An Introduction to Description Logics: Techniques, Properties, and Applications

— Modularity —

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Thanks again to Thomas Schneider in Bremen for these slides

Plan for today

1. What is modularity good for?
2. Modules for reuse
3. Summary and Outlook

And now . . .

1. What is modularity good for?
2. Modules for reuse
3. Summary and Outlook

What can I do with my ontology?

Building and using an ontology often requires

- fast reasoning
  expressivity $\leftrightarrow$ complexity; optimisations, incremental reasoning
- collaborative development
- version control
- *efficient* reuse
- an understanding of the ontology's content and structure
  comprehension

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(M) = modularity helps

A priori vs. a posteriori modularisation

A priori (not covered today)

- At first, a modular structure is decided on.
- Then, the ontology is developed and used according to that structure.

A posteriori

- The ontology is regarded as a monolithic entity.
- At some point, a module is extracted
  or the ontology is decomposed into several modules.

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Comparing two ontologies

Assume that . . .

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- Well, compare $\{A \sqsubseteq B, \; B \sqsubseteq A\}$ vs. $\{A \equiv B\}$
- or $\{A \sqsubseteq B, \; B \sqsubseteq A \sqcup \neg A, \; A \sqcap \neg A \sqsubseteq B\}$ vs. $\{A \equiv B\}$

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Possible A: Number of entailments? Number of models?

Ontologies and their entailments

Think of axioms as **generating entailments** – e.g.:

$$\left.\begin{array}{l} A \sqsubseteq \exists r.B \\ \exists r.\top \sqsubseteq C \sqcap D \end{array}\right\} \models \quad A \sqsubseteq D$$

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A: 0? 1? 2? ... n? ... 2^n? ... ∞?

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$$A \sqsubseteq D \quad A \sqsubseteq D \sqcup A \quad A \sqsubseteq D \sqcup (A \sqcap D), \quad \ldots$$

Ontologies and their models

Think of axioms as **restricting possible models**

Axioms "filter out" unwanted models – e.g.:

- $\mathtt{Hand} \sqsubseteq \exists\, \mathtt{hasPart.Finger}$

  $\rightsquigarrow$ models cannot have instances of Hand with no hasPart-edge to an instance of Finger

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Next attempt at “more” entailments/models

We cannot compare *numbers* of entailments or models

But we can use set inclusion:

“$\mathcal{O}$ knows at most as much as $\mathcal{O}'$” if

- every entailment of $\mathcal{O}$ is one of $\mathcal{O}'$:
  $\{\eta \mid \mathcal{O} \models \eta\} \subseteq \{\eta \mid \mathcal{O}' \models \eta\}$ or
- every model of $\mathcal{O}'$ is one of $\mathcal{O}$:
  $\{\mathcal{I} \mid \mathcal{I} \models \mathcal{O}'\} \subseteq \{\mathcal{I} \mid \mathcal{I} \models \mathcal{O}\}$

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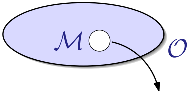
Problem:

How do we test these conditions?

Knowledge w.r.t. a signature



Let's reformulate the initial dialogue.

Assume that ...

- you want to buy **a subset of** a medical ontology $\mathcal{O}$ from me that covers the subdomain of, say, diseases
- I offer two subsets $\mathcal{M}_1$ and $\mathcal{M}_2$

Q: which one do you choose?

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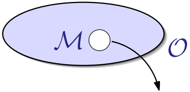

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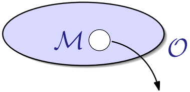

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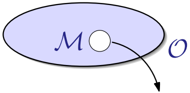

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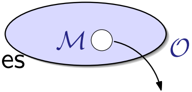
Possible A: the one that "knows more" about diseases!

Q: which is the **best subset** I can offer?

