
COMP20121 The Implementation and Power of Computer Languages

'Power' Part

<http://www.cs.man.ac.uk/~petera/2121/index.html> .

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LECTURE SEVEN

Section 3: Turing machines

Turing: The man

Who was Alan Turing?

Turing: The man

Who was Alan Turing?

- Founder of computer science,
- mathematician, philosopher,
- codebreaker, strange visionary

Turing: The man

Who was Alan Turing?

1912 (23 June): Birth, Paddington, London

1931-34: Undergraduate at King's College, Cambridge
University

1935: Elected fellow of King's College, Cambridge

1936: The Turing machine, computability, universal machine

Turing: The man

Who was Alan Turing?

1936-38: Princeton University. Ph.D. Logic, algebra, number theory

1938-39: Return to Cambridge. Introduced to German Enigma cipher machine

1939-40: The Bombe, machine for Enigma decryption

1939-42: Breaking of U-boat Enigma, saving battle of the Atlantic

Turing: The man

Who was Alan Turing?

1943-45: Chief Anglo-American crypto consultant. Electronic work.

1945: National Physical Laboratory, London

1946: Computer and software design leading the world.

1947-48: Programming, neural nets, and artificial intelligence

1948: Manchester University

Turing: The man

Who was Alan Turing?

1949: First serious mathematical use of a computer

1950: The Turing Test for machine intelligence

1951: Elected FRS. Non-linear theory of biological growth

1952: Arrested as a homosexual, loss of security clearance

1953-54: Unfinished work in biology and physics

1954 (7 June): Death (suicide) by cyanide poisoning,
Wilmslow, Cheshire.

Overview

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- These automata have read/write memory, so they can access data they have stored in arbitrary order.

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- We finish by giving an overview over a **hierarchy of languages**.

Automata with random-access memory

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- to keep the idea of ‘state’;

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Idea: With read-write random-access memory, we can store input string in memory.

Transition function

A transition should depend on:

- the current state;

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- the content of memory cell(s?).

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(Compare PDAs.)

Actions

What **actions** should the machine be able to carry out?

- Read from memory;

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- write to memory;

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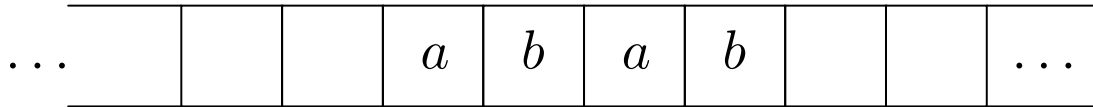
The easiest way of doing this is to assume that the memory is arranged linearly.

The memory

- We think of the memory as an infinite tape.

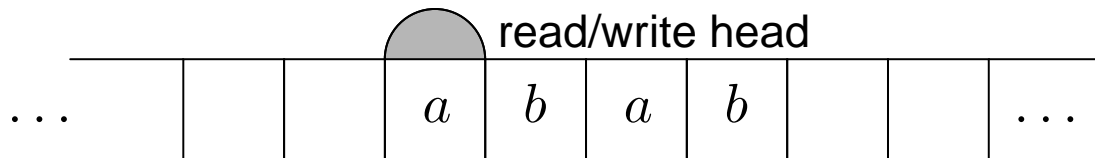
The memory

- We think of the memory as an **infinite** **tape**.



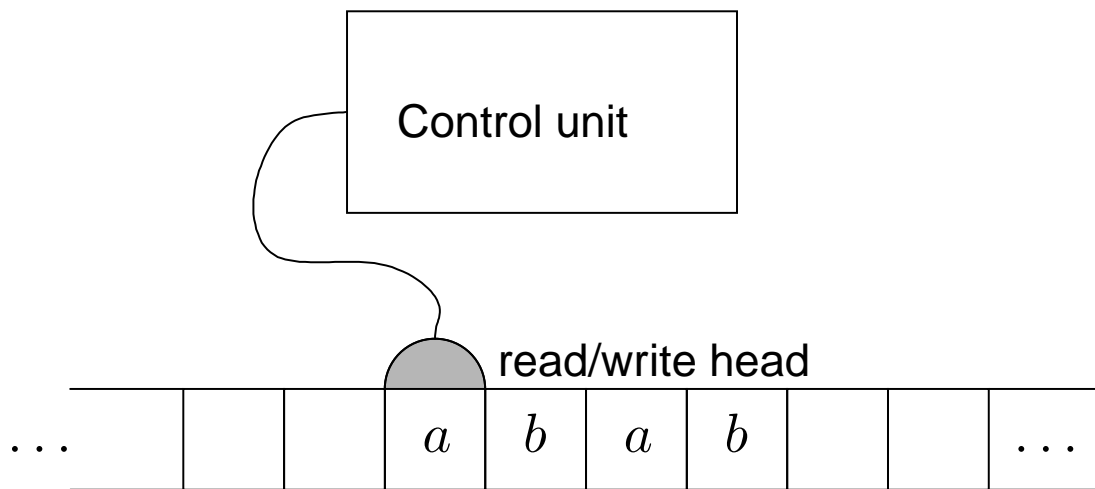
The memory

- Can do without memory addresses if we assume there is one **read/write head** pointing at the 'current' memory cell and which can move along the tape.



