COMP20121 The Implementation and Power of Computer Languages

‘Power’ Part

http://www.cs.man.ac.uk/~petera/2121/index.html .

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LECTURE EIGHT

Section 3: Turing machines (ctd)

Informal description

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- Go to the right until you find a blank. (This is the end of the input string.)

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- If the symbol to the left of that blank is the one remembered, erase it. If it is a blank, stop in an accepting state (0). Otherwise stop (in a non-accepting state).

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- If the symbol to the left of that blank is the one remembered, erase it. If it is a blank, stop in an accepting state (0). Otherwise stop (in a non-accepting state).
- Go to the left until you find a blank. (This is the start of what's left of the input string.)
- Start over with the head pointing at the symbol to the right of that blank.

Informal description–II

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- If the first letter and the last letter of the string were not equal, the machine has stopped in a non-accepting state.
- If the first and the last letter were equal, they have now both been erased and the head is pointing at the new first letter (the old second letter). The machine will now start over.

Informal description–II

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palindromes.

Accepting languages

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We can code this with two states, for ‘odd’ and ‘even’.

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The machine moves to the right until it finds a blank, switching between these two states, where 0 is the sole accepting state.

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0	$(1, a, R)$	stop
1	$(0, a, R)$	stop

Accepting languages–II

Now consider instead the Turing machine given by

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Now consider instead the Turing machine given by

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This machine moves to the right as well, switching between two states coding for ‘odd’ and ‘even’.

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If the machine finds a blank then

- it will stop (in an accepting state) if the current state is the one which is for ‘even’;
- it will keep moving to the right forever if the current state is the one for ‘odd’.

Accepting languages–III

The machines

δ	a	$\sqcup$
0	$(1, a, R)$	stop
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and

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both with sole accepting state 0 accept the same language.

Accepting languages–III

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Which one is more useful?

Recognizable *versus* decidable

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A language is Turing-decidable (or recursive) if there is a Turing machine which recognizes it which halts for all inputs.

Recognizable *versus* decidable–II

- Every Turing-decidable language is

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Recognizable *versus* decidable–II

- Every Turing-decidable language is Turing-recognizable, but not *vice versa*.
- In order to show that a language is Turing-decidable, we have to design a machine which halts for all inputs. Such a machine will give us a definite answer, ‘yes’ or ‘no’, for every word.

Recognizable *versus* decidable–II

- In order to show that a language is Turing-recognizable, we merely have to design a machine which halts for all inputs which it accepts.

Recognizable *versus* decidable–II

- In order to show that a language is Turing-recognizable, we merely have to design a machine which halts for all inputs which it accepts.
- With a Turing-recognizable language, we may never be certain that some given word really does not belong to the language.

C-F languages are Turing-decidable

Proposition 3.1 *Every context-free language is Turing-decidable.*

Biggest problem: Create machine which terminates for all inputs.

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  - We haven't covered this, so we will only describe the properties of this normal form here.

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The proof requires the following.

- Convert grammar into *Chomsky normal form (Chomsky nf)*.
  - Then a string of length n will be generated after at most $2n-1$ applications of a production rule.

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- Convert grammar into *Chomsky nf*.
- There is a TM to do this.

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- Given an input string α of length n, let the machine generate all words of the language which require no more than $2n - 1$ applications of a production rule.

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- Convert grammar into *Chomsky nf*.
- There is a TM to do this.
- Given an input string α of length n, let the machine generate all words of the language which require no more than $2n - 1$ applications of a production rule. If α is generated along the way, accept, otherwise reject.

Turing-decidability & complementation

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Proving this result is an exercise.

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For this it just starts producing words, starting with an empty tape.

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Theorem 3.3 *A language is Turing-recognizable if and only if there is a Turing machine enumerating it.*

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Turing-dec.	✓	✓	✓	✓	✓
Turing-rec.		✓	✓	✓	✓

Chomsky's hierarchy

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With (some of) these languages there are corresponding machines and grammars.