Possible A: a **module for diseases**

- $\mathcal{M} \subseteq \mathcal{O}$ that knows as much as $\mathcal{O}$ about diseases:
  $\mathcal{M}$ **indistinguishable** from $\mathcal{O}$ w.r.t. all terms relevant for diseases
- $\mathcal{M}$ as small as possible

Inseparability w.r.t. a signature



Definition

- **Signature** Σ = a set of concept/role names
- The signature of axiom (ontology) X
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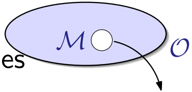

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  for all $\eta \in \mathcal{L}$ with $\text{sig}(\eta) \subseteq \Sigma$,

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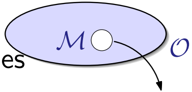

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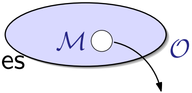

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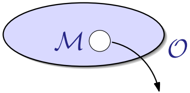
  Alternative names:
  - $\mathcal{M}$ **covers** $\mathcal{O}$ for Σ w.r.t. $\mathcal{L}$
  - $\mathcal{M}$ **is a module of** $\mathcal{O}$ for Σ w.r.t. $\mathcal{L}$

Choosing the signature Σ

Definition (repeated from previous slide)



$\mathcal{O}$ is a Σ-module of $\mathcal{M}$ w.r.t. $\mathcal{L}$
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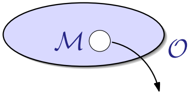
The signature Σ ...

- can be seen as a "topic"
- that the module is required to cover
- is difficult to formulate:

  Q: how many interesting entailments in $\Sigma = \{\texttt{Disease}\}$ can $\mathcal{O}$ possibly have?

Choosing the logic $\mathcal{L}$

Definition (repeated from previous slide)



$\mathcal{O}$ is a Σ-module of $\mathcal{M}$ w.r.t. $\mathcal{L}$
if $\mathcal{M} \subseteq \mathcal{O}$ and $\mathcal{M} \equiv^{\mathcal{L}}_{\Sigma} \mathcal{O}$

Choice of $\mathcal{L}$ depends on your usage of the module:

- for ontology design: subsumptions betw. (complex?) concepts
- for ontology usage: your favourite query language

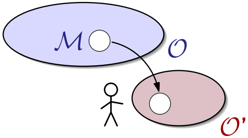
Modules for reuse

If we want to reuse module $\mathcal{M}$,

we need a stronger guarantee:

$$\mathcal{M} \cup \mathcal{O}' \equiv^{\mathcal{L}}_{\Sigma} \mathcal{O} \cup \mathcal{O}' \quad \text{for all } \mathcal{O}'$$

Q: is this reasonable to expect?



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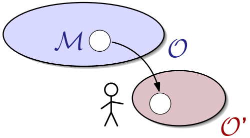

Q: is this reasonable to expect? **A:** no! Consider

$$\mathcal{O} = \{A \sqsubseteq B,\ A \sqsubseteq \exists r.C\} \quad \Sigma = \{A, r, C\} \quad \mathcal{O}' = \{B \sqsubseteq C\}$$

Then $\mathcal{M} = \{A \sqsubseteq \exists r.C\} \equiv^{\mathcal{ALC}}_{\Sigma} \mathcal{O}$,
but $\mathcal{M} \cup \mathcal{O}' \not\equiv^{\mathcal{ALC}}_{\Sigma} \mathcal{O} \cup \mathcal{O}'$,
because $\mathcal{O} \cup \mathcal{O}' \models A \sqsubseteq C$

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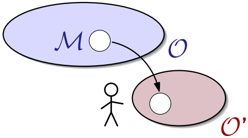


Q: is this reasonable to expect? **A:** no!

Solution:

Lemma

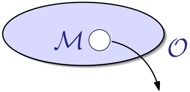
If $\mathcal{M} \equiv^{\mathcal{L}}_{\Sigma} \mathcal{O}$, then $\mathcal{M} \cup \mathcal{O}' \equiv^{\mathcal{L}}_{\Sigma} \mathcal{O} \cup \mathcal{O}'$, for

- every $\mathcal{O}'$ with $\text{sig}(\mathcal{O}) \cap \text{sig}(\mathcal{O}') \subseteq \Sigma$,
- expressive enough $\mathcal{L}$, e.g. $\mathcal{SROIQ}$ (OWL).