The memory

- This head is connected to a **control unit** which tells it what to do (depending on the current state and the content of the current memory cell).



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- We need to be able to denote in the text that a memory cell might be **empty**.
- For that we use the symbol \sqcup .
- Sometimes there are other symbols we want to be able to write onto the tape.

Alphabets

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 - all symbols of Σ are contained in T and
 - the symbol \sqcup is contained in T .

Note that T might contain other symbols.

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Therefore the transition function will take

- the current state

from Q ,

Transition function

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- the current state from Q ,
- the content of the current cell from T ,

Transition function

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- the current state from Q ,
- the content of the current cell from T ,
- and will return

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 - a direction from $\{L, R, N\}$.

A move is one step in that direction.

Transition function

Therefore the transition function will take

- the current state from Q ,
- the content of the current cell from T ,
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- a new state from Q ,
- a symbol to write onto the tape from T ,
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A move is one step in that direction.

- Alternatively, the machine might stop.

Turing machine

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- a finite set of states Q ;

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- a finite set of states Q ;
- a start state $q_{\bullet} \in Q$;
- a set $F \subseteq Q$ of accepting states;
- a transition function that maps $Q \times T$ to $(Q \times T \times \{L, R, N\}) \cup \{\text{stop}\}$.

Comparison: TMs *versus* PDAs

Unlike a pushdown automaton, a Turing machine can

- manipulate the input string;

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Unlike a pushdown automaton, a Turing machine can

- manipulate the input string;
- process the input string several times;
- write to a random-access memory.

Like a pushdown automaton, a Turing machine may run forever (on some input strings).

Example

We typically give a Turing machine *via* its transition table.

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For example:

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To see what this machine does with an input string, we first have to define what it means for a TM to **accept an input string**.

Acceptance of a string by a TM.

Definition 15 *We say that a string α is accepted by a Turing machine*

if, when

- *started in the initial state*
- *with the head positioned on the first (left-most) character of α and*
- *the tape otherwise empty*

the Turing machine halts and the state at that time is an accepting state.

Example-II

Assume that the Turing machine given by the transition table

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has only one accepting state, namely 0.

Example-II

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Then the language accepted by the machine is the language of all words consisting entirely of as .

Example-II

δ	a	b	\sqcup
0	$(0, a, R)$	$(1, b, R)$	stop
1	stop	stop	stop

Then the language accepted by the machine is **the language of all words consisting entirely of as .**

At the end of the calculation, the input string is untouched.

Example–III

Assume that the Turing machine given by the transition table

δ	a	b	\sqcup
0	$(0, \sqcup, R)$	$(1, \sqcup, R)$	stop
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Example–III

δ	a	b	\sqcup
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Example–III

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Example–III

δ	a	b	\sqcup
0	$(0, \sqcup, R)$	$(1, \sqcup, R)$	stop
1	stop	stop	stop

Then the language accepted by the machine is **the language of all words consisting entirely of as .**

At the end of the calculation, the input string has been destroyed.

A more sophisticated example

Consider the following Turing machine with only accepting state 0.

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
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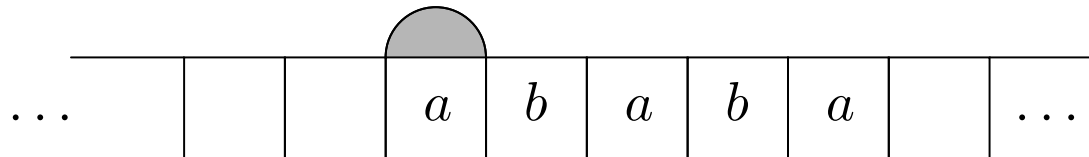
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What is the language accepted by this Turing machine? **Not so easy!**

