COMP20121 The Implementation and Power of Computer Languages

‘Power’ Part

http://www.cs.man.ac.uk/~petera/2121/index.html .

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LECTURE SEVEN

Section 3: Turing machines

Turing: The man

Who was Alan Turing?

Turing: The man

Who was Alan Turing?

- Founder of computer science,
- mathematician, philosopher,
- codebreaker, strange visionary

Turing: The man

Who was Alan Turing?

1912 (23 June): Birth, Paddington, London

1931-34: Undergraduate at King's College, Cambridge University

1935: Elected fellow of King's College, Cambridge

1936: The Turing machine, computability, universal machine

Turing: The man

Who was Alan Turing?

1936-38: Princeton University. Ph.D. Logic, algebra, number theory

1938-39: Return to Cambridge. Introduced to German Enigma cipher machine

1939-40: The Bombe, machine for Enigma decryption

1939-42: Breaking of U-boat Enigma, saving battle of the Atlantic

Turing: The man

Who was Alan Turing?

1943-45: Chief Anglo-American crypto consultant. Electronic work.

1945: National Physical Laboratory, London

1946: Computer and software design leading the world.

1947-48: Programming, neural nets, and artificial intelligence

1948: Manchester University

Turing: The man

Who was Alan Turing?

1949: First serious mathematical use of a computer

1950: The Turing Test for machine intelligence

1951: Elected FRS. Non-linear theory of biological growth

1952: Arrested as a homosexual, loss of security clearance

1953-54: Unfinished work in biology and physics

1954 (7 June): Death (suicide) by cyanide poisoning,

Wilmslow, Cheshire.

Overview

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- These automata have read/write memory, so they can access data they have stored in arbitrary order.

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- We finish by giving an overview over a hierarchy of languages.

Automata with random-access memory

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Idea: With read-write random-access memory, we can store input string in memory.

Transition function

A transition should depend on:

- the current state;

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- the content of memory cell(s?).

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(Compare PDAs.)

Actions

What actions should the machine be able to carry out?

- Read from memory;

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- Read from memory;
- write to memory;
- move to different memory cell.

The easiest way of doing this is to assume that the memory is arranged linearly.

The memory

- We think of the memory as an infinite tape.

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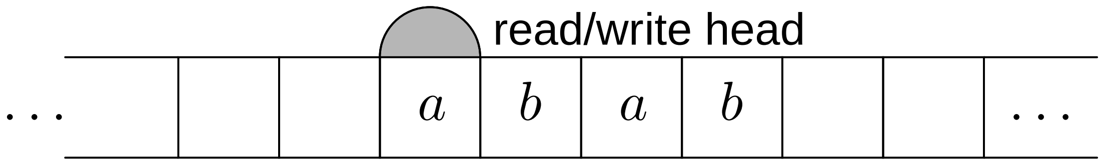
...				a	b	a	b			...

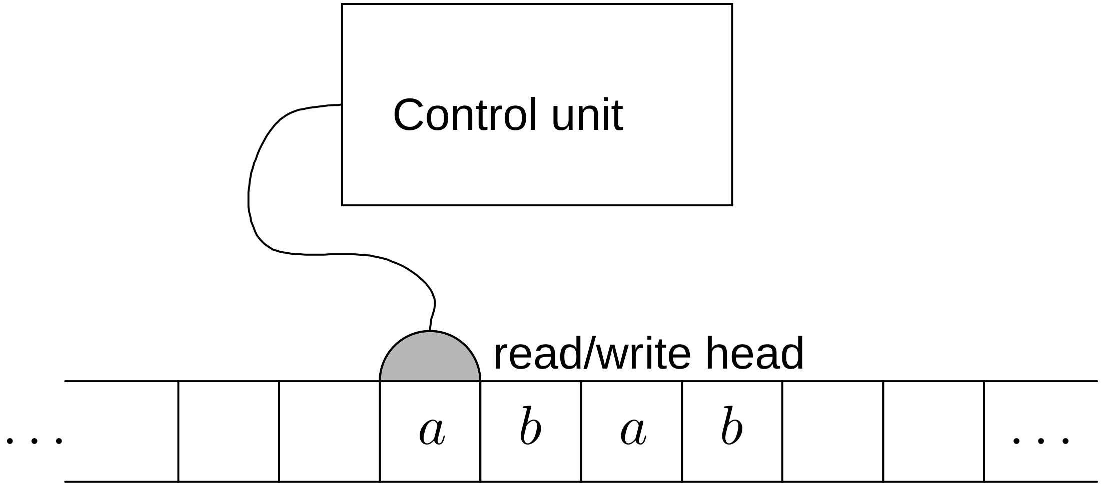
The memory

- Can do without memory addresses if we assume there is one read/write head pointing at the 'current' memory cell and which can move along the tape.



The memory

- This head is connected to a control unit which tells it what to do (depending on the current state and the content of the current memory cell).



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- We need to be able to denote in the text that a memory cell might be empty.
- For that we use the symbol $\sqcup$.
- Sometimes there are other symbols we want to be able to write onto the tape.

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  - the symbol $\sqcup$ is contained in T.

