Terminating Minimal Model Generation Procedures for Propositional Modal Logics

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(Minimal) Model Generation

Useful for several tasks:

- hardware and software verification
- fault analysis
- commonsense reasoning
- query answering
- ...

Minimality criteria:

- domain minimality
- minimisation of a certain set of predicates
- minimal Herbrand models
- In this talk: models minimal modulo subset-simulation

Aims

- a new minimality criterion based on subset-simulation
- minimal model generation procedures for all sublogics of **S5**
  - sound
  - refutationally complete
  - minimal model complete
  - minimal model sound
- blocking techniques for all the logics under consideration

Propositional Modal Logic

Syntax: $\phi ::= \bot \mid \top \mid p_i \mid \neg\phi \mid \phi_1 \vee \phi_2 \mid \phi_1 \wedge \phi_2 \mid \Box\phi \mid \Diamond\phi$

Kripke Semantics: An interpretation $\mathcal{I}$ is a tuple (W, R, V).

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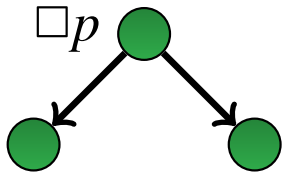

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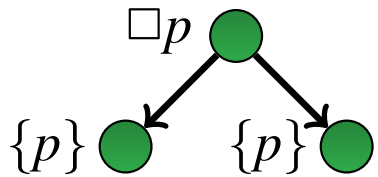

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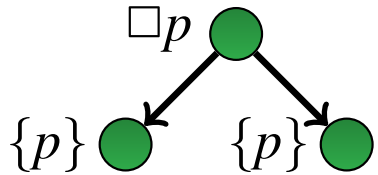

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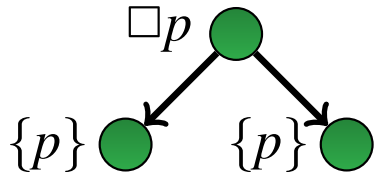

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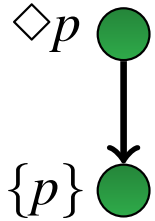

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Frame Properties

Fifteen possible logics:

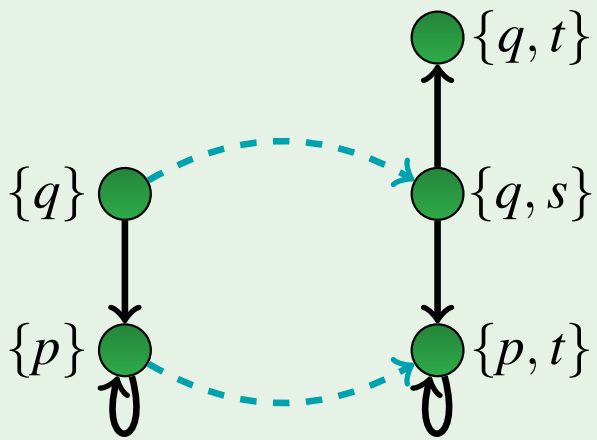
K, **KT**, **KB**, **KTB**, **KD**, **KDB**, **K4**, **K5**, **KT4**, **KD4**, **KD5**, **K45**, **KD45**, **KB4** and **KT5**$(=$ **S5**$)$

$\Box$	Axiom	Frame condition	First-order representation
K			
T	$\Box p \to p$	reflexivity	$\forall x R(x,x)$
B	$p \to \Box\Diamond p$	symmetry	$\forall x \forall y (R(x,y) \to R(y,x))$
D	$\Box p \to \Diamond p$	seriality	$\forall x \exists y R(x,y)$
4	$\Box p \to \Box\Box p$	transitivity	$\forall x \forall y \forall z (R(x,y) \wedge R(y,z) \to R(x,z))$
5	$\Diamond p \to \Box\Diamond p$	Euclideanness	$\forall x \forall y \forall z (R(x,y) \wedge R(x,z) \to R(y,z))$

Subset-Simulation $S_{\subseteq}$

Relation between nodes of two models $\mathcal{I} = (W, R, V)$ and $\mathcal{I}' = (W', R', V')$ s.t. for any two worlds $u \in W$ and $u' \in W'$, if uSu' then the following hold.

