We know (nearly) nothing!

But can we learn?

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Abstract

The greatest source of progress in automated theorem proving in the last 30 years has been the development of better search heuristics, usually based on developer experience and empirical evaluation, but increasingly also using automated optimization techniques. Despite this progress, we still know very little about proof search. We have mostly failed to identify good features for characterizing homogeneous problem classes, or for identifying interesting and relevant clauses and formulas.

I propose the challenge of bringing together inductive techniques (generalization and learning) and deductive techniques to attack this problem. Hardware and software have finally evolved to a point that we can reasonably represent and analyze large proof searches and search decisions, and where we can hope to achieve order-of-magnitude improvements in the efficiency of the proof search.

1 Introduction

During the last 30 years, automated theorem provers for first-order logic have reached relative maturity, with some progress in refining calculi, and significant progress towards more robust and efficient implementations. However, the greatest source of progress in automated theorem proving has been the development of better search heuristics. As an example, our recent paper [15] shows a 60% improvement in the number of proofs found for current clause selection heuristics compared to the simple heuristics that were state-of-the-art 30 years ago.

Most of the leading modern theorem provers are based on variants of the superposition calculus with optional negative literal selection [1]. This includes e.g. SPASS [17], Vampire [8, 5], Prover9 [7] and E [11, 12]. These provers are based on saturation, using variants of the given-clause algorithm. The proof by contradiction is attempted by systematically deriving new clauses, using redundancy elimination (in particular rewriting and subsumption) to reduce the search space. The main heuristic choice points are selection of a good term ordering for a given proof task, selection of inference literals, or of a selection strategy, and selection of the *given clause*, i.e. the clause that is one of the premises of all generating inferences in a given iteration of the main loop.

Heuristics for these choice points have historically been developed based on developer experience and hunches, backed by, often extensive, experimental evaluation. However, increasingly automated or semi-automated mechanisms come into play. Clause selection heuristics for E have been created using hill-climbing methods [16] and genetic algorithms [9]. Vampire is using look-ahead to find good literal selection schemes [4].

2 Challenge: Using induction to guide deduction

We live in one of two realities. In the one reality, proof search in first-order logic is inherently unpredictable, even for classes of proof problems of particular interest to users. In the alternative reality, it is possible, at least in theory, to find strategies that are more likely than others to succeed on defined classes of problems.

There are several reasons to believe in the second case. The success of manual and semi-automatic tuning hints that proof search is not entirely unpredictable. There is some existing work to obtain search control knowledge automatically from examples of successful proofs, using patterns [2, 13, 3, 10] or neural networks [14, 6], with moderate success.

However, we believe that we have now reached a level of robustness and efficiency of the software and capability of available hardware that such attempts should yield significantly greater rewards than in the past. There are a number of interesting research questions, including the following:

- How do we represent search control experiences? What are relevant features of the proof state, of formulas, clauses, terms, and inferences?
- What automated learning methods are likely to be useful? Deep Learning is attractive because it has already tackled many problems that have been hard to formalize. However, it requires significant hardware, and even then is quite slow. Also, it is non-trivial to understand and communicate knowledge from such highly distributed representations.
- Can we represent clauses, formula, proof states, proof problems by relatively small sets of simple features while maintaining the information necessary for proof guidance? If yes, how?
- Can we automatically cluster proof problems in a way that problems in a given cluster have similar search properties? If yes, can we characterize these clusters in a way that allows us to assign problems to them a-priori?
- Can we dynamically detect if any given proof attempt is unlikely to succeed and switch to a different strategy? Can we extract useful intermediate results from failed proof attempts and reuse them, maybe in a way some propositional CDCL solvers keep learned lemmata during a restart?

3 Conclusion

Anyone who has seriously studied mathematics will have made the experience that there are at least two levels of learning to reason in a given domain. On the one hand, one can consciously learn and understand the definitions, theorems, and proofs. But, as is obvious from observing first-year students during open-book exams, this knowledge is not sufficient to effectively perform mathematics. There is a second, mostly subconscious level of learning to perform the right steps at the right place during reasoning, of using the right piece of knowledge in a constructive way.

I believe that to achieve human-level performance on hard problems, theorem provers likewise must be equipped with *soft* knowledge, in particular soft knowledge automatically gained from previous proof experiences. I also suspect that this will be one of the most fruitful areas of research in automated theorem proving. And one of the hardest.

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