Note that T might contain other symbols.

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Therefore the transition function will take

- the current state from Q,

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- the content of the current cell from T,

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- and will return

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  - a new state from Q,

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  - a direction from $\{L, R, N\}$.

A move is one step in that direction.

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A move is one step in that direction.

- Alternatively, the machine might stop.

Turing machine

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- *a finite set of* states Q*;*
- *a* start state $q_\bullet \in Q$*;*
- *a set* $F \subseteq Q$ *of* accepting states*;*
- *a* transition function *that maps* $Q \times T$ *to* $(Q \times T \times \{L, R, N\}) \cup \{\text{stop}\}$.

Comparison: TMs *versus* PDAs

Unlike a pushdown automaton, a Turing machine can

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Unlike a pushdown automaton, a Turing machine can

- manipulate the input string;
- process the input string several times;
- write to a random-access memory.

Like a pushdown automaton, a Turing machine may run forever (on some input strings).

Example

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For example:

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To see what this machine does with an input string, we first have to define what it means for a TM to accept an input string.

Acceptance of a string by a TM.

Definition 15 *We say that a string* α

is accepted by a Turing machine

if, when

- *started in the initial state*
- *with the head positioned on the first (left-most) character of* α *and*
- *the tape otherwise empty*

the Turing machine halts and the state at that time is an accepting state.

Example–II

Assume that the Turing machine given by the transition table

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has only one accepting state, namely 0.

Example–II

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δ	a	b	$\sqcup$
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Then the language accepted by the machine is the language of all words consisting entirely of as.

Example–II

δ	a	b	$\sqcup$
0	$(0, a, R)$	$(1, b, R)$	stop
1	stop	stop	stop

Then the language accepted by the machine is the language of all words consisting entirely of as.

At the end of the calculation, the input string is untouched.

Example–III

Assume that the Turing machine given by the transition table

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Example–III

δ	a	b	$\sqcup$
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δ	a	b	$\sqcup$
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Example–III

δ	a	b	$\sqcup$
0	$(0, \sqcup, R)$	$(1, \sqcup, R)$	stop
1	stop	stop	stop

Then the language accepted by the machine is the language of all words consisting entirely of as.

At the end of the calculation, the input string has been destroyed.

A more sophisticated example

Consider the following Turing machine with only accepting state 0.

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
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What is the language accepted by this Turing machine? Not so easy!

Running a Turing machine

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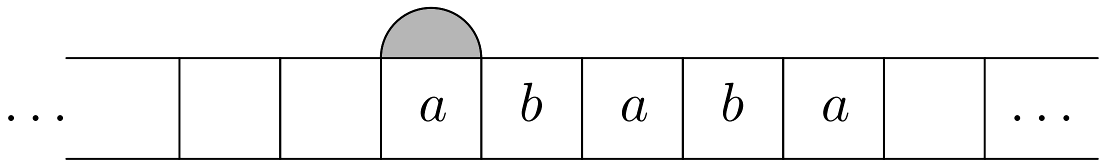

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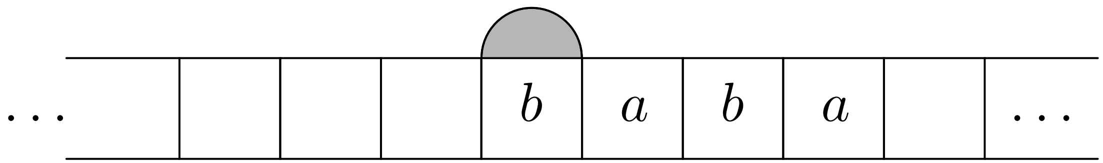

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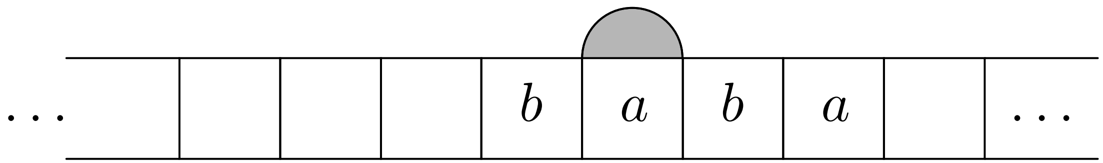

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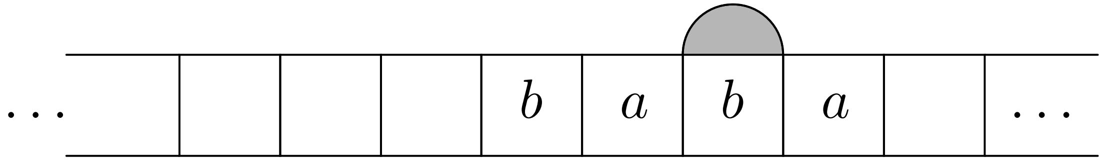