- $V(u) \subseteq V'(u')$, and
- if uRv, then there exists a $v' \in W'$ such that $u'R'v'$ and vSv'.



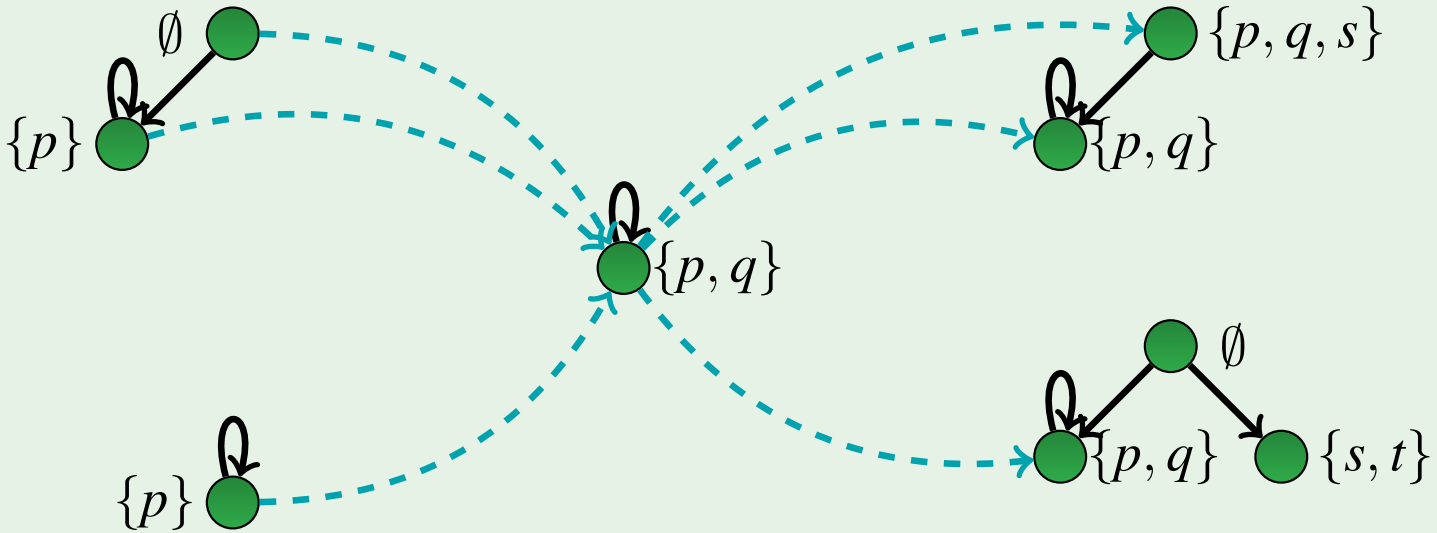
If for all $u \in W$ there is at least one $u' \in W'$ such that uSu', then we call $S_{\subseteq}$ a full subset-simulation from $\mathcal{I}$ to $\mathcal{I}'$ ($\mathcal{I} \leq_{\subseteq} \mathcal{I}'$).

Models Minimal Modulo Subset-Simulation

Subset-simulation is a preorder on models.

Definition

A model $\mathcal{I}$ of a modal formula φ is minimal modulo subset-simulation iff for any model $\mathcal{I}'$ of φ, if $\mathcal{I}' \leq_{\subseteq} \mathcal{I}$, then $\mathcal{I} \leq_{\subseteq} \mathcal{I}'$.