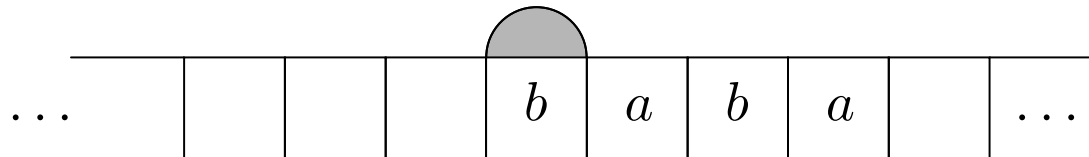
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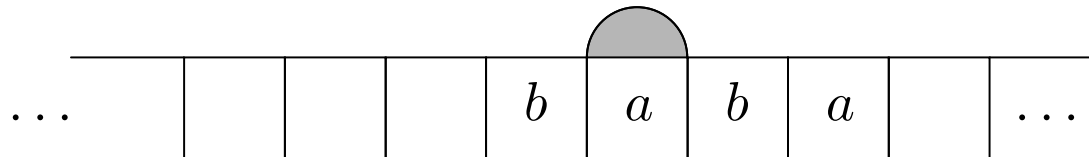
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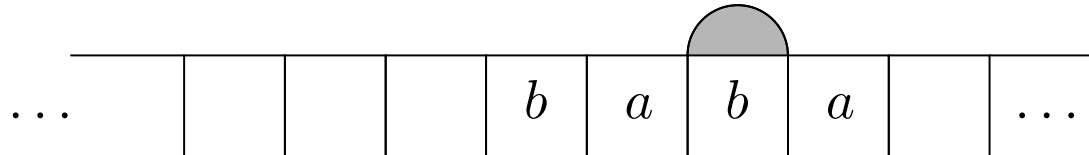
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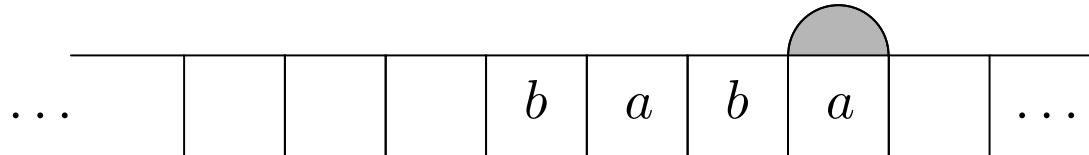
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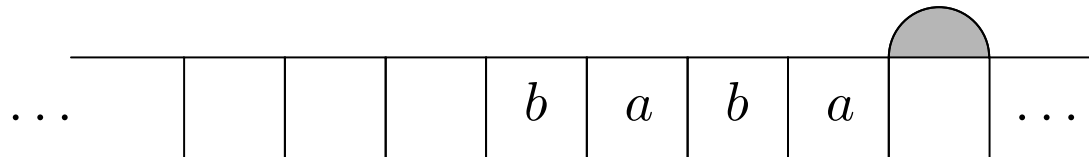
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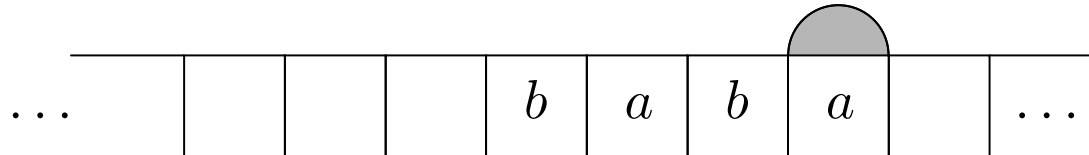
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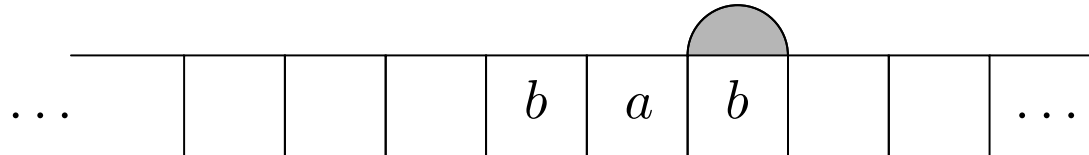
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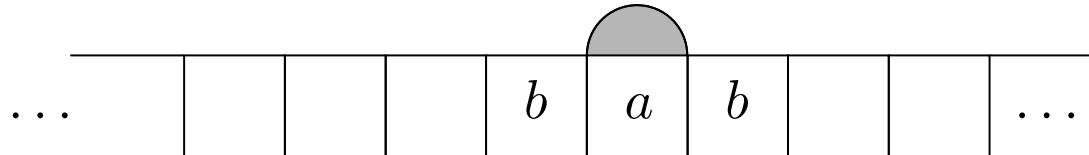