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Languages		

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Turing-recognizable	Γ any string containing non-terminals Δ any string	Turing machines

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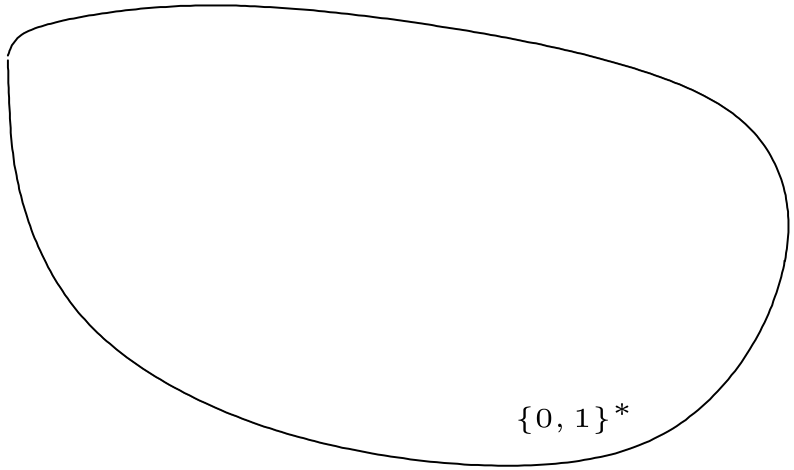
- Turing machines are more powerful than PDAs.
- Languages recognized by a Turing machine are called Turing-recognizable, and these are the same languages as those which are enumerated by a Turing machine.
- If there is a Turing machine recognizing a language which halts for all inputs we say the language is Turing-decidable. (Rather than just having to accept words belonging to the language, the machine must effectively reject words that do not.)

Languages over $\{0, 1\}$



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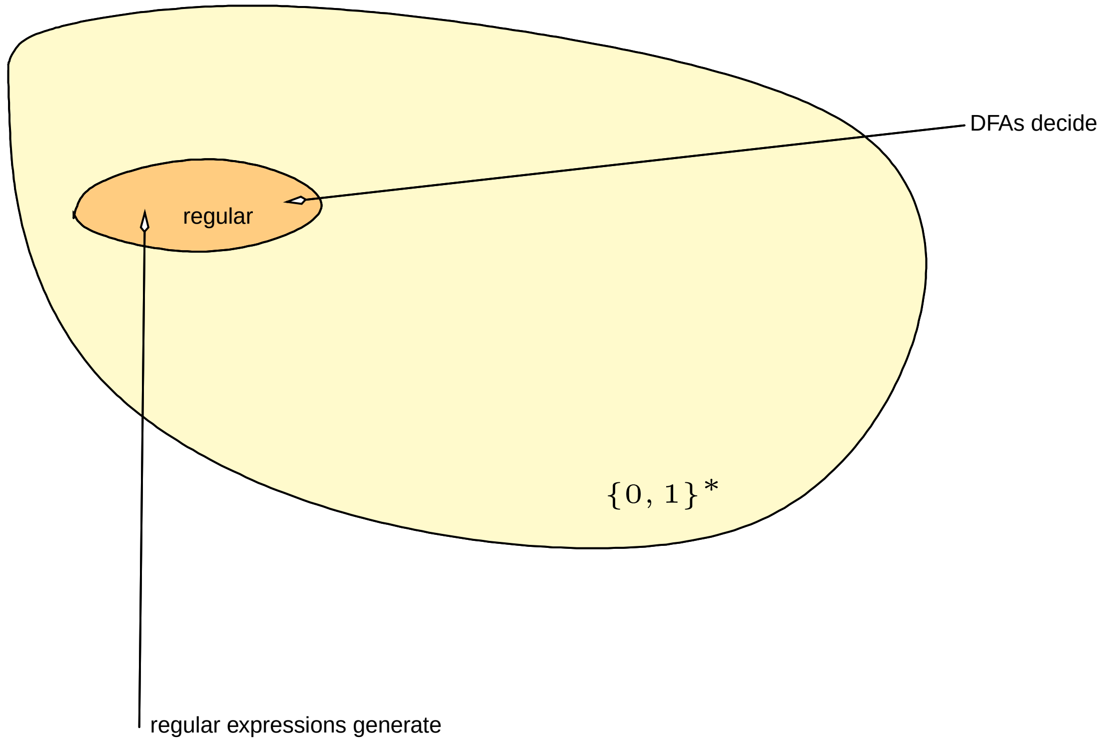