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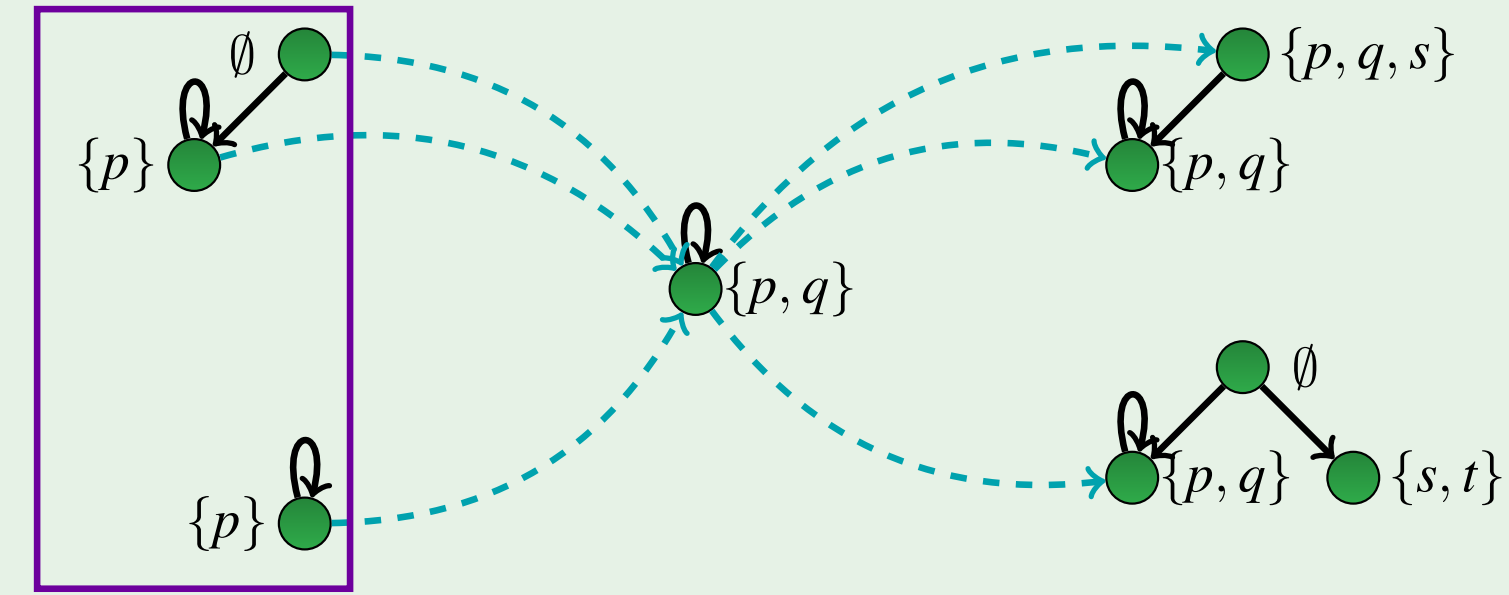