Consequence:

we can safely import $\mathcal{M}$ into any $\mathcal{O}'$ that reuses only terms from Σ

How is a minimal Σ-module extracted?



Simple module extraction algorithm:

- $\mathcal{M} \leftarrow \mathcal{O}$
- While $\mathcal{M} \setminus \{\alpha\} \equiv^{\mathcal{L}}_{\Sigma} \mathcal{O}$, for some $\alpha \in \mathcal{M}$,
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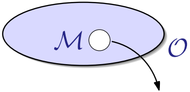

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Observation:

Different orders of choosing α
can lead to different minimal modules

Example

Let $\Sigma = \{\text{Knee}, \text{HingeJoint}\}$. Suppose *Galen* contains:

$$
\begin{aligned}
\text{Knee} &\equiv \text{Joint} \sqcap \exists \text{hasPart.Patella} \sqcap && (1)\\
&\quad \exists \text{hasFunct.Hinge} && \\
\text{Patella} &\sqsubseteq \text{Bone} \sqcap \text{Sesamoid} && (2)\\
\text{Ginglymus} &\equiv \text{Joint} \sqcap \exists \text{hasFunct.Hinge} && (3)\\
\text{Joint} \sqcap \exists \text{hasPart.}(\text{Bone} \sqcap \text{Sesamoid}) &\sqsubseteq \text{Ginglymus} && (4)\\
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\text{Meniscus} &\equiv \text{FibroCartilage} \sqcap \exists \text{locatedIn.Knee} && (6)
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$\subseteq$-Minimal module for Σ ?

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$$\text{Patella} \sqsubseteq \text{Bone} \sqcap \text{Sesamoid} \tag{2}$$

$$\text{Joint} \sqcap \exists \text{hasPart.}(\text{Bone} \sqcap \text{Sesamoid}) \sqsubseteq \text{Ginglymus} \tag{4}$$

$$\text{Ginglymus} \equiv \text{HingeJoint} \tag{5}$$

$\subseteq$-Minimal module for Σ? $\{(1), (2), (4), (5)\}$

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$\subseteq$-Minimal module for Σ? $\{(1), (2), (4), (5)\}$ and $\{(1), (3), (5)\}$

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$\subseteq$-Minimal module for Σ? $\{(1),(2),(4),(5)\}$ and $\{(1),(3),(5)\}$

Note that a module for Σ does not necessarily contain

- all axioms that use terms from Σ
- only axioms that only use terms from Σ

Bad news for expressive ontology languages?

Big, sad theorem

Let $\mathcal{O}_1, \mathcal{O}_2$ be ontologies in $\mathcal{L}$ and Σ a signature.

Determining whether $\mathcal{O}_1 \equiv^{\mathcal{L}}_{\Sigma} \mathcal{O}_2$ is

ExpTime-complete	for	$\mathcal{L} = \mathcal{EL}$
2ExpTime-complete	for	$\mathcal{ALC} \leqslant \mathcal{L} \leqslant \mathcal{ALCQI}$, and
undecidable	for	$\mathcal{L} \geqslant \boldsymbol{\mathcal{ALCQO}}$, including OWL

(even if $\mathcal{O}_1, \mathcal{O}_2$ are in $\mathcal{ALC}$).

Consequences for modules of expressive DLs

Extracting modules is highly complex for expressive DLs.

What to do?

1. Give up? No: modules clearly too important
2. Reduce expressivity of logic? Yes! (Not covered here.)
3. Approximate for expressive logics? Yes – but from the *right* direction!