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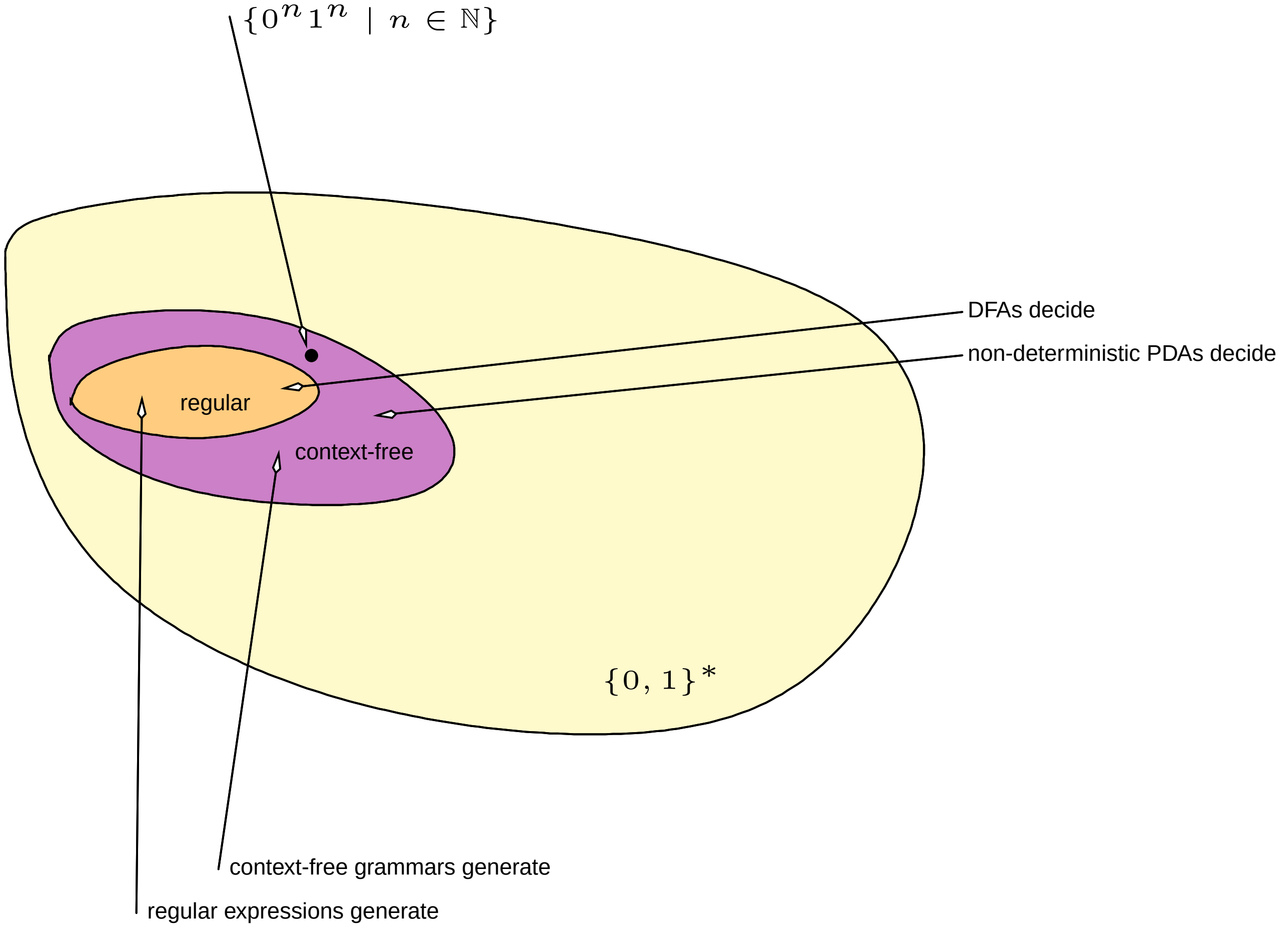