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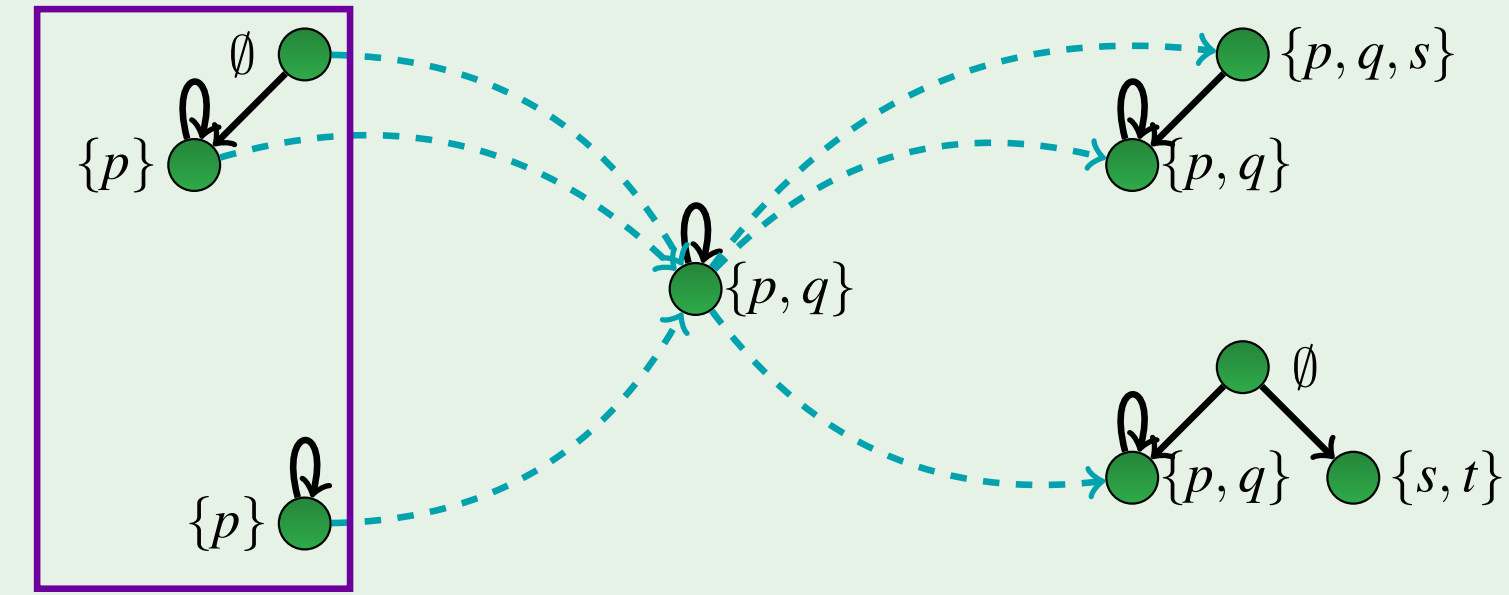


Infinitely many minimal models can belong to a symmetry class.

Minimal Model Soundness and Completeness

Minimal Model Soundness

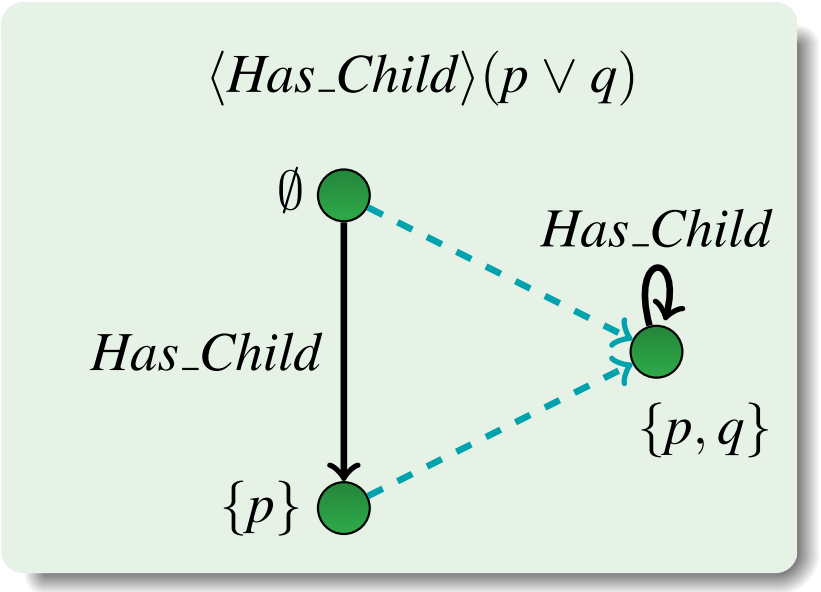
A procedure is minimal model sound if it generates only models minimal modulo subset-simulation.