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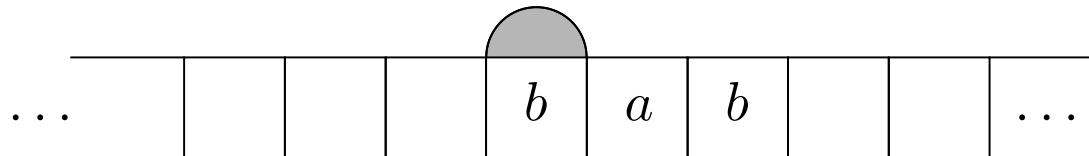
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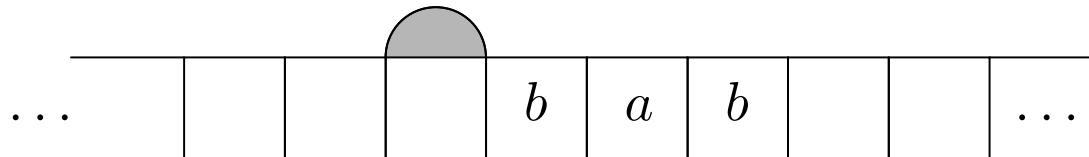
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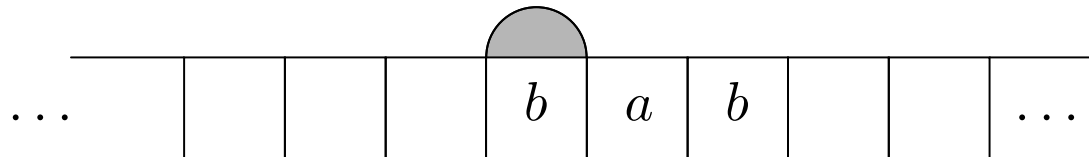
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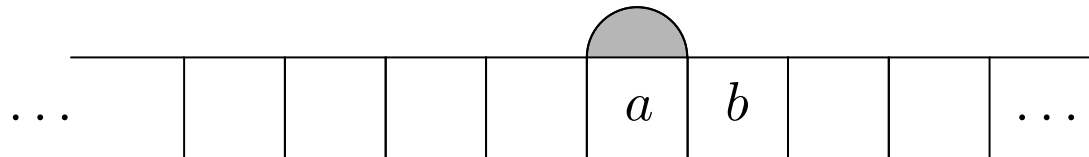
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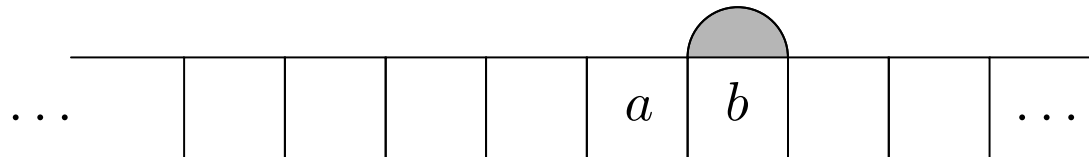
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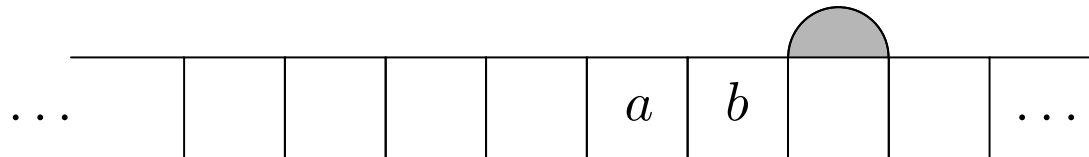
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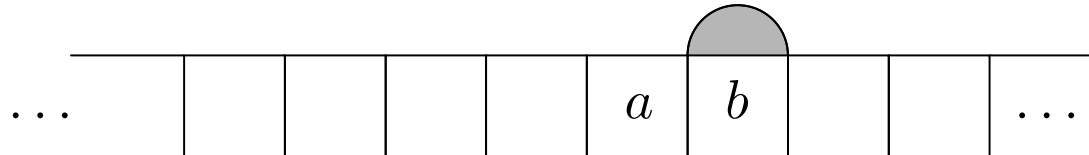
Running a Turing machine