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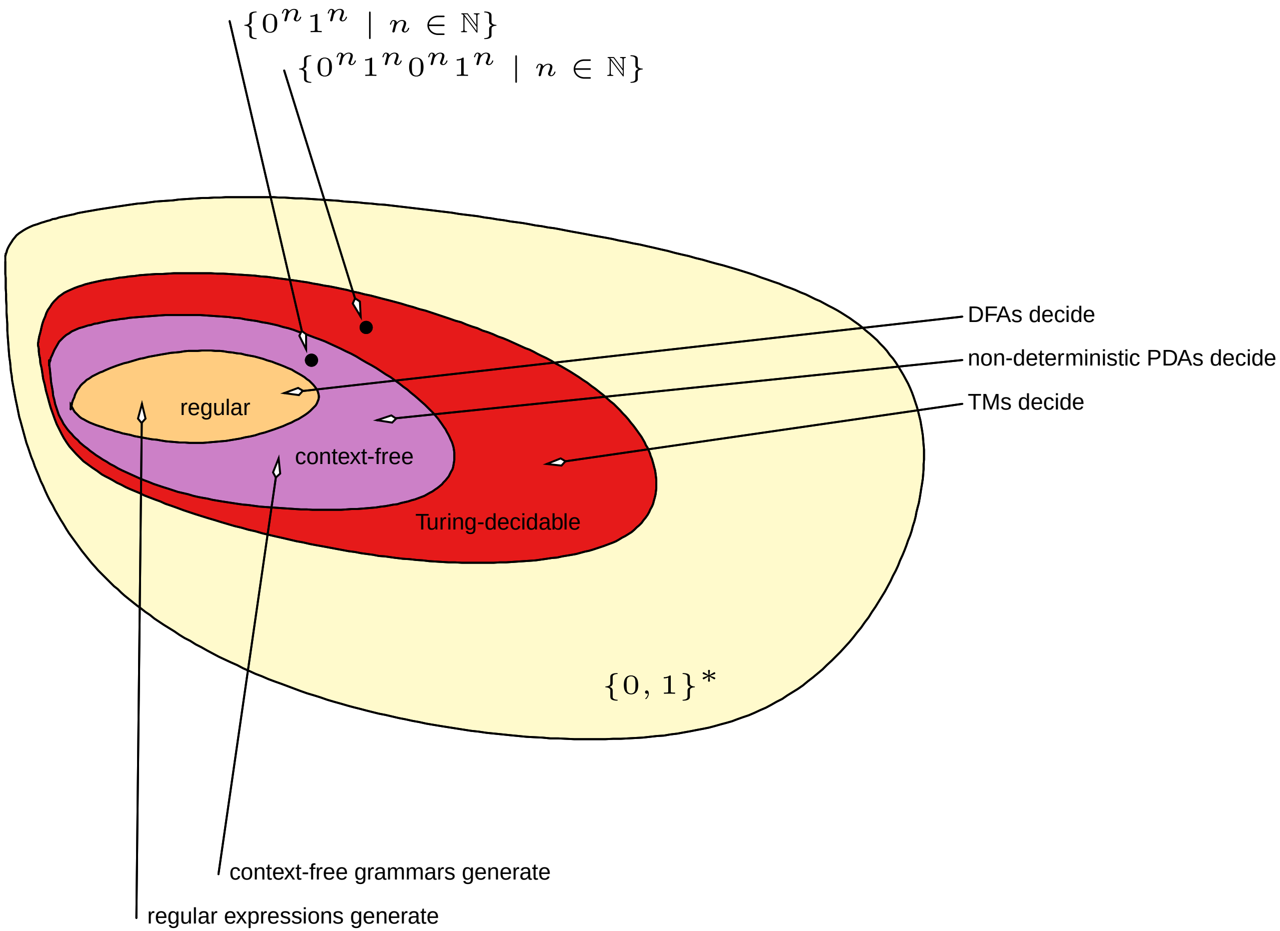