Minimal Model Completeness

A procedure is minimal model complete if it generates at least one model minimal modulo subset-simulation per symmetry class.

Properties of the Minimality Criterion

- loop free models are preferred
- syntax independent
- minimisation of the valuation function
- suitable for many non-classical logics



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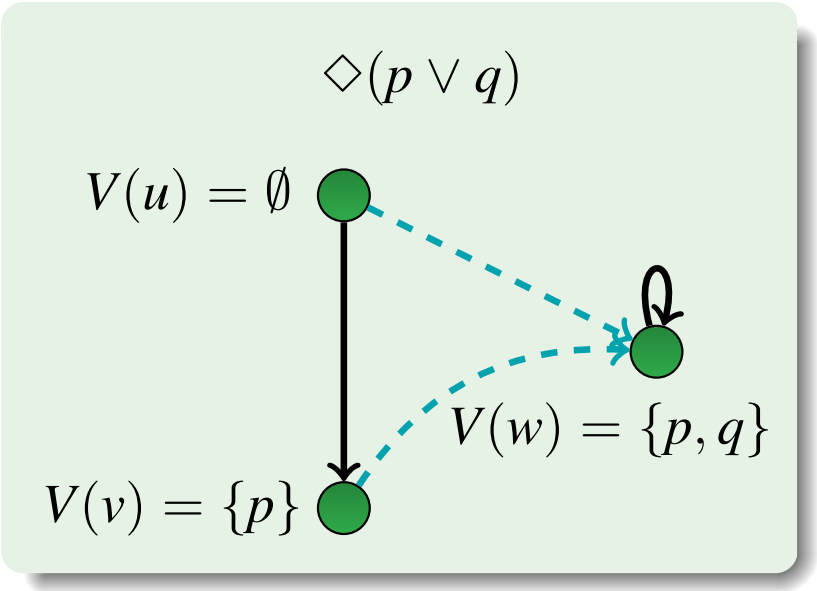

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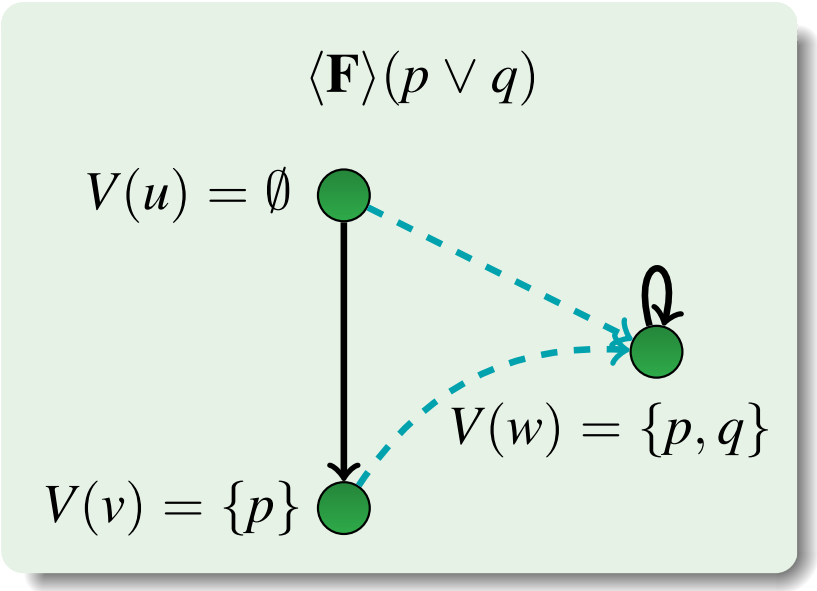

Procedures for Computing Minimal Models

Combination of tableaux calculi and a minimality test.

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- goal-oriented rules
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- termination
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Subset-simulation test

- closes unwanted branches of a tableau
- logic independent
- ensures minimal model soundness

Tableau Calculus

Input: a modal formula in negation normal form.

