

# Minimal Models for Modal Logics

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## Introduction

(Minimal) model generation is useful for several tasks such as hardware and software verification, fault analysis, and commonsense reasoning.

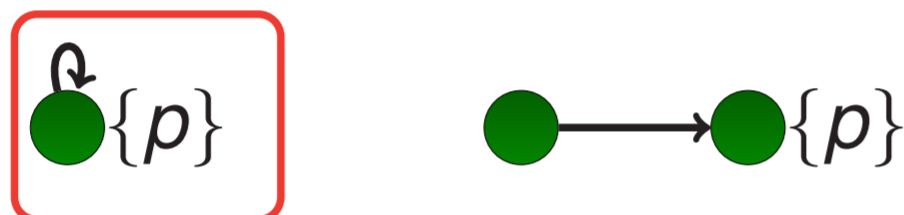
For classical logics, several minimality criteria have already been studied (**domain minimality**, **minimisation of a certain set of predicates**, **minimal Herbrand models**).

These minimality criteria can be applied to modal logics, and it is also possible to adopt a more “modal” criterion: **minimality with respect to bisimulation**.

## Domain Minimality

Minimality is based on the size of the domain.

Example for  $\diamond p$

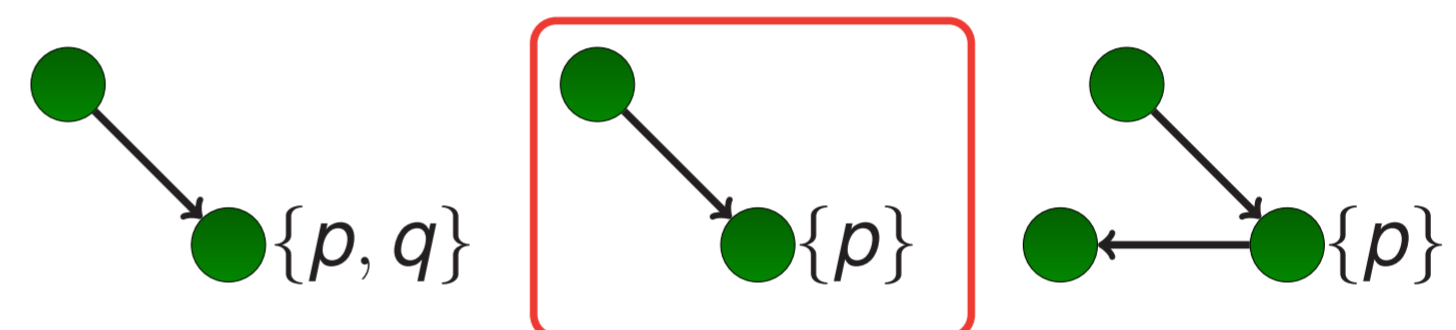


- 😊 Termination for logics with the finite model property is easily achievable
- 😞 All possible diamond expansions must be tried
- 😞 Minimal model completeness is neither easy to achieve nor desirable

## Minimal Modal Herbrand Models

Minimality is based on the subset relation over the extensions of all predicates.

Example for  $\diamond p \wedge \Box(p \vee q \vee \diamond T)$

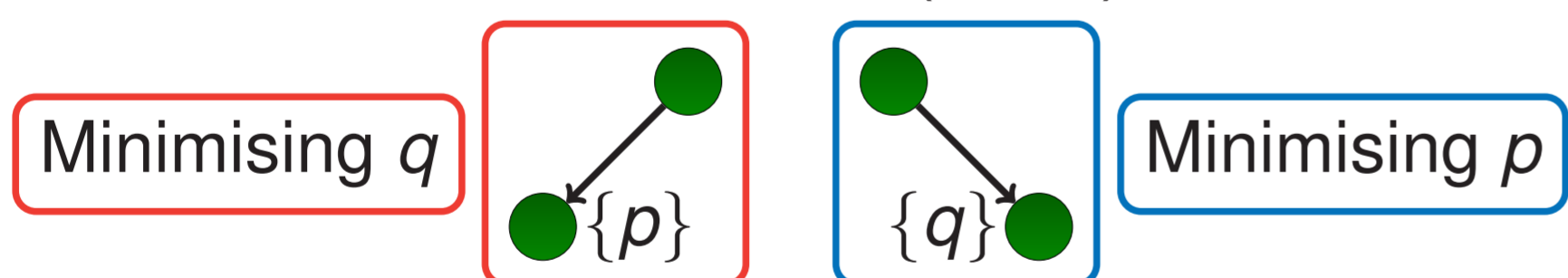


- 😊 Diamond expansions are completely deterministic
- 😊 Herbrand models are widely used in automated reasoning
- 😞 A blocking technique is necessary for termination
- 😞 Domains of different models must be comparable

## Minimisation of a Set of Predicates

Minimality is based on the subset relation over the extensions of a specific set of predicates.

Example for  $\diamond(p \vee q)$

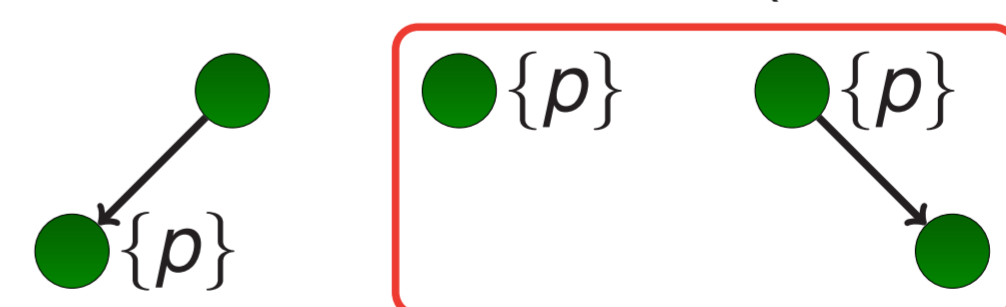


- 😊 In theory: no constraint for expanding diamond formulae is needed
- 😞 In practice: constraints are necessary for minimal model completeness
- 😞 Domains of different models must be comparable

## Minimal Under Bisimulation

Minimality is based on the existence of bisimulation between (sub)models.

Example for  $p \vee \diamond p \vee (p \wedge \diamond T)$



- 😊 It is more semantic than other minimality criteria
- 😞 Bisimulation is too strong, it only closes models that contain *generated submodels*

## References

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