was overly optimistic. Have to adjust preliminary value $v$ to $w$, maybe allow a larger range. This is known as ‘failing low’.
Modified alpha-beta search

Search will report new value $w$ and the following cases may arise:

- Value $w$ is between $\alpha$ and $\beta$. This will be the correct value.
- Value $w$ is larger than $\beta$. Means our original upper bound $\beta$ was too low and $v$ too pessimistic. Have to adjust our preliminary value $v$ to $w$, and might consider allowing a larger range.

This is known as ‘failing high’.

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This is known as ‘failing low’.

This technique is known as aspiration search.
Benefits of aspiration search

In the best case: best move explored first, considered range contains correct value. Then:

\[ \text{Total size of the tree searched reduced to } (p^b d) \]

where \(b\): branching factor of the tree and \(d\): depth of search.

So might be able to search twice as deeply in the same time—in the best case.

This algorithm is implemented in most game-playing programs. Good idea to include the current values of \(p\) and \(d\) in the hash table of previously searched positions.
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Often it is sufficient to make sure the first few moves are the best candidates, because the others may be pruned. Algorithms like **HeapSort** or **SelectionSort** deliver sorted items one by one.

Can search the first move(s) with big window for potential value (see aspiration search), and later moves with smaller ones. This is known as **principal variation search**.
Not winning from winning positions

But:

Let's assume Black moves to d8. Then moving the king back to d6 again gives White a winning position.

But if Black now moves back to e8, we are back where we started and our program might go into a loop. This will lead to a draw since there are rules about repeating the same position.

Can avoid this by assigning slightly lower values to winning positions, for example 1000 number of moves req'd to get win:

Then alpha-beta search will work properly.
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If White moves the king to e6 (one field to the right) then he is still in a winning position, with Black’s only valid moves being to d8 and f8.
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The horizon effect

General problem: The computer cannot see anything which is beyond its horizon, that is, that happens below its search depth.

Solutions:

- Add knowledge so that program can detect when piece is trapped.
- Increase overall depth of search in such situations so that horizon is widened.
- Whenever piece is threatened, search to deeper level selectively.
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In order to avoid, say, the capture of one of its pieces the program may try pointless moves which merely postpone the inevitable—typically these moves do not progress the program’s play.

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Selective extension

Many games do not search to fixed depth everywhere. Instead the **select an appropriate depth**, which is greater whenever there is reason to believe that the current value for a position is inaccurate or when the current line of play is particularly important. For example, when currently set depth is reached search deeper for all moves which are likely to lead to change of evaluation considerably (Chess: capturing moves, check moves). This is known as **quiescent search**. Alternatively one might increase search depth whenever the currently explored line contains a capturing move. Can only do this in a limited way, or the program will keep looking deeper and deeper! Many programs search deeper on what they think is the best move (see **principal variation search**).
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Mistakes or weaknesses in a program can be explored over and over (until the creator finds a chance to fix this, since these programs don’t learn). Many tournaments between various programs seemed to be more about who could discover whose built-in faults, rather than whose program genuinely played best!
Chess-playing programs
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Shannon thought this would be a useful application for computers, and would give insights into how one makes intelligent decisions.
The first Chess programs

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1974: First world computer Chess championships. Repeated every three years.
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Search to **variable depth**, depending on whether the current position is judged to be ‘tricky’ or relatively straight-forward.
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The early, strong claims regarding the possibilities of AI turned to out to be vastly exaggerated. Today, Artificial Intelligence often is about search techniques and the machine learning is very different from human learning!
Further improvements

By the mid- to late eighties, the following had been achieved.

- Inclusion of large opening databases covering most approaches.
- Development of endgame databases; all five piece endgames were solved. Some of these solutions were genuinely new, in that people didn’t realize that one player could force a win in some of these. As a result, the official Chess rules were changed.
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Number of positions examined in three minutes, official ranking. (Note logarithmic scale along horizontal axis!) Where is perfect play?
Depth of search

To give some idea of how much strength a program gains by searching to a greater depth, here are the results of a program (called ‘Belle’) playing against copies of itself which searched to a different depth (late seventies).

<table>
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<tr>
<th>Rating</th>
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</table>
To give some idea of how much strength a program gains by searching to a greater depth, here are the results of a program (called ‘Belle’) playing against copies of itself which searched to a different depth (late seventies).

<table>
<thead>
<tr>
<th></th>
<th>4</th>
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Three or four levels more of search means outclassing one’s opponent!
Another way of measuring whether increasing the depth of search improves the program is to check whether search to a higher depth leads to a different move being chosen.
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<tr>
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<td>1796</td>
</tr>
<tr>
<td>7</td>
<td>29.5</td>
<td>2037</td>
</tr>
<tr>
<td>8</td>
<td>26.0</td>
<td>2249</td>
</tr>
<tr>
<td>9</td>
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## Hardware for Chess

The following table gives an overview over Chess-playing programs and the hardware they were running on.

<table>
<thead>
<tr>
<th>Name</th>
<th>Year</th>
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<tbody>
<tr>
<td>Ostrich</td>
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Chess-playing programs have done very little to improve our understanding of how humans think and make decisions.
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**Go**-playing programs currently are way below even good amateurs, let alone professionals.
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- **Alpha-beta search** assigns a value to a position by searching the game tree below it and eventually applying the evaluation function. Searching to greater depth will result in a better program, so any gain in speed goes into searching to a greater depth. There are many tricks to try to only search the relevant parts of the game tree; in particular ordering moves to search the most promising ones first.
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- Most effort so far has gone into creating Chess-playing programs. They have profited from faster hardware, and many improvements have been made which are very Chess-specific: better heuristics, opening and endgame libraries, and the like.