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Selection-based resolution:

- closure rule
- removes negative information from disjunctions

$$(SBR)\ \frac{u : p_1 \ \ldots \ u : p_n \qquad u : \neg p_1 \vee \ldots \vee \neg p_n \vee \Phi^+_\alpha}{u : \Phi^+_\alpha}$$

Φ^+_α: a disjunction where no disjunct is of the form $\neg p_i$.

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Lazy clausification:

- avoids preprocessing steps
- results in less inferences

$$(\alpha)\ \frac{u : (\phi_1 \wedge \ldots \wedge \phi_n) \vee \Phi^+_\alpha}{\begin{array}{c} u : \phi_1 \vee \Phi^+_\alpha \\ \vdots \\ u : \phi_n \vee \Phi^+_\alpha \end{array}}$$

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Tableau Calculus (cont'd)

Complement splitting:

- variation of the standard β rule
- detects trivially non-minimal models

$$(\beta)\ \frac{u : \mathcal{A} \vee \Phi^+}{\begin{array}{l|l} u : \mathcal{A} & u : \Phi^+ \\ u : neg(\Phi^+) & \end{array}}$$

$$\mathcal{A} ::= p \mid \Diamond\phi \mid \Box\phi$$
$$neg(\Phi^+) = \neg p_1 \wedge \ldots \wedge \neg p_n$$

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Expansion of diamond and box formulae:

$$(\Diamond)\ \frac{u : \Diamond\phi}{\begin{array}{c}(u,v) : R \\ v : \phi\end{array}}$$

$$(\Box)\ \frac{(u,v) : R \qquad u : \Box\phi}{v : \phi}$$

v is a fresh new world

Φ^+: a disjunction where no disjunct is of the form $\neg p_i$ or is a conjunction.

Frame Properties Rules

$$(\mathbf{T})\ \frac{}{(u,u) : R} \qquad (\mathbf{B})\ \frac{(u,v) : R}{(v,u) : R}$$

$$(\mathbf{4})\ \frac{(u,v) : R \qquad (v,w) : R}{(u,w) : R} \qquad (\mathbf{5})\ \frac{(u,v) : R \qquad (u,w) : R}{(v,w) : R}$$

$$(\mathbf{D})\ \frac{}{u : \Diamond \top}$$

Properties of the Tableaux Calculi

- sound
- refutationally complete
- minimal model complete
- NOT minimal model sound

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Subset-Simulation Test

Early closure of “non-minimal” branches

A partial model $\mathcal{I}$ subset-simulates an extracted model $\mathcal{I}'$ ($\mathcal{I}' \leq_{\subseteq} \mathcal{I}$).

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Backward closure of branches - minimal model refining

$\mathcal{I} =$ newly extracted model, $S =$ current set of minimal models.

- if there is an $\mathcal{I}' \in S$ s.t. $\mathcal{I}' \leq_{\subseteq} \mathcal{I}$, close the current branch
- close all the branches from which an $\mathcal{I}'$ s.t. $\mathcal{I} \leq_{\subseteq} \mathcal{I}'$ was extracted

Properties of the Procedures (so far)

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They might not terminate!

Termination

A procedure terminates if the tableau calculus has strong termination.

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K, **KT**, **KB** and **KTB**: already terminating.

Main Problem

Finding blocking techniques that preserve minimal model completeness.