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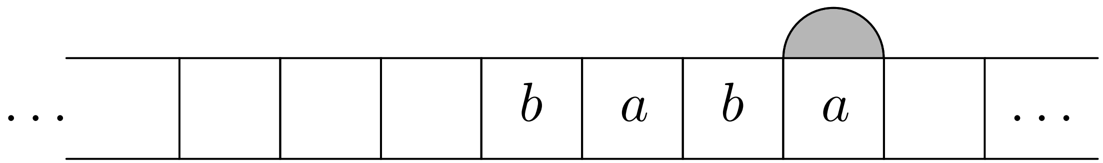

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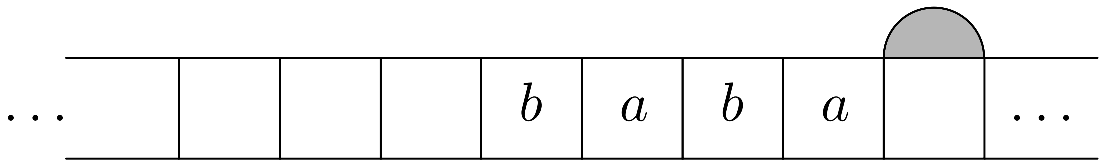

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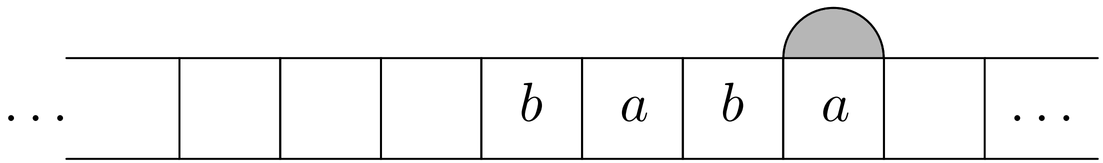

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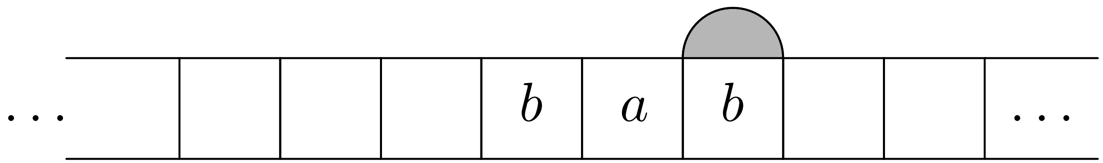

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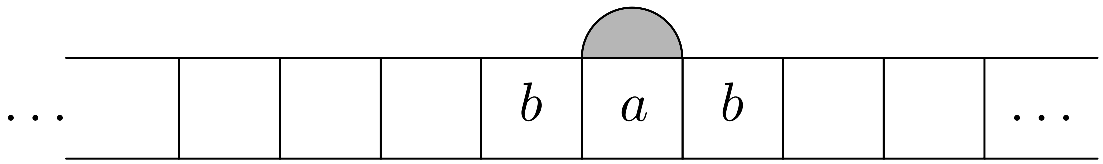

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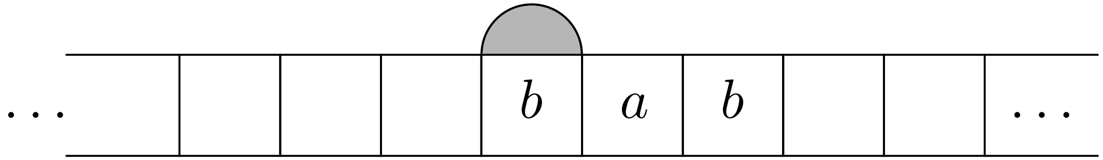

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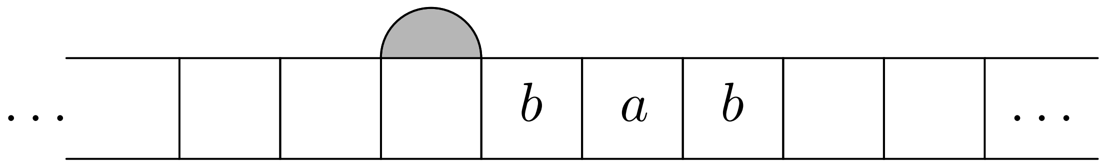

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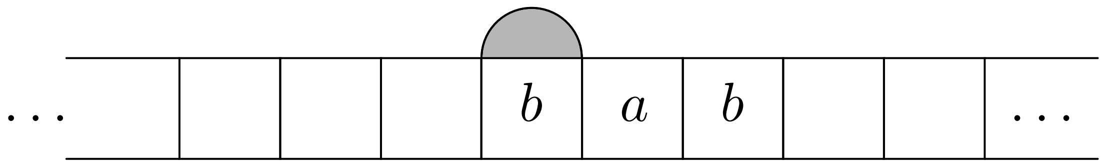

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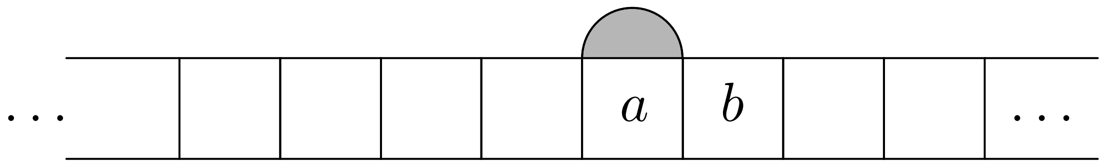