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



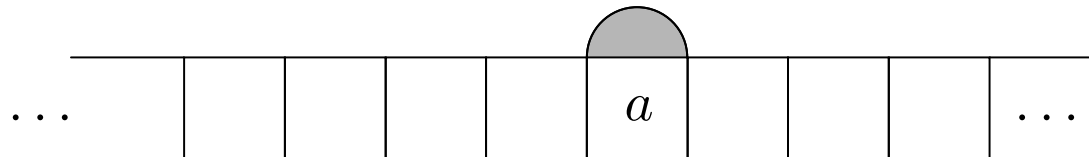
Running a Turing machine

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



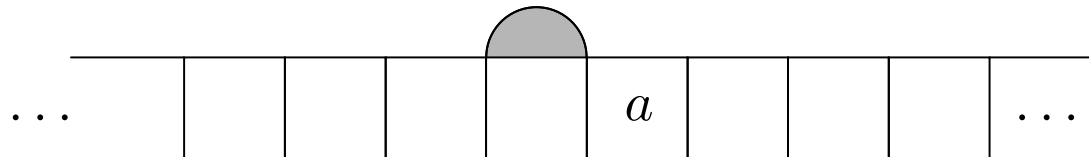
Running a Turing machine

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



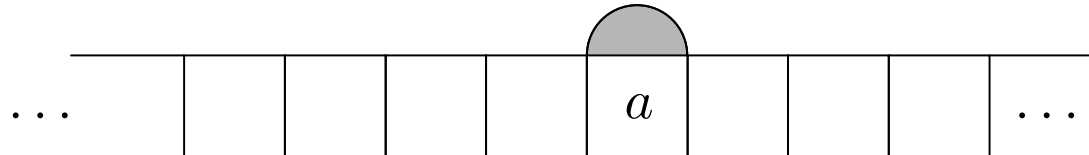
Running a Turing machine

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



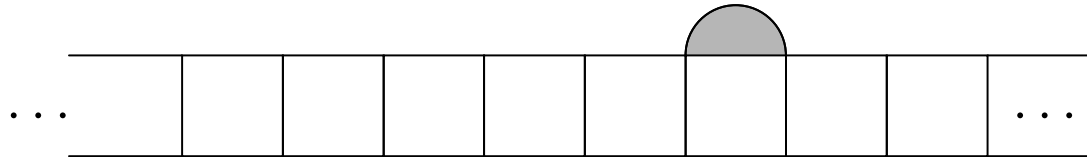
Running a Turing machine

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



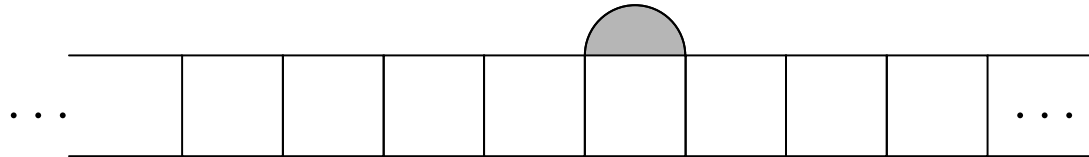
Running a Turing machine

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



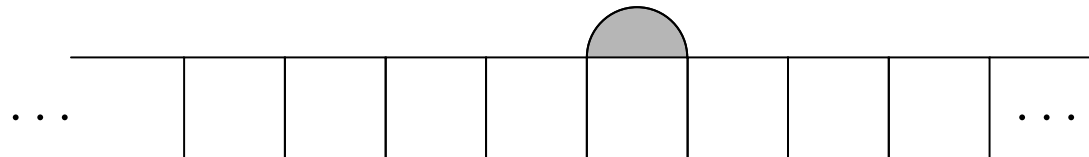
Running a Turing machine

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



Running a Turing machine

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



Hence the word *ababa* is accepted by the machine.

Configurations

We use a configuration

$$x_1 x_2 \cdots x_i q x_{i+1} \cdots x_n$$

to describe a situation where

Configurations

We use a configuration

$$x_1 x_2 \cdots x_i q x_{i+1} \cdots x_n$$

to describe a situation where

- the content of the tape is $x_1 \cdots x_n$ (where the x_j are from T), and all cells to the left of x_1 and to the right of x_n are blank;

Configurations

We use a configuration

$$x_1 x_2 \cdots x_i q x_{i+1} \cdots x_n$$

to describe a situation where

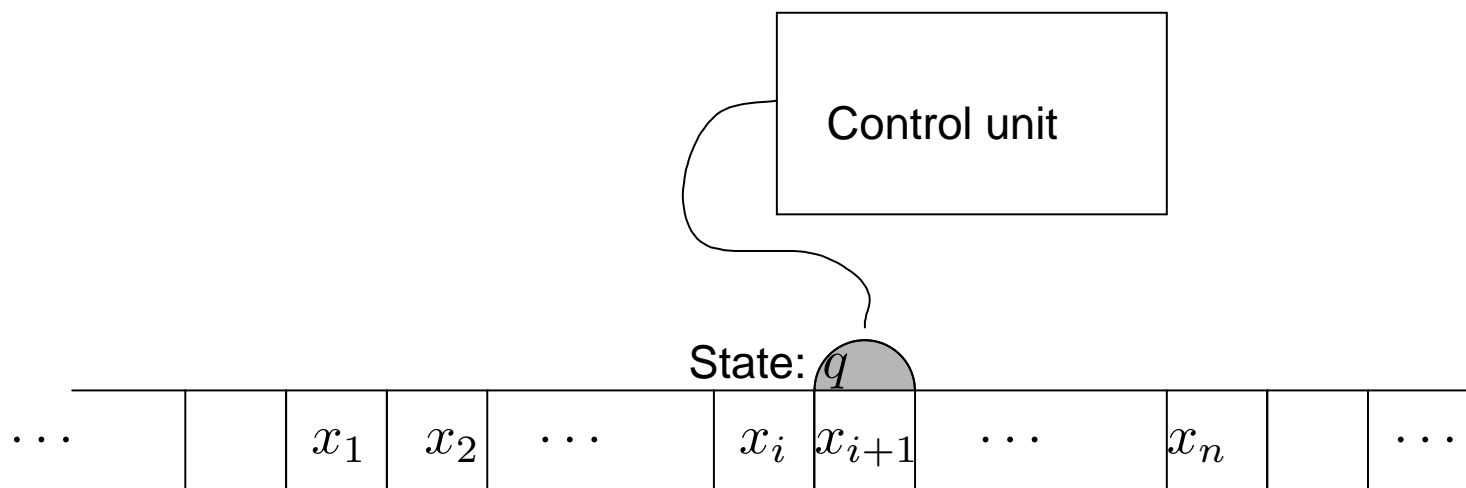
- the current state is q ;
- the head currently points at the cell holding x_{i+1} .

