An Introduction to Description Logics: Techniques, Properties, and Applications

— Day 1, Part 1 —

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Welcome!

Thanks for having me!

I hope we have good time and learn a lot.

Thanks to **Thomas Schneider**: I am recycling some of our slides from an ESSLLI course.

Welcome!

Let me know if you

... have questions. – **Do ask** them at any time.

... have difficulties understanding us/reading our writing/...

In this course, we'll

... ask you to **think** a lot

... ask you to **work** through numerous examples

... talk about **complex** stuff with many **interesting** facets!

What's in this course?

1. Introduction
   - The basic DL $\mathcal{ALC}$, reasoning problems
   - Relation with other logics, ontologies, examples and exercises
2. DLs, ontologies, and OWL: applications and tools
3. Core reasoning via a tableau algorithm
4. Complexity of selected DLs
   - upper bounds, lower bounds, undecidability
   - a polynomial DL: $\mathcal{EL}$
5. Reasoner optimisation and behaviour
   - sources of complexity
   - classification versus consistency
   - performance hetero/homogeneity
6. Understanding entailments
   - Justifications and more

Plan for today

1. DL basics
2. Relationship with other logics
3. Ontologies
4. OWL and DLs

DLs: the core

Core part of a DL: its **concept language**, e.g.:

$$\texttt{Animal} \sqcap \exists\, \texttt{hasPart.Feather}$$

describes all animals that are related via "`hasPart`" to a feather.

Syntactic ingredients of a concept language:

- **Concept names stand for sets of elements**, e.g., `Animal`
- **Role names** stand for binary relations between elements, e.g., `hasPart`
- **Constructors** to build **concept expressions**, e.g., $\sqcap$, $\exists$

Syntax and semantics of $\mathcal{ALC}$

Semantics given by means of an **interpretation** $\mathcal{I} = (\Delta^\mathcal{I}, \cdot^\mathcal{I})$, where

- $\Delta^\mathcal{I}$ is a nonempty set (the **domain**), and
- $\cdot^\mathcal{I}$ is a mapping (the **interpretation function**) as follows:

Constructor	Syntax	Example	Semantics
concept name	A	`Human`	$A^\mathcal{I} \subseteq \Delta^\mathcal{I}$
role name	r	`likes`	$r^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$

For C, D concepts and R a role name:

conjunction	$C \sqcap D$	`Human` $\sqcap$ `Male`	$C^\mathcal{I} \cap D^\mathcal{I}$
disjunction	$C \sqcup D$	`Nice` $\sqcup$ `Rich`	$C^\mathcal{I} \cup D^\mathcal{I}$
negation	$\neg C$	$\neg$`Meat`	$\Delta^\mathcal{I} \setminus C^\mathcal{I}$
restrictions:			
existential	$\exists r.C$	$\exists$`hasChild.Human`	$\{x \mid \exists y.(x,y) \in r^\mathcal{I} \wedge y \in C^\mathcal{I}\}$
value	$\forall r.C$	$\forall$`hasChild.Blond`	$\{x \mid \forall y.(x,y) \in r^\mathcal{I} \Rightarrow y \in C^\mathcal{I}\}$

Understanding syntax and semantics of $\mathcal{ALC}$

We can "draw" interpretations ...

(similarly to Kripke models if you happen to know modal logic)

Exercise 1: Formulate $\mathcal{ALC}$ concepts that describe

1. happy pet owners
2. unhappy pet owners who own an old cat
3. pet owners who own a cat, a dog, and only cats and dogs
4. pet owners who own a cat, a dog, and no other animals
5. everything (abbreviated by $\top$ with $\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$)
6. nothing (abbreviated by $\bot$ with $\bot^{\mathcal{I}} = \emptyset^{\mathcal{I}}$)

For each of your concepts (1)–(4),
"draw" an interpretation with an instance of that concept.

Basic reasoning problems in $\mathcal{ALC}$

Definition: let C, D be $\mathcal{ALC}$ concepts. We say that

- $e \in C^{\mathcal{I}}$ is **an instance of** C in $\mathcal{I}$.
- C is **satisfiable** if there is an interpretation $\mathcal{I}$ with $C^{\mathcal{I}} \neq \emptyset$.
- C is **subsumed by** D (written $\emptyset \models C \sqsubseteq D$) if:
  for every interpretation $\mathcal{I}$, we have that $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$.

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Exercise 2: Which of the following concepts is satisfiable?
Which is subsumed by which?

(1) $\exists r.(A \sqcap B)$ (2) $\exists r.(A \sqcup B)$
(3) $\forall r.(A \sqcap B)$ (4) $\exists r.(A \sqcap \neg A)$

(5) $\exists r.A \sqcap \forall r.B$ (6) $\exists r.A$
(7) $\exists r.A \sqcap \forall r.\neg A$ (8) $\exists r.A \sqcap \forall s.\neg A$

The TBox

Definition

- A **general concept inclusion (GCI)** has the form $C \sqsubseteq D$, for C, D (possibly complex) concepts
- A **general TBox** is a finite set of GCIs: $\mathcal{T} = \{C_i \sqsubseteq D_i \mid 1 \leqslant i \leqslant n\}$
- $\mathcal{I}$ **satisfies** $C \sqsubseteq D$ if $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ (written $\mathcal{I} \models C \sqsubseteq D$)
- $\mathcal{I}$ is a **model of TBox** $\mathcal{T}$ if $\mathcal{I}$ satisfies every $C_i \sqsubseteq D_i \in \mathcal{T}$
- We use $C \equiv D$ to abbreviate $C \sqsubseteq D$, $D \sqsubseteq C$

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Example:

$$\begin{aligned} \{ \quad \texttt{Father} &\equiv \texttt{Man} \sqcap \exists\, \texttt{hasChild.Human}\,, \\ \texttt{Human} &\equiv \texttt{Mammal} \sqcap \forall\, \texttt{hasParent.Human}\,, \\ \exists\, \texttt{favourite.Brewery} &\sqsubseteq \exists\, \texttt{drinks.Beer} \quad \} \end{aligned}$$

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Exercise 3: Draw a model of the above TBox.
Draw an interpretation that is **not** a model of it.