Next: 2 approximations, i.e., sufficient conditions for inseparability

1. based on semantic locality
2. based on syntactic locality

Model-theoretic inseparability

Remember: $\mathcal{O}_1 \equiv_\Sigma^\mathcal{L} \mathcal{O}_2$ if:

for all $\eta \in \mathcal{L}$ with $\text{sig}(\eta) \subseteq \Sigma$,

$$\mathcal{O}_1 \models \eta \quad \text{iff} \quad \mathcal{O}_2 \models \eta$$

Good news: $\Uparrow$

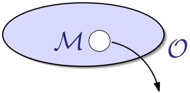
$$\{\mathcal{I}|_\Sigma \mid \mathcal{I} \models \mathcal{O}_1\} = \{\mathcal{I}|_\Sigma \mid \mathcal{I} \models \mathcal{O}_2\}$$

- i.e., $\mathcal{O}_1$ and $\mathcal{O}_2$ have the same models modulo Σ ($\mathcal{I}|_\Sigma$ is the restriction of $\mathcal{I}$ to Σ)
- shorthand: $\mathcal{O}_1 \equiv_\Sigma^{\text{sem}} \mathcal{O}_2$ (model-inseparable)
- this notion does not depend on $\mathcal{L}$

Bad news: $\mathcal{O}_1 \equiv_\Sigma^{\text{sem}} \mathcal{O}_2$ is undecidable already for $\mathcal{ALC}$!

Semantic locality

We can approximate model-inseparability, exploiting that $\mathcal{M}$ is a subset of $\mathcal{O}$



$$\mathcal{M} \equiv_{\Sigma}^{\mathsf{sem}} \mathcal{O}$$

$$\Updownarrow$$

every $\mathcal{I} \models \mathcal{M}$ can be extended to $\mathcal{J} \models \mathcal{O}$ with $\mathcal{I}|_\Sigma = \mathcal{J}|_\Sigma$
and $\forall X \notin \Sigma : X^{\mathcal{J}} = \emptyset$
(every $\mathcal{I} \models \mathcal{O}$ is a model of $\mathcal{M}$ since $\mathcal{M} \subseteq \mathcal{O}$)

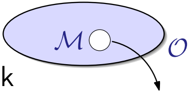
$$\Uparrow$$

every $\mathcal{I} \models \mathcal{M}$ can be extended to $\mathcal{J} \models \mathcal{O}$ with $\mathcal{I}|_\Sigma = \mathcal{J}|_\Sigma$

$$\Updownarrow$$

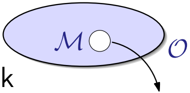
every $\alpha \in \mathcal{O} \setminus \mathcal{M}$ is **semantically local w.r.t. $\Sigma \cup \mathsf{sig}(\mathcal{M})$**:
α, with all terms not in $\Sigma \cup \mathsf{sig}(\mathcal{M})$ replaced by $\bot$, is a tautology

From semantic to syntactic locality



- Semantic locality involves tautology check
  - $\leadsto$ can be tested using a reasoner
  - $\leadsto$ has the same complexity as standard reasoning

From semantic to syntactic locality



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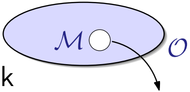
- A syntactic approximation that can be tested in poly-time: **syntactic locality**

  (describes "obviously" sem. local axioms via a grammar)

From semantic to syntactic locality



- Semantic locality involves tautology check
  $\rightsquigarrow$ can be tested using a reasoner
  $\rightsquigarrow$ has the same complexity as standard reasoning

- A syntactic approximation that can be tested in poly-time:
  syntactic locality

  (describes "obviously" sem. local axioms via a grammar)

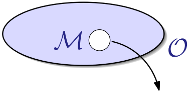
- Both notions lead to modules that are
  - $(\Sigma \cup \mathsf{sig}(\mathcal{M}))$-inseparable from $\mathcal{O}$
  - not necessarily minimal

Examples of syntactically (non)-local axioms

$\overline{B} \sqsubseteq A$	form $C \sqsubseteq C^{\emptyset} \leadsto$ not $\{\overline{B}, \ldots\}$-local
$A \sqsubseteq \overline{B} \sqcap \exists r.\overline{C}$	form $C^{\emptyset} \sqsubseteq C \leadsto \{\overline{B}, \overline{C}\}$-local
$X \sqcap A \sqsubseteq Y$	is Σ-local if, e.g., $A \notin \Sigma$
$\overline{B} \sqcap \exists r.\overline{C} \sqsubseteq A$	is $\{\overline{B}, \overline{C}\}$-local
$\overline{A} \sqsubseteq \overline{A} \sqcup \overline{B}$	is not $\{\overline{A}, \overline{B}\}$-local, yet a tautology!