Configurations

We use a configuration

$$x_1 x_2 \cdots x_i q x_{i+1} \cdots x_n$$

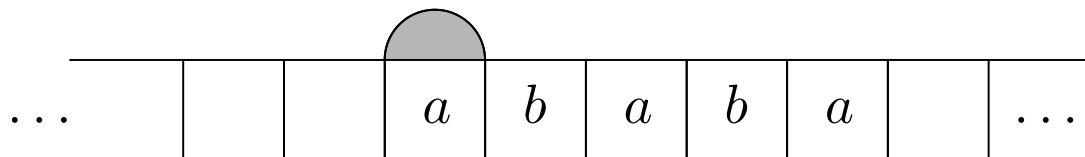
to describe a situation where



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

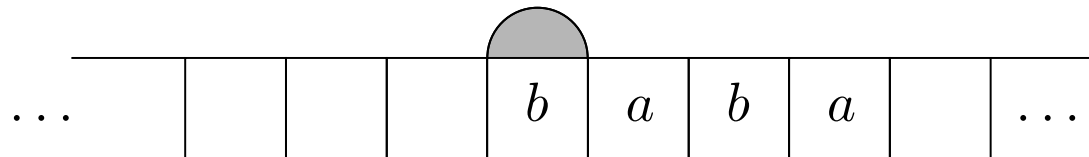
0ababa



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

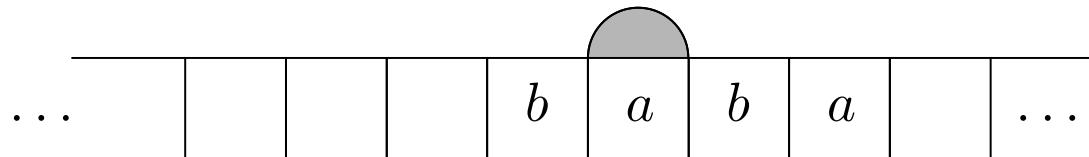
1baba



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

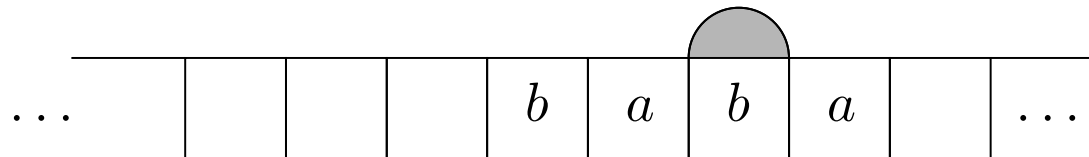
$b1aba$



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

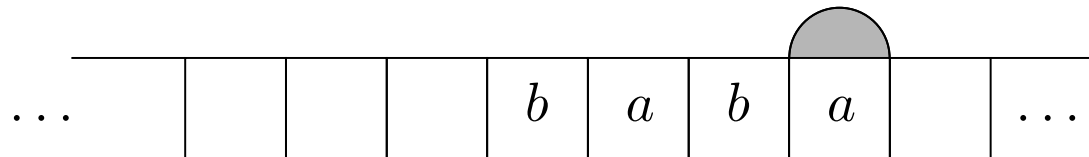
ba1ba



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

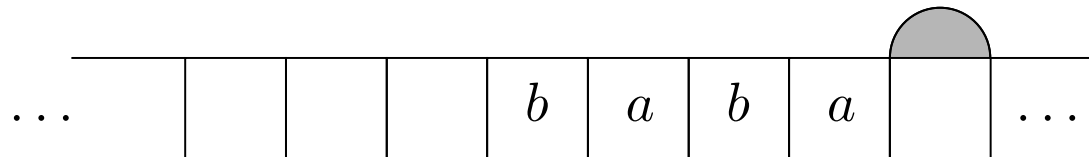
bab1a



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

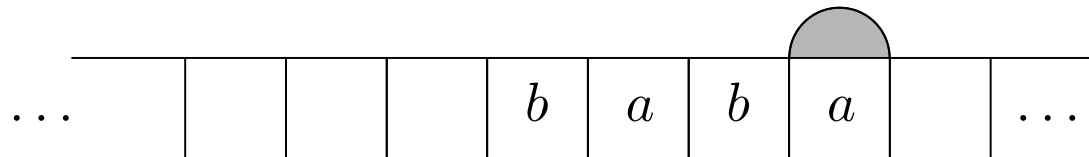
baba1



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

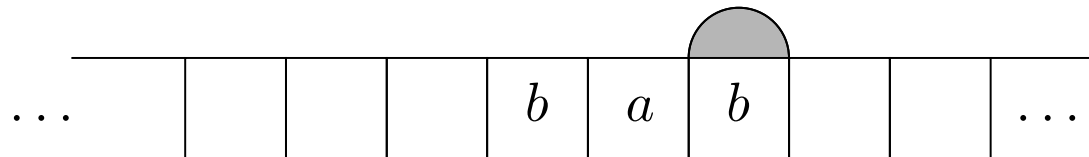
bab3a



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

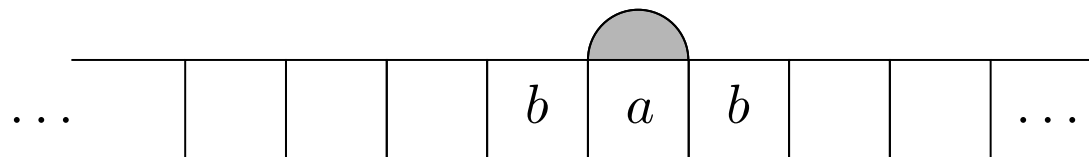
$ba5b$



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

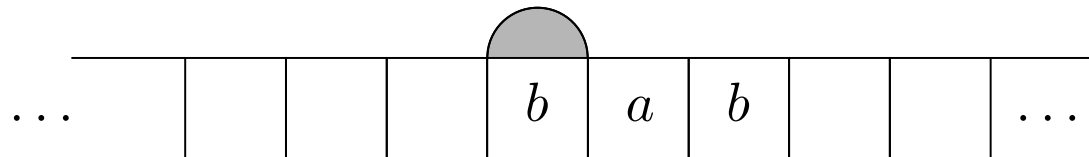
$b5ab$



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