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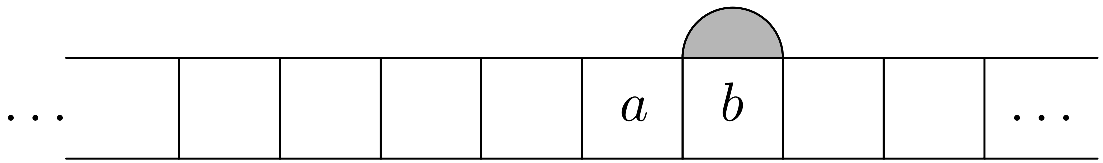
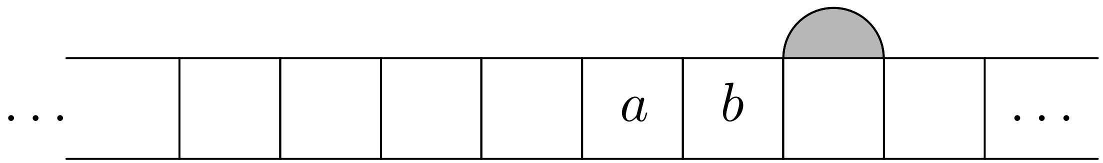
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



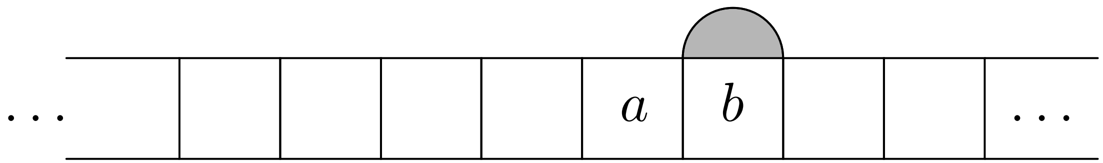
Running a Turing machine

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



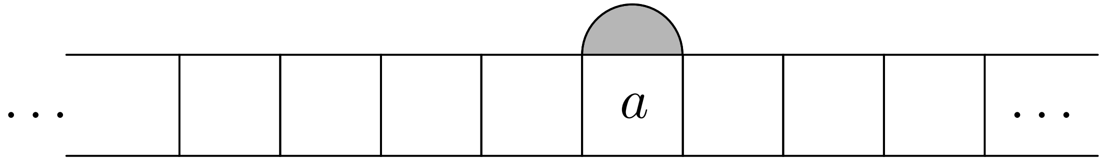
Running a Turing machine

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



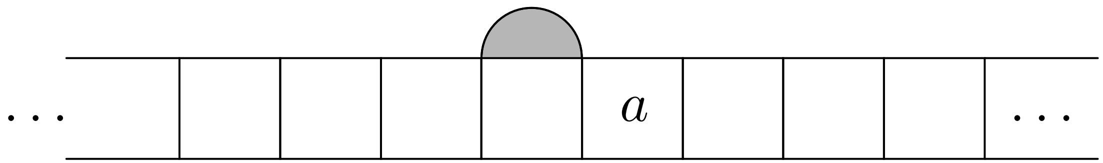
Running a Turing machine

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



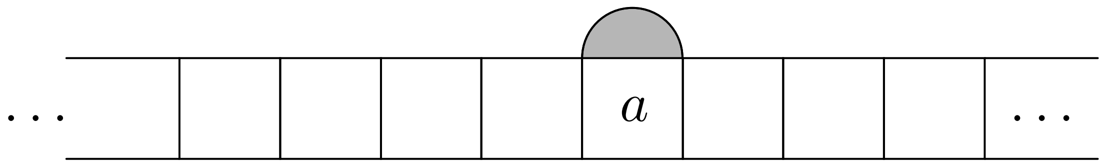
Running a Turing machine

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



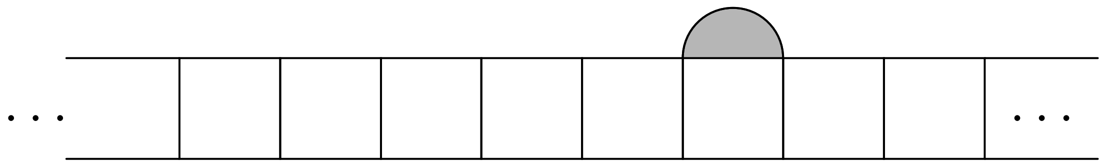
Running a Turing machine

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



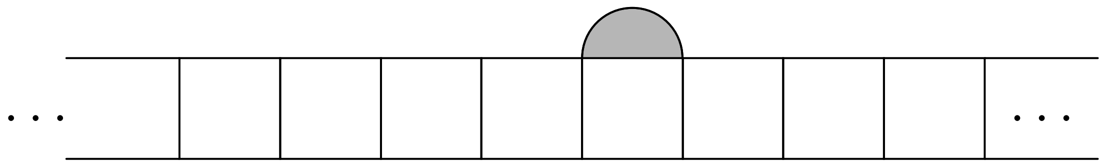
Running a Turing machine

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



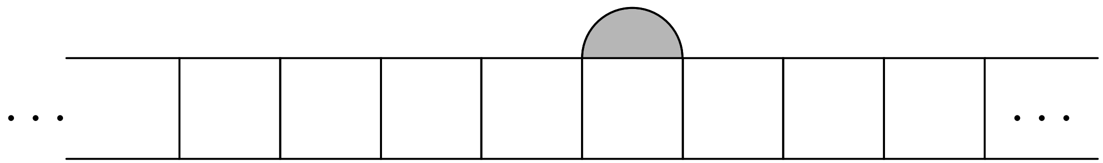
Running a Turing machine

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