Reasoning problems with respect to a TBox

Definition: let C, D be concepts, $\mathcal{T}$ a TBox. We say that

- C is **satisfiable w.r.t. $\mathcal{T}$**
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Example:

$$\mathcal{T} = \{ \quad A \sqsubseteq B \sqcap \exists r.C \,, \quad \exists r.\top \sqsubseteq \neg A \quad \}$$

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Example:

$$\mathcal{T} = \{ \begin{array}{rcl} A & \sqsubseteq & B \sqcap \exists r.C, \\ \exists r.\top & \sqsubseteq & \neg A \end{array} \}$$

Exercise 4: Does $\mathcal{T}$ have a model?
Are all concept names in $\mathcal{T}$ satisfiable?
Any subsumptions that you can point out?

How many models does a TBox have?

The ABox

TBox
- captures knowledge on a general, conceptual level
- contains concept def.s + general axioms about concepts

ABox
- captures knowledge on an individual level
- is a finite set of
  - **concept assertions** $a : C$ e.g., $\texttt{John} : \texttt{Man}$, and
  - **role assertions** $(a, b) : r$ e.g., $(\texttt{John}, \texttt{Mary}) : \texttt{hasChild}$

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Semantics: an interpretation $\mathcal{I}$

- maps each **individual name** e to some $e^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
- satisfies a concept assertion $a : C$ if $a^{\mathcal{I}} \in C^{\mathcal{I}}$
- satisfies a role assertion $(a, b) : r$ if $(a^{\mathcal{I}}, b^{\mathcal{I}}) \in r^{\mathcal{I}}$
- is a **model** of an ABox $\mathcal{A}$ if $\mathcal{I}$ satisfies each assertion in $\mathcal{A}$

$a : C$ is **entailed by** $\mathcal{A}$ if every model of $\mathcal{A}$ satisfies $a : C$

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repeated from previous slide

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Example:

$$\mathcal{A} = \{ \quad a : (B \sqcap \exists r.C), \quad b : (A \sqcap \neg P \sqcap \forall s.\forall r.F), \quad (b, a) : s \}$$

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Exercise 5: Does $\mathcal{A}$ have a model? – Describe some of them. Can you see any entailments?

(Later) Can you translate this into FOL? ML?

Ontologies: TBox and ABox

Definition: an **ontology** consists of

- a TBox that captures knowledge on a general, conceptual level
- an ABox that captures knowledge on an individual level and **uses terms described in the TBox**

Notation: $(\mathcal{T}, \mathcal{A})$ or $\mathcal{T} \cup \mathcal{A}$ – no difference!

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Notation: $(\mathcal{T}, \mathcal{A})$ or $\mathcal{T} \cup \mathcal{A}$ – no difference!

Semantics:

- Int. $\mathcal{I}$ is a **model** of $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ (written $\mathcal{I} \models \mathcal{O}$) if $\mathcal{I}$ satisfies each assertion and axiom in $\mathcal{O}$

  alternatively: $\mathcal{I} \models \mathcal{T}$ and $\mathcal{I} \models \mathcal{A}$
- $\mathcal{O}$ is **consistent** if it has a model
- $\mathcal{O}$ is **coherent** if each conc. name A in $\mathcal{O}$ is satisfiable w.r.t. $\mathcal{O}$
- $C \sqsubseteq D$ is **entailed by** $\mathcal{O}$ if every model of $\mathcal{O}$ satisfies $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
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Ontologies: TBox and ABox

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Exercise 6: Does $\mathcal{O}$ have a model? – Describe some of them.
Can you see any entailments?
What about $\mathcal{O} \cup \{b : C\}$ or $\mathcal{O} \cup \{b : A\}$?

Ontologies: TBox and ABox

Semantics:

repeated from previous slide

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Lemma

$C \sqsubseteq D$ is entailed by $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ iff $C \sqsubseteq D$ is entailed by $\mathcal{T}$.

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Lemma

$C \sqsubseteq D$ is entailed by $\mathcal{O} = (\mathcal{T}, \mathcal{A})$ iff $C \sqsubseteq D$ is entailed by $\mathcal{T}$.

Proof: for "$\Leftarrow$", note that every model of $\mathcal{O}$ is one of $\mathcal{T}$.

For "$\Rightarrow$", use contraposition; distinguish between $\mathcal{O}$ being inconsistent (trivial) and consistent (combine a model witnessing $\mathcal{T} \not\models C \sqsubseteq D$ and one of $\mathcal{O}$ to one witnessing $\mathcal{O} \not\models C \sqsubseteq D$). □

. . .

Let's switch to next slide set for **Ontologies and other logics**. . .