Module extraction

Module extraction algorithm:



- $\mathcal{M} \leftarrow \emptyset$
- While α not local w.r.t. $\Sigma \cup \text{sig}(\mathcal{M})$, for some $\alpha \in \mathcal{O} \setminus \mathcal{M}$,
  do $\mathcal{M} \leftarrow \mathcal{M} \cup \{\alpha\}$
- Output $\mathcal{M}$

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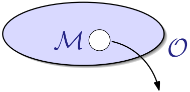

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Variations:

- this notion: (semantic/syntactic) $\bot$-module
- dual notion: (semantic/syntactic) $\top$-module
- smaller modules by nesting $\top$- and $\bot$-module extraction: $\top\bot$*-modules

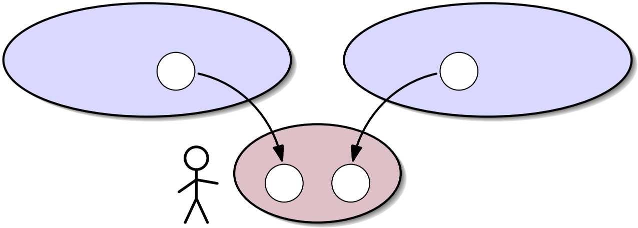
And now . . .

Summary

- Inseparability/coverage is a guarantee relevant (not only) for reuse
- Approximation of minimal covering modules via locality
- Modules based on syntactic locality can be extracted efficiently in logics up to $\mathcal{SROIQ}$ (OWL 2)
- Tool support for extracting modules:
  http://owl.cs.manchester.ac.uk/modularity
  http://owlapi.sourceforge.net/
- This line of research is rather new for DLs and ontology languages, and many questions are (half)open.

An import/reuse scenario

"Borrow" knowledge from external ontologies

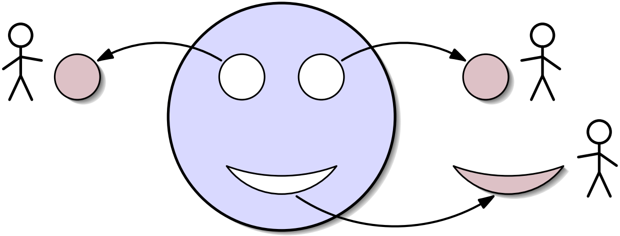


- Provides access to well-established knowledge
- Doesn't require expertise in external disciplines

This scenario is well-understood and implemented.

A collaboration scenario

Collaborative ontology development

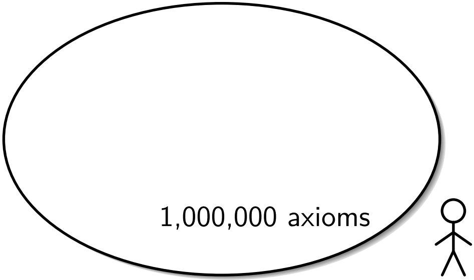


- Developers work (edit, classify) locally
- Extra care at re-combination
- Prescriptive/analytic behaviour

This approach is mostly understood, but not implemented yet.

Understanding and/or structuring an ontology

Compute the modular structure of an ontology



Understanding and/or structuring an ontology

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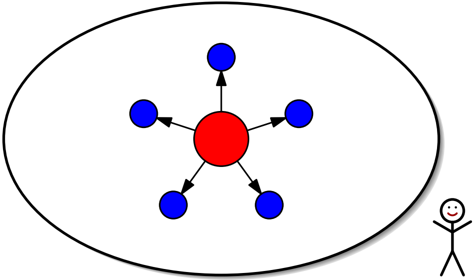

This is work in progress.

See also . . .

. . . slides from ESSLLI 2011 course “Modularity in Ontologies”

http://www.informatik.uni-bremen.de/~ts/teaching

. . . references . . .