Running a Turing machine

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



Hence the word $ababa$ is accepted by the machine.

Configurations

We use a configuration

$$x_1 x_2 \cdots x_i q x_{i+1} \cdots x_n$$

to describe a situation where

Configurations

We use a configuration

$$x_1 x_2 \cdots x_i q x_{i+1} \cdots x_n$$

to describe a situation where

- the content of the tape is $x_1 \cdots x_n$ (where the x_j are from T), and all cells to the left of x_1 and to the right of x_n are blank;

Configurations

We use a configuration

$$x_1 x_2 \cdots x_i q x_{i+1} \cdots x_n$$

to describe a situation where

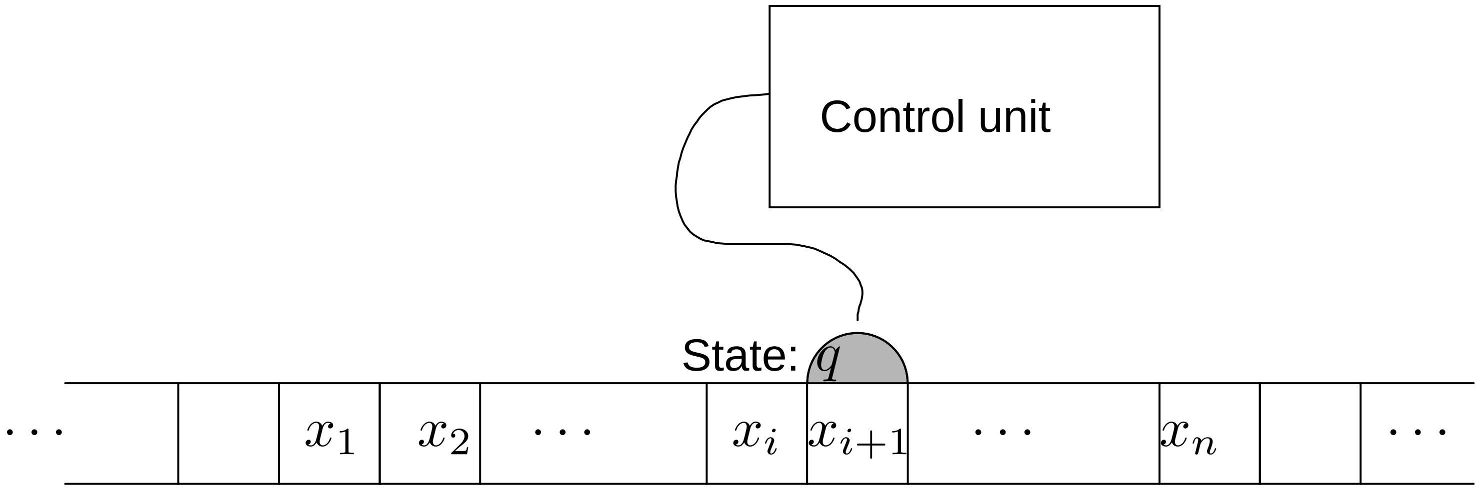
- the current state is q;
- the head currently points at the cell holding x_{i+1}.

Configurations

We use a configuration

$$x_1 x_2 \cdots x_i q x_{i+1} \cdots x_n$$

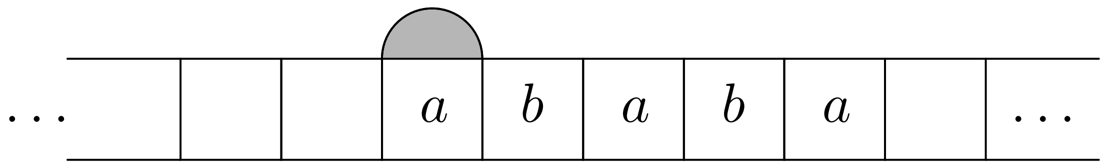
to describe a situation where



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

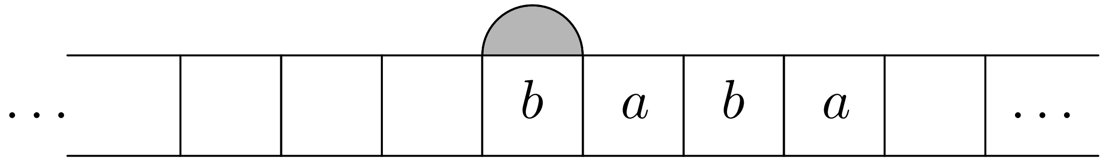
$$0ababa$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