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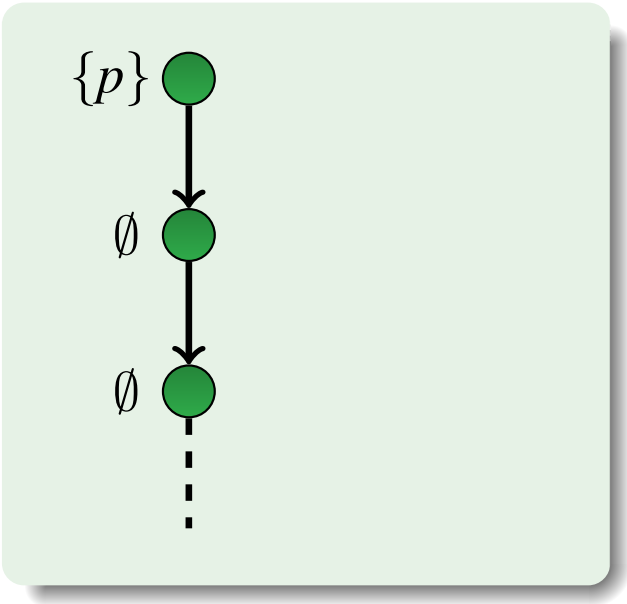
KD and KDB

- non-termination is caused by seriality
- infinite paths $u_1, \ldots, u_i$ where $V(u_j) = \emptyset$



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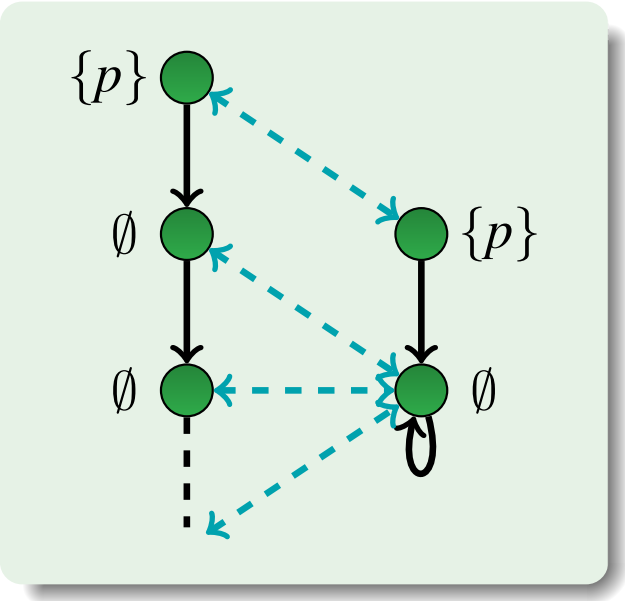
  $\Rightarrow$ create a loop on u_1



Termination (cont'd)

K4, **KT4** and **KD4**

Known blocking techniques are

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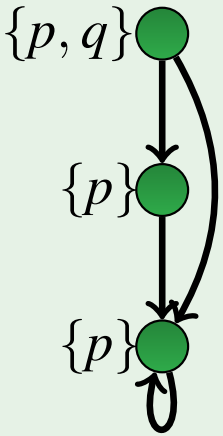

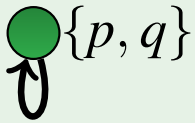

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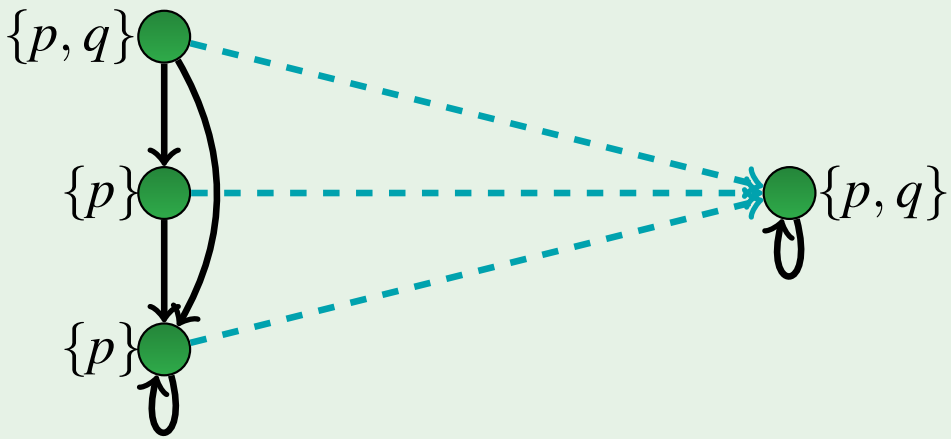