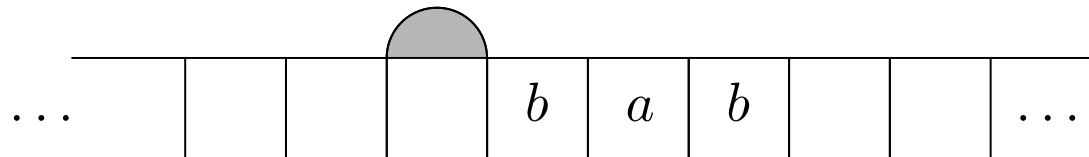
$5bab$



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

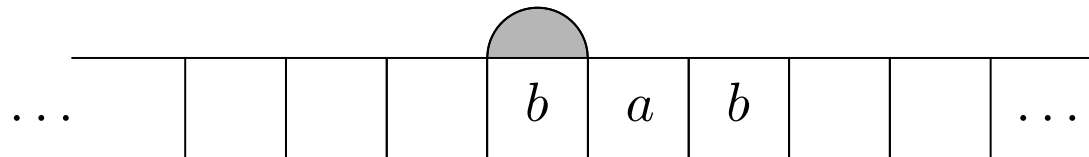
5 \sqcup \sqcup bab



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

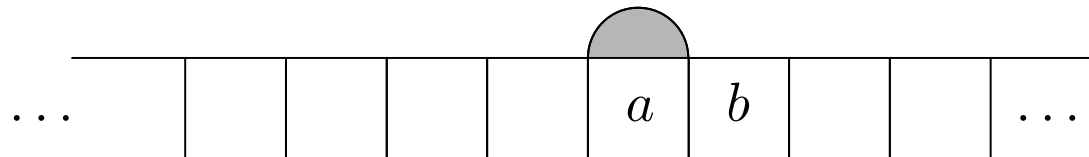
$0bab$



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

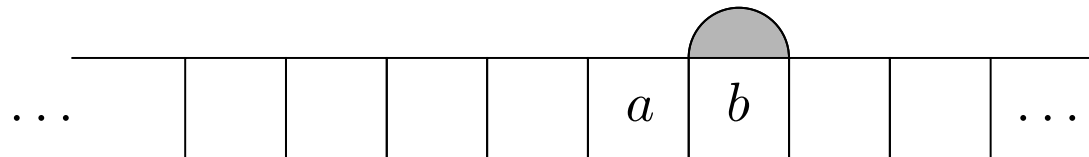
$2ab$



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

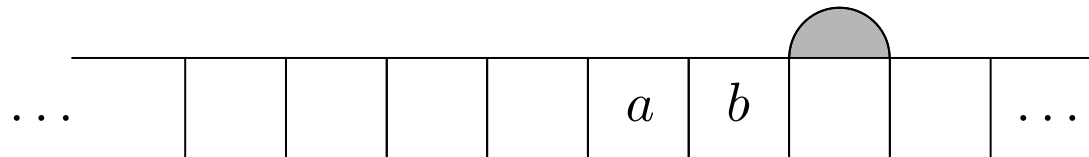
$a2b$



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

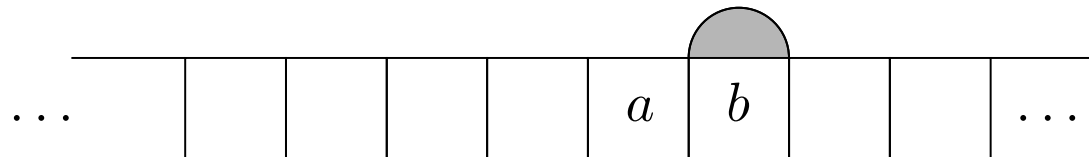
$ab2$



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

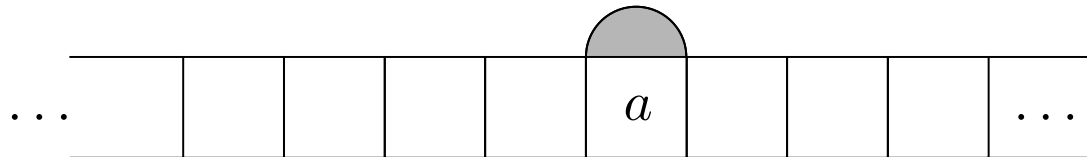
$a4b$



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

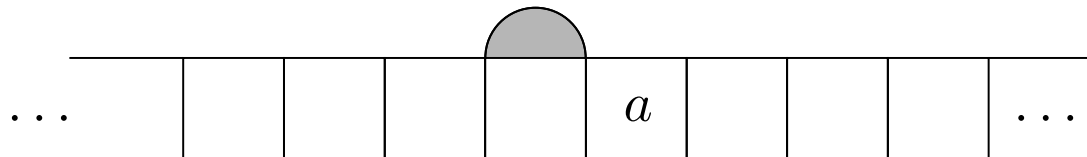
5a



Running a Turing machine–II

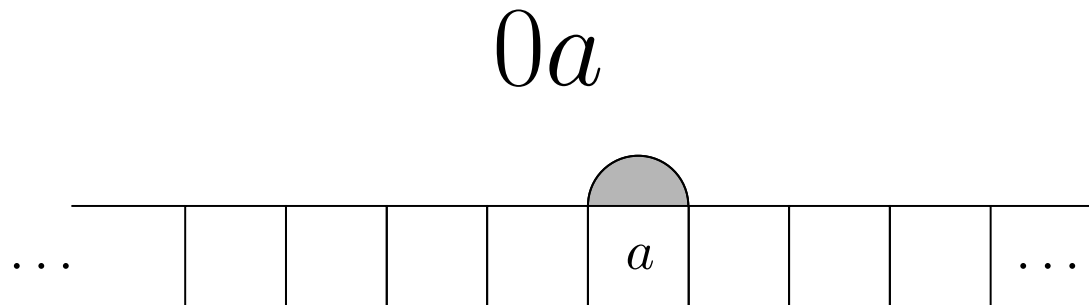
δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

5 \sqcup a



Running a Turing machine–II

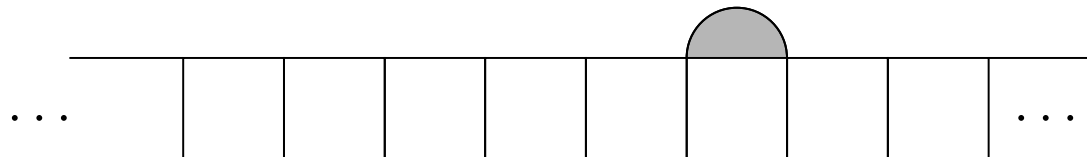
δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



Running a Turing machine–II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

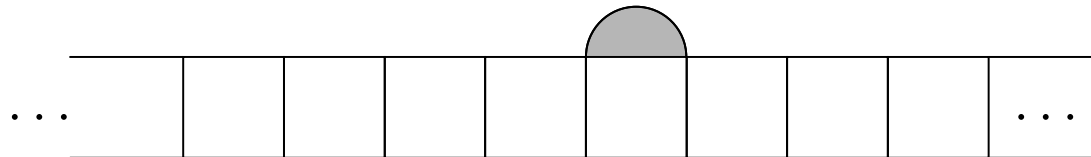
1