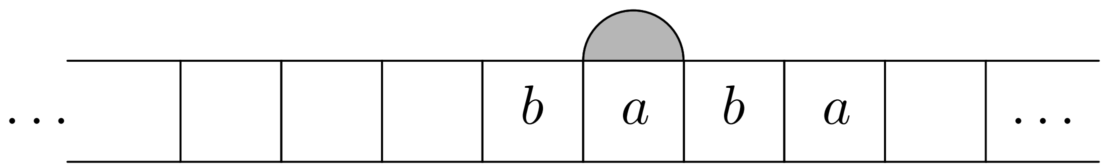
$$1baba$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

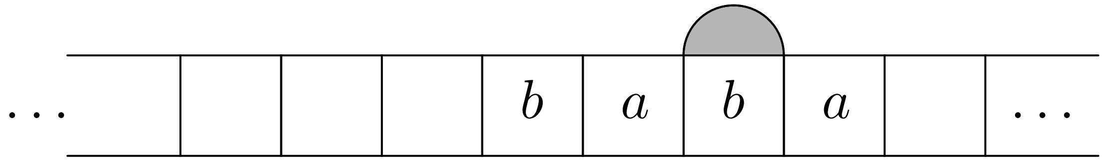
$$b1aba$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

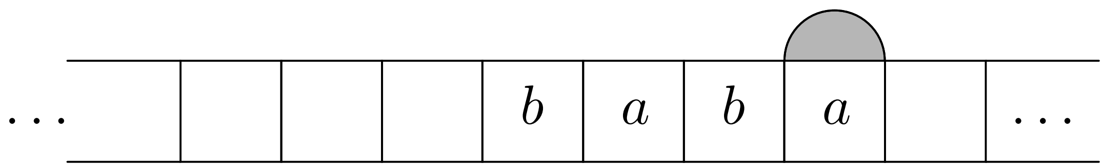
$$ba1ba$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

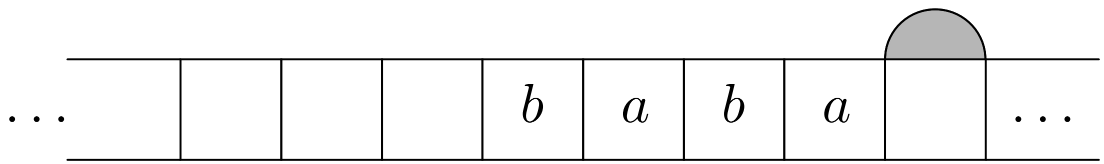
$$bab1a$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

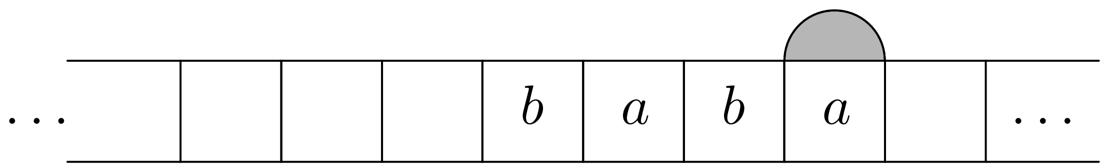
$$baba1$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

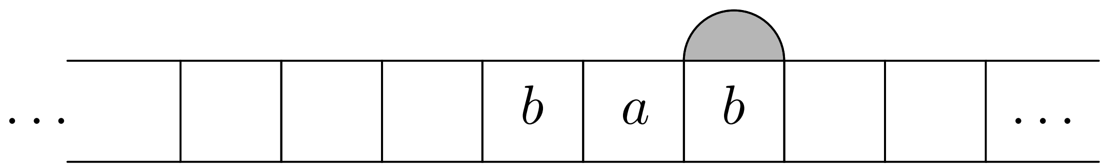
$$bab3a$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

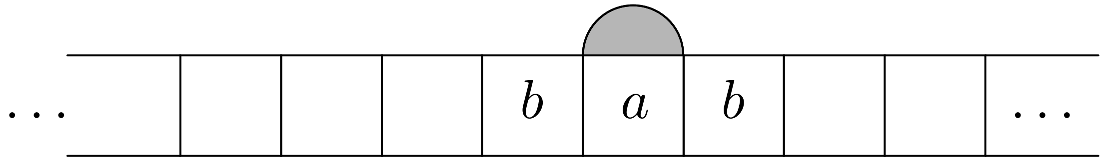
$$ba5b$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

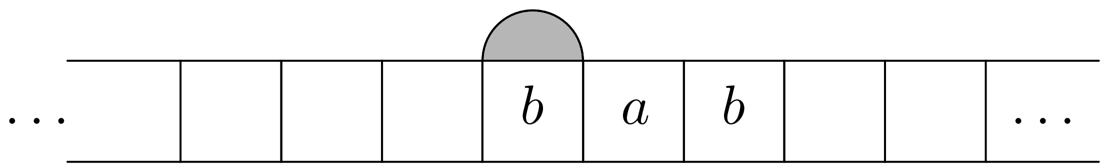
$$b5ab$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

