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Minimal Models for Modal Logics

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Introduction

(Minimal) model generation is useful for several tasks such as hardware and software verification, fault analysis, and commonsense reasoning.

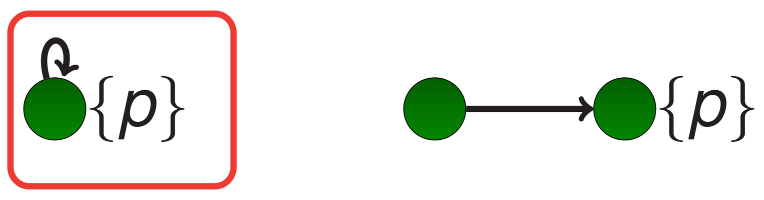
For classical logics, several minimality criteria have already been studied (**domain minimality**, **minimisation of a certain set of predicates**, **minimal Herbrand models**).

These minimality criteria can be applied to modal logics, and it is also possible to adopt a more "modal" criterion: **minimality with respect to bisimulation**.

Domain Minimality

Minimality is based on the size of the domain.

Example for $\Diamond p$

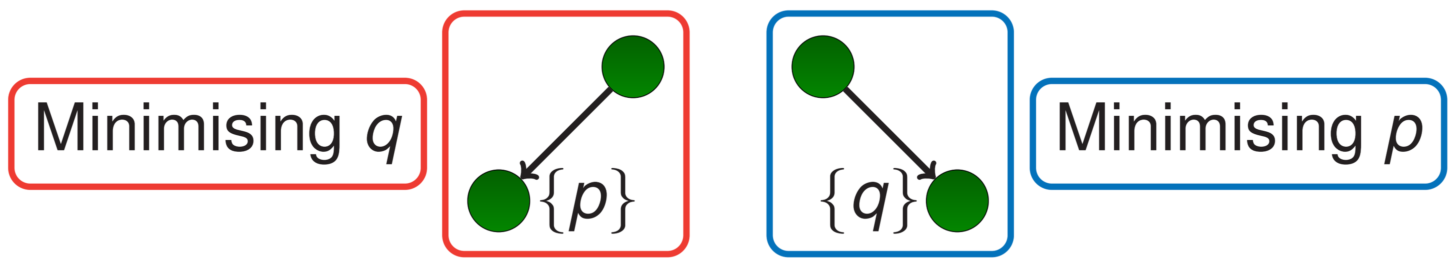


- ☺ Termination for logics with the finite model property is easily achievable
- ☹ All possible diamond expansions must be tried
- ☹ Minimal model completeness is neither easy to achieve nor desirable

Minimisation of a Set of Predicates

Minimality is based on the subset relation over the extensions of a specific set of predicates.

Example for $\Diamond(p \lor q)$

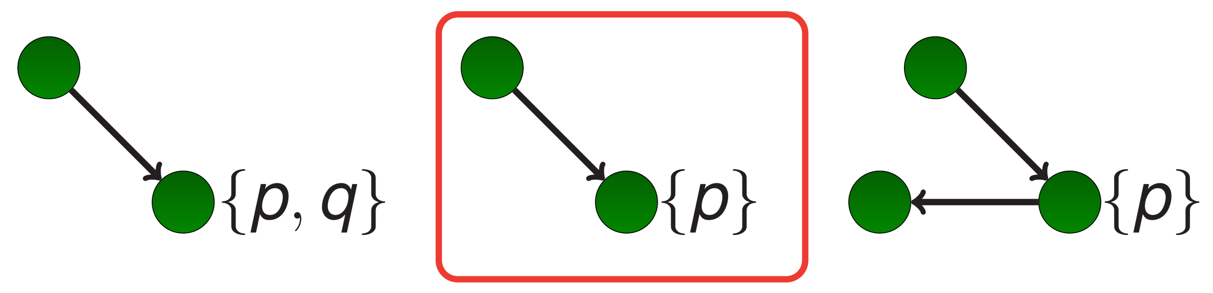


- ☺ In theory: no constraint for expanding diamond formulae is needed
- ☹ In practice: constraints are necessary for minimal model completeness
- 😐 Domains of different models must be comparable

Minimal Modal Herbrand Models

Minimality is based on the subset relation over the extensions of all predicates.

Example for $\Diamond p \land \Box(p \lor q \lor \Diamond\top)$

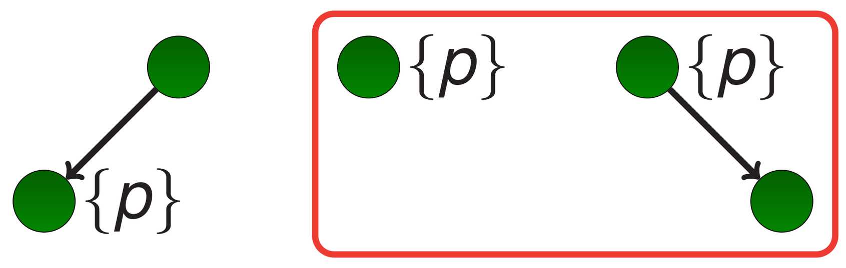


- ☺ Diamond expansions are completely deterministic
- ☺ Herbrand models are widely used in automated reasoning
- ☹ A blocking technique is necessary for termination
- 😐 Domains of different models must be comparable

Minimal Under Bisimulation

Minimality is based on the existence of bisimulation between (sub)models.

Example for $p \lor \Diamond p \lor (p \land \Diamond\top)$



- ☺ It is more semantic than other minimality criteria
- ☹ Bisimulation is too strong, it only closes models that contain *generated submodels*

References

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