Cooperating Proof Attempts in Vampire

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Outline

- Motivation
- Interleaving
- AVATAR
- Cooperation via AVATAR
- Experiment
- Conclusions
Simple Idea

• Very simple idea:
  Run more than one proof attempt, have them cooperate

• Lots of previous work
  • Strategy selection in Gandelf with clause reuse
  • Parallel proving with clause sharing in DISCOUNT
  • ...

• But these lacked a good vehicle for cooperation

• This work is about cooperation between **concurrently** running proof attempts
  
  ...but supporting **parallelism** is a goal

• We didn’t use these ideas in this year’s CASC competition

• Firstly, why multiple proof attempts?
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Vampire Strategies

• In CASC 2015 we tried 351 unique strategies

• What do they use?
  • 303 use saturation (128 dis, 128 lrs, 57 ott), 32 instgen, 6 fmb
  • 231 use AVATAR
  • On average vary 13 options, the longest varies 25
  • Time limits: shortest 0.1s, longest 600s, mean 16.1 with sdev 42.4, median 4.3

• What do they solve?
  • 933 solutions, 372 use 1 strategy (561 use more)
  • Mean 3.9 with sdev 5.6, median 2, max 53
  • 152 unique strats (prove mean 6.1 sdev 13, median 2, max 91)

• Observations
  • Very short strategies are useful
  • Lots of complementary strategies are required
Vampire Strategies

- In CASC 2015 we found solutions with 152 unique strategies
- What do they use?
  - 133 use saturation (61 dis, 44 lrs, 28 ott), 13 instgen, 6 fmb
  - 105 use AVATAR
  - On average vary 12 options, the longest varies 25
  - Time limits: shortest 0.1s, longest 600s, mean 26.4 with sdev 61.4, median 5.6
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Observations
- Very short strategies are useful
- Lots of complementary strategies are required
This talk

• This works focuses on organising the cooperation of multiple Vampire proof attempts employing different strategies

• In this setting we consider two techniques for ‘cooperation’
  1. Interleaving of proof attempts to find the short proofs from a single strategy faster
  2. Sharing splitting decisions to prevent a proof attempt from exploring parts of the search space shown not to contain a proof by another proof attempt
Running multiple Proof Attempts...

- ... at the same time required us to rewrite quite a bit of Vampire... and introduce an input format for specifying multiple strategies

- Long-term plans to allow proof attempts to run in parallel but currently their execution is interleaved
Interleaving Strategies

- Generally if a strategy finds a proof it finds it quickly
- By interleaving strategies we can find the quick proofs faster
Experiment with just Interleaving

Number of solved problems vs. seconds

- Sequential
- Pseudo-concurrent
Scheduling

- Lots of variables to play with - still an area of experimentation

- An obvious variable is *granularity* of interleaving
  - Too small and we get bad memory issues
  - Too big and we don’t get the benefit we want

- Other ideas
  - Changing priorities
  - Resource limiting
  - Online learning of ‘good’ kinds of proof attempts
  - Offline identification of complementary strategies
Proof Search by Saturation

- Vampire is a saturation based prover
  - Saturate (up to redundancy) an input set of clauses $C$ with respect to a set of inferences $I$
    - Pragmatically this involves a growing search space from which clauses are selected and have inferences applied to generate new clauses.
    - If we derive false then $C$ was unsatisfiable.
    - If we saturate (and $I$ was complete) then $C$ was satisfiable
Splitting

- The search space can become full of long and heavy clauses
- A solution is splitting
  - For variable disjoint clauses $C_1$ and $C_2$
  - $S \cup (C_1 \lor C_2)$ is unsat iff both $S \cup C_1$ and $S \cup C_2$ are
  - Consider $S \cup C_1$ and $S \cup C_2$ separately

- For each clause we assert each non-splittable component in turn until all have been refuted or one branch is saturated without refutation
The AVATAR Approach

- The idea: represent the splitting decisions as a SAT problem

- To do this
  1. Name each clause component with a SAT variable
  2. Pass the corresponding SAT clause to a SAT solver
  3. Ask for a model and use this to make splitting decisions
  4. Carry around these assumptions in the first-order part
  5. On a refutation with assumptions, add these refuted assumptions to the SAT solver and recompute the model
AVATAR Architecture