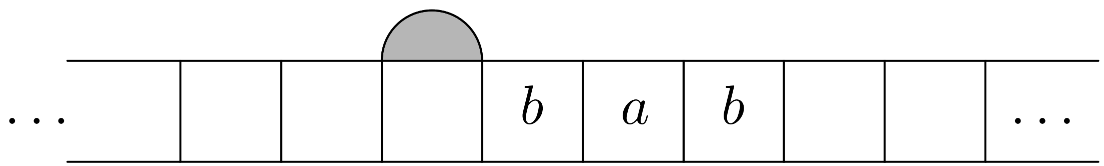
$$5bab$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

$$5 \sqcup bab$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
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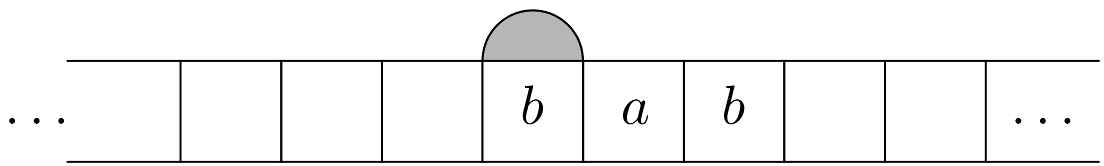
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

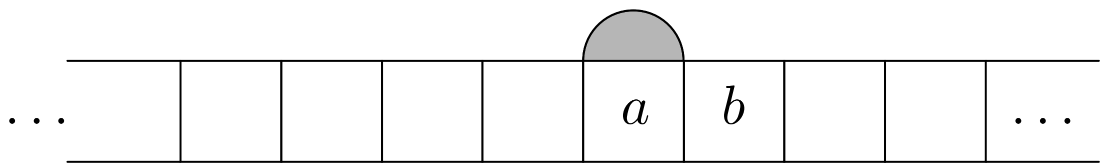
$$0bab$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

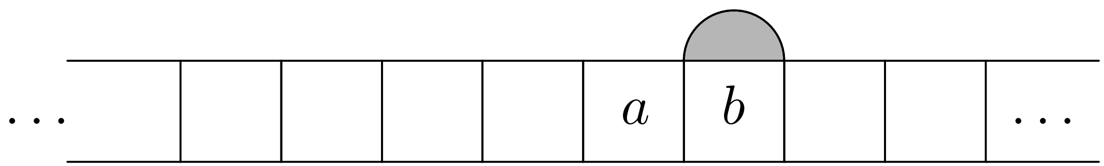
$$2ab$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

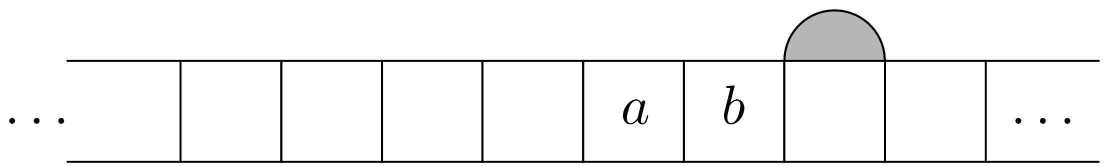
$$a2b$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

$$ab2$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
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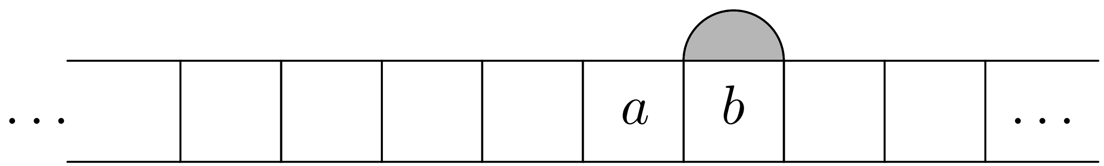
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

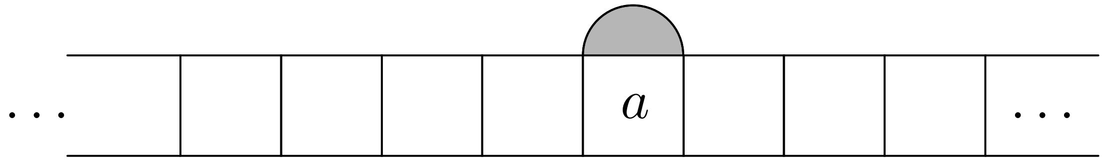
$$a4b$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

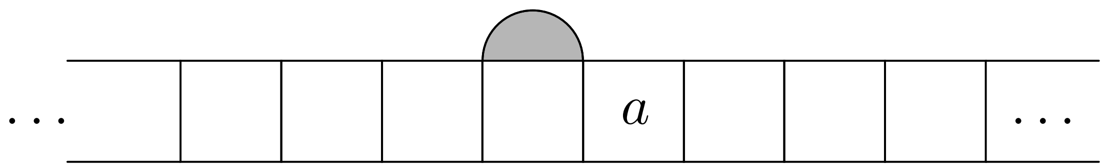
$$5a$$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