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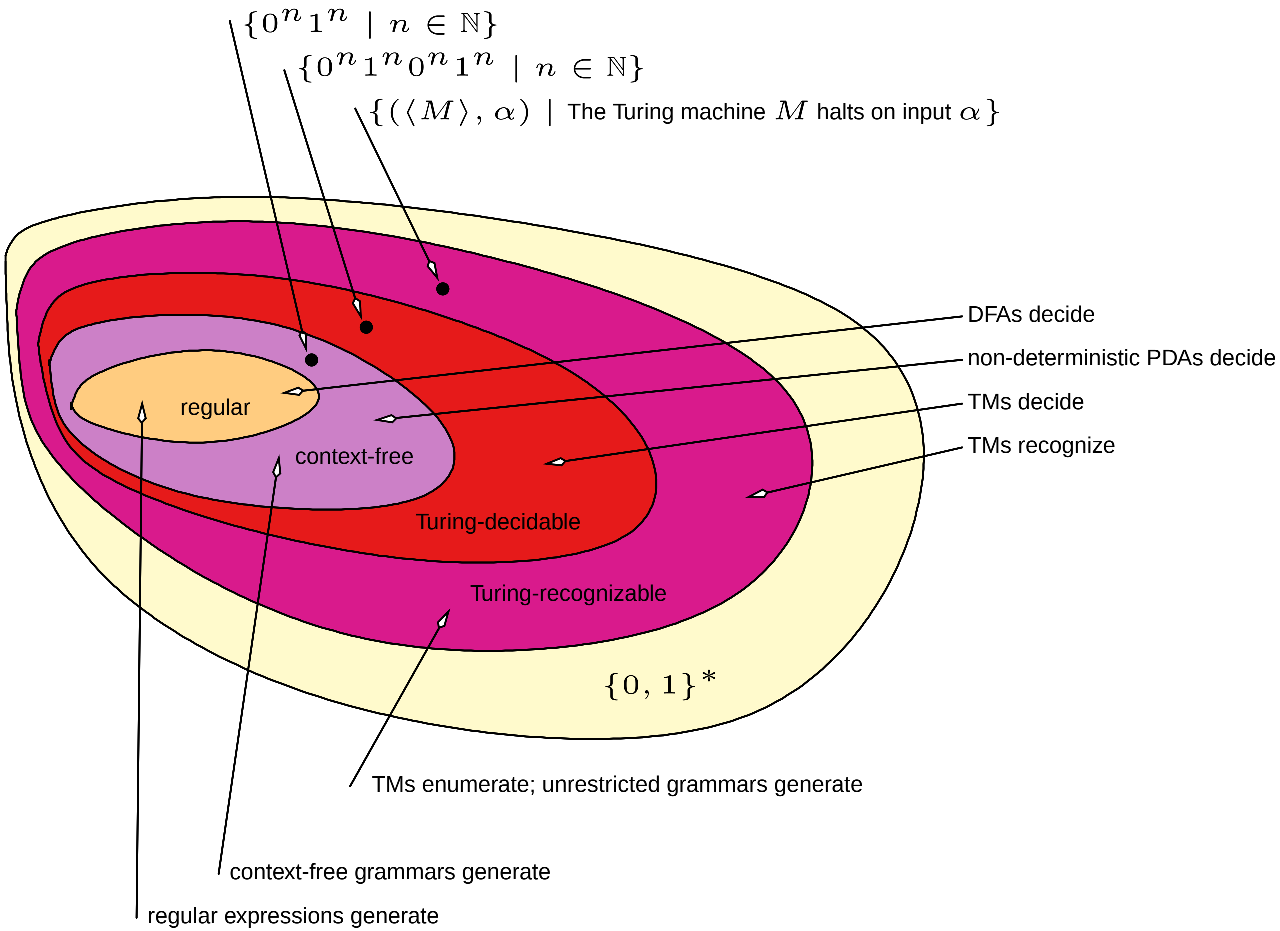

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