FO prover

assert($C \leftarrow [C]$)
reinsert($D \leftarrow A$)
remove($D \leftarrow A$)

allProcessed
new($C_1 \lor \ldots \lor C_n \leftarrow [C'_1] \land \ldots \land [C'_m]$)
contradict($\bot \leftarrow [C_1] \land \ldots \land [C_m]$)

Splitting Interface

variant index
component records
current model

Solve

$[C_1] \lor \ldots \lor [C_n] \lor \neg[C'_1] \lor \ldots \lor \neg[C'_m]$ (split clause)
$\neg[C_1] \lor \ldots \lor \neg[C_m]$ (contradiction clause)

model

Unsatisfiable

SAT solver
Communicating Splitting Decisions

• Idea: if one proof attempt shows a part of the splitting space to be inconsistent then another proof attempt doesn’t need to explore it

• Very easy to share such splitting decisions via AVATAR - just share the SAT solver

• Has the effect of allowing proof attempts to explore the search space much faster
Exploring the Search Space Together

- Proof attempt 1 shows that assuming a component of a clause leads to contradiction
- Proof attempt 2 can ignore any splitting branch containing this component
Motivation Interleaving AVATAR Cooperation via AVATAR Experiment Conclusions

Shared AVATAR Architecture

Proof attempt 1 \[\uparrow\text{splitting decisions}\] \[\downarrow\text{new clauses, contradictions}\] \[\cdots\]

Splitting Interface

\begin{align*}
\text{variant index, component records, individual models}
\end{align*}

Interpretation or \textit{Unsatisfiable} \[\uparrow\text{split and contradiction clauses}\]

Proof attempt \textit{n} \[\uparrow\text{splitting decisions}\] \[\downarrow\text{new clauses, contradictions}\]

SAT solver
Shared AVATAR Architecture

Proof attempt 1

splitting decisions

new clauses, contradictions

Proof attempt n

splitting decisions

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new clauses, contradictions

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splitting decisions

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Splitting Interface

variant index, component records, individual models

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Proof attempt 1

\[\text{splitting decisions} \quad \downarrow \quad \text{new clauses, contradictions}\]

\[\uparrow\]

\text{Splitting Interface}

\[\text{variant index, component records, individual models}\]

Interpretation or \textit{Unsatisfiable}

\[\downarrow\]

\[\text{split and contradiction clauses}\]

SAT solver

\[\uparrow\]

Proof attempt \(n\)

\[\text{splitting decisions} \quad \downarrow \quad \text{new clauses, contradictions}\]
Experiment

- We took
  - 1747 very hard first-order problems from TPTP
  - 30 random ‘sensible’ strategies
- And ran
  - Each strategy independently for 10 seconds
  - All 30 together with a per-strategy 10 second time limit
- We found
  - Problems were solved on average 1.53 times faster, in some cases it was much higher than this
  - Sharing splitting decisions led to 63 more problems being solved, often quickly. It also led to previously unsolved problems being solved - this is significant.
  - However some problems were lost. There are two explanations
    - SAT solver overhead goes up 20%
    - Loss of memory locality
Experiment

The graph shows the number of solved problems over time for two different methods:
- **Sequential** (dashed line)
- **Pseudo-concurrent** (solid line)
- **Difference** (dotted line)

The x-axis represents time in seconds, ranging from 0 to 300, while the y-axis represents the number of solved problems, ranging from 0 to 400.
Replacing the SAT solver with a SMT solver

- A big advantage of this architecture is that we can replace the SAT solver with a SMT solver and only search models that satisfy some set of theories.

- This only requires ground components to be passed directly instead of being represented by a SAT variable.

- We are currently experimenting with incorporating Z3 for this purpose and the results are encouraging.
Conclusions

• A very promising direction to prove more problems and prove them faster

• Plugging in a SMT solver will make this approach highly applicable to problems with quantifiers and theories

• Still lots of ways we can extend the architecture i.e. cooperating via other data structures

• Some engineering problems still to solve

Thank you for listening