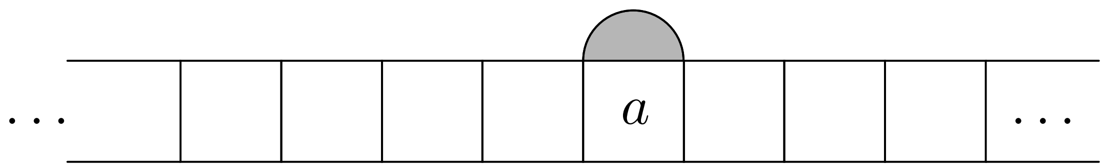
$$5 \sqcup a$$



Running a Turing machine–II

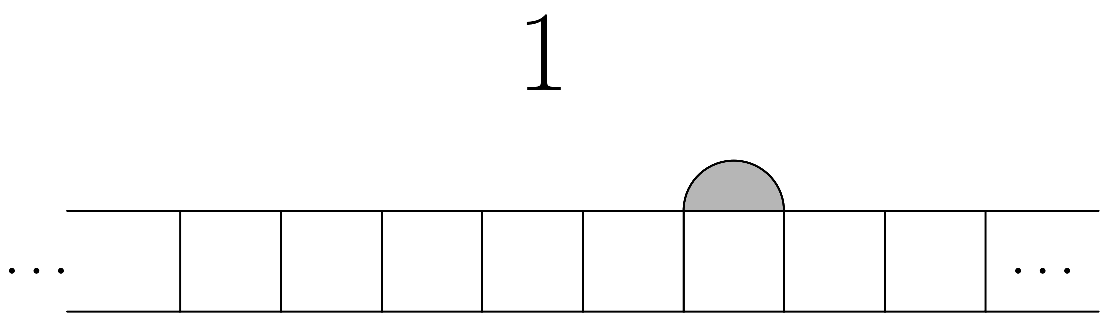
δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

$$0a$$



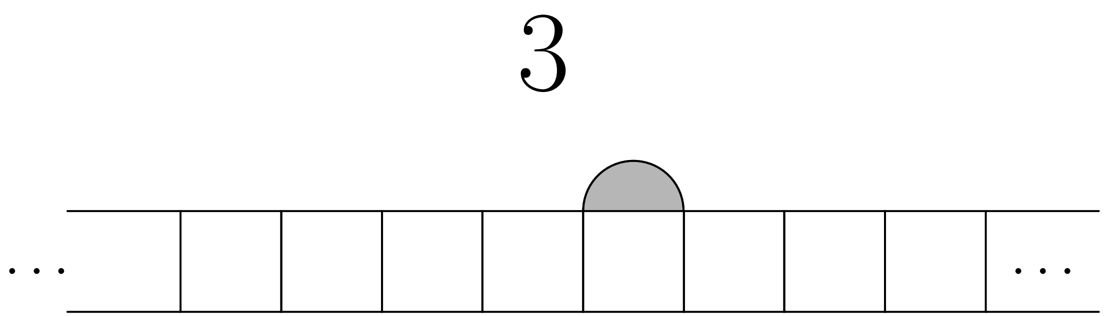
Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



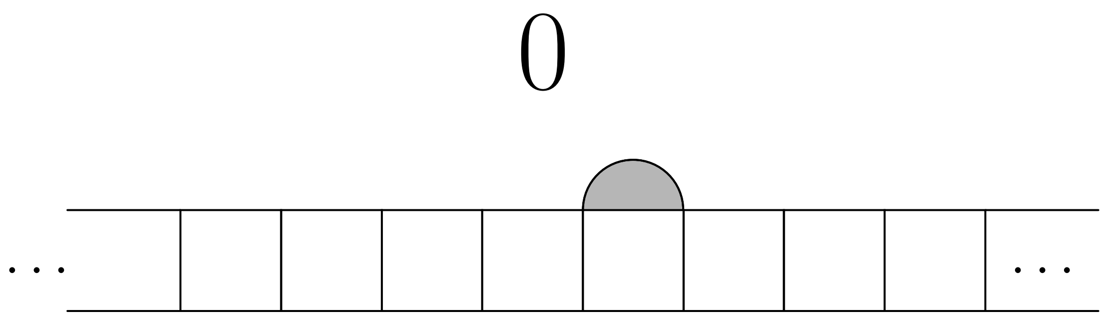
Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



Running a Turing machine–II

δ	a	b	$\sqcup$
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



Running a Turing machine–II

Hence the word $ababa$ is accepted by the machine.

We normally would not keep repeating the transition function, just give the sequence of configurations the machine goes through.

Running a Turing machine–II

$$0ababa \to 1baba \to 1baba \to 1baba \to$$

$$1baba \to 1baba \to b1aba \to ba1ba \to$$

$$bab1a \to baba1 \to bab3a \to ba5b \to$$

$$b5ab \to 5bab \to 5\sqcup bab \to 0bab \to$$

$$2ab \to a2b \to ab2 \to a4b \to 5a \to$$

$$5\sqcup a \to 0a \to 1 \to 3 \to 0 \text{ stop}$$