Running a Turing machine–II

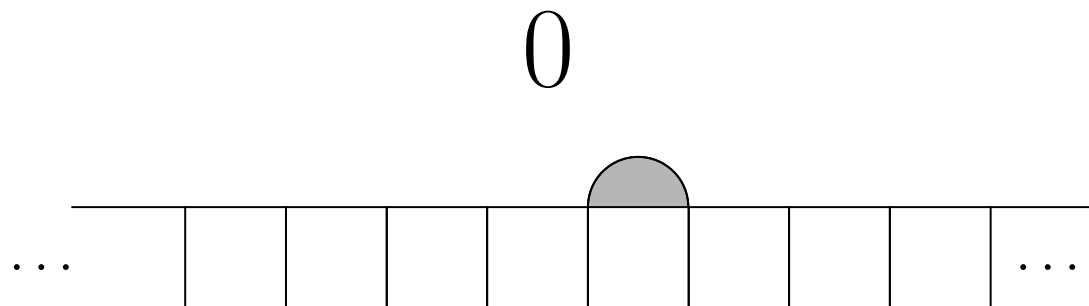
δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$

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Running a Turing machine-II

δ	a	b	\sqcup
0	$(1, \sqcup, R)$	$(2, \sqcup, R)$	stop
1	$(1, a, R)$	$(1, b, R)$	$(3, \sqcup, L)$
2	$(2, a, R)$	$(2, b, R)$	$(4, \sqcup, L)$
3	$(5, \sqcup, L)$	stop	$(0, \sqcup, N)$
4	stop	$(5, \sqcup, L)$	$(0, \sqcup, N)$
5	$(5, a, L)$	$(5, b, L)$	$(0, \sqcup, R)$



Running a Turing machine–II

Hence the word *ababa* is accepted by the machine.

We normally would not keep repeating the transition function, just give the sequence of configurations the machine goes through.

Running a Turing machine–II

$0ababa \rightarrow 1baba \rightarrow 1baba \rightarrow 1baba \rightarrow$
 $1baba \rightarrow 1baba \rightarrow b1aba \rightarrow ba1ba \rightarrow$
 $bab1a \rightarrow baba1 \rightarrow bab3a \rightarrow ba5b \rightarrow$
 $b5ab \rightarrow 5bab \rightarrow 5\sqcup bab \rightarrow 0bab \rightarrow$
 $2ab \rightarrow a2b \rightarrow ab2 \rightarrow a4b \rightarrow 5a \rightarrow$
 $5\sqcup a \rightarrow 0a \rightarrow 1 \rightarrow 3 \rightarrow 0$ **stop**