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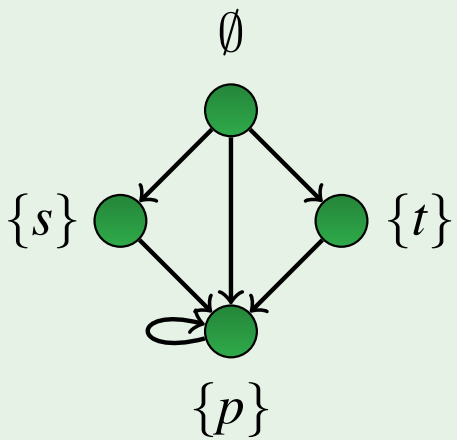

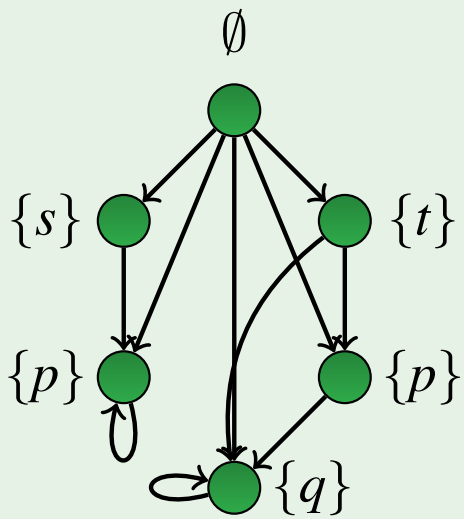

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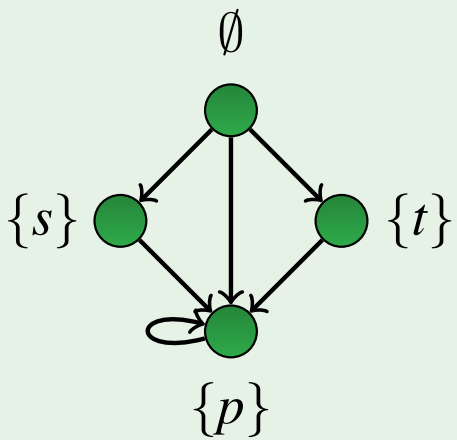

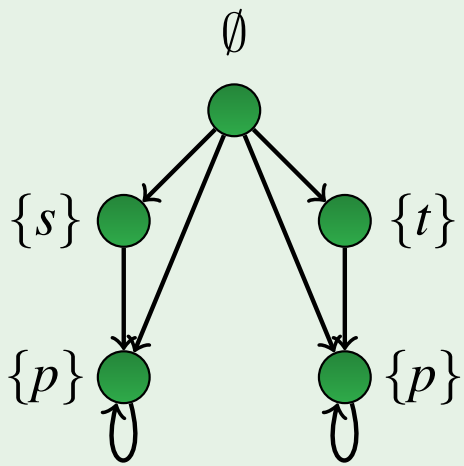

Termination (cont'd)

K5, **KD5**, **K45**, **KD45**, **KB4** and **KT5**: dynamic anywhere equality blocking.

For any two non-root worlds u and v

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Theorem

All the normal modal logics between **K** and **KT5** have finitely many symmetry classes of models minimal modulo subset-simulation.

Conclusion and Further Work

- a new minimality criterion based on subset-simulation
- procedures for all the sublogics of **KT5**
  - sound
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  - terminating

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  - multi-modal logics
  - universal modalities
  - inclusion axioms
  - converse relation
- generalisation to fragments of first-order logic
  - guarded fragment
  - guarded negation
- implementation