A Modal Tableau Approach for Minimal Model Generation

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Model generation/minimization

Model generation is used in computer science areas like

- system verification and debugging
- validation and debugging of data models
- non-monotonic reasoning

Model minimization can be categorized in:

- minimization of the domain
- minimization of specific predicates
- minimization of all predicates

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Minimal Modal Herbrand Model

Modal Herbrand model $\Longleftrightarrow$ Herbrand model for the first-order logic translation

Example

Modal Herbrand models of $p_1 \wedge (\Diamond p_2 \vee p_3)$:

$I_1 = \{w : p_1, w : p_3\}$

$I_2 = \{w : p_1, f_{\Diamond p_2}(w) : p_2, (w, f_{\Diamond p_2}(w)) : R\}$

$I_3 = \{w : p_1, w : p_3, f_{\Diamond p_2}(w) : p_2, (w, f_{\Diamond p_2}(w)) : R\}$

I_3 is not minimal because $I_1 \subset I_3$ and $I_2 \subset I_3$

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Minimal Modal Model Generation (3MG) Calculus

Input: a set of modal clauses

Box miniscoping during the conjunctive normal form transformation: $\Box(\phi_1 \wedge \phi_2) \Rightarrow \Box\phi_1 \wedge \Box\phi_2$

Expansion strategy: depth-first left-to-right strategy

Output:

- the input is unsatisfiable (closed tableau)
- all and only minimal modal Herbrand models, each model exactly once
  (open branch $\leadsto$ minimal modal Herbrand model)

Rules of the 3MG calculus

Four expansion rules:

- $(\Diamond)$: expands diamond formulae
- $(\vee)_E$: prepares the input for the other rules
- (CS) – complement splitting rule:
  - avoids model duplication
  - ensures that the first model is minimal
- $(PUHR)$:
  - expands clauses with negative literals
  - can close a branch

Model constraint propagation rule:
blocks the generation of non-minimal model

A derivation example

Derivation for:

$$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$$

A derivation example

$w : \Diamond p$

$w : q$

$w : \neg q \vee \Box s$

$w : \Diamond p \vee \Diamond s$

$w : \neg q \vee w : \Box s$

$w : \Diamond p \vee w : \Diamond s$

$(w, f_{\Diamond p}(w)) : R$

$f_{\Diamond p}(w) : p$

$f_{\Diamond p}(w) : s$

$w : \Diamond p$

$(w, f_{\Diamond s}(w)) : \neg R$

$(w, f_{\Diamond p}(w)) : R$

$f_{\Diamond p}(w) : p$

$w : \Diamond s$

MC

$\perp$

Derivation for:
$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$

Set of clauses in input

A derivation example

$w : \Diamond p$

$w : q$

$w : \neg q \vee \Box s$

$w : \Diamond p \vee \Diamond s$

$w : \neg q \vee w : \Box s$

$w : \Diamond p \vee w : \Diamond s$

$(w, f_{\Diamond p}(w)) : R$

$f_{\Diamond p}(w) : p$

$f_{\Diamond p}(w) : s$

$w : \Diamond p$	$w : \Diamond s$
$(w, f_{\Diamond s}(w)) : \neg R$	MC
$(w, f_{\Diamond p}(w)) : R$	$\bot$
$f_{\Diamond p}(w) : p$	

Derivation for:

$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$

$(\vee)_E$ rule

A derivation example

$w : \Diamond p$
$w : q$
$w : \neg q \vee \Box s$
$w : \Diamond p \vee \Diamond s$
$w : \neg q \vee w : \Box s$
$w : \Diamond p \vee w : \Diamond s$
$(w, f_{\Diamond p}(w)) : R$
$f_{\Diamond p}(w) : p$
$f_{\Diamond p}(w) : s$

$w : \Diamond p$	$w : \Diamond s$
$(w, f_{\Diamond s}(w)) : \neg R$	MC
$(w, f_{\Diamond p}(w)) : R$	$\bot$
$f_{\Diamond p}(w) : p$	

Derivation for:
$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$

$(\vee)_E$ rule

A derivation example

$w : \Diamond p$
$w : q$
$w : \neg q \vee \Box s$
$w : \Diamond p \vee \Diamond s$
$w : \neg q \vee w : \Box s$
$w : \Diamond p \vee w : \Diamond s$
$(w, f_{\Diamond p}(w)) : R$
$f_{\Diamond p}(w) : p$
$f_{\Diamond p}(w) : s$

$w : \Diamond p$
$(w, f_{\Diamond s}(w)) : \neg R$
$(w, f_{\Diamond p}(w)) : R$
$f_{\Diamond p}(w) : p$

$w : \Diamond s$
MC
$\bot$

Derivation for:
$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$

$(\Diamond)$ rule

A derivation example

$w : \Diamond p$
$w : q$
$w : \neg q \vee \Box s$
$w : \Diamond p \vee \Diamond s$
$w : \neg q \vee w : \Box s$
$w : \Diamond p \vee w : \Diamond s$
$(w, f_{\Diamond p}(w)) : R$
$f_{\Diamond p}(w) : p$
$f_{\Diamond p}(w) : s$

$w : \Diamond p$
$(w, f_{\Diamond s}(w)) : \neg R$
$(w, f_{\Diamond p}(w)) : R$
$f_{\Diamond p}(w) : p$

$w : \Diamond s$
MC
$\bot$

Derivation for:
$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$

$(PUHR)$ rule

A derivation example

$$w : \Diamond p$$
$$w : q$$
$$w : \neg q \vee \Box s$$
$$w : \Diamond p \vee \Diamond s$$
$$w : \neg q \vee w : \Box s$$
$$w : \Diamond p \vee w : \Diamond s$$
$$(w, f_{\Diamond p}(w)) : R$$
$$f_{\Diamond p}(w) : p$$
$$f_{\Diamond p}(w) : s$$

$$w : \Diamond p \qquad\qquad w : \Diamond s$$
$$(w, f_{\Diamond s}(w)) : \neg R \qquad MC$$
$$(w, f_{\Diamond p}(w)) : R \qquad \bot$$
$$f_{\Diamond p}(w) : p$$

Derivation for:
$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$

(CS) rule

A derivation example

$$w : \Diamond p$$
$$w : q$$
$$w : \neg q \vee \Box s$$
$$w : \Diamond p \vee \Diamond s$$
$$w : \neg q \vee w : \Box s$$
$$w : \Diamond p \vee w : \Diamond s$$
$$(w, f_{\Diamond p}(w)) : R$$
$$f_{\Diamond p}(w) : p$$
$$f_{\Diamond p}(w) : s$$

$$w : \Diamond p$$
$$(w, f_{\Diamond s}(w)) : \neg R$$
$$(w, f_{\Diamond p}(w)) : R$$
$$f_{\Diamond p}(w) : p$$

$$w : \Diamond s$$
$$MC$$
$$\perp$$

Derivation for:
$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$

$(\Diamond)$ rule

A derivation example

$w : \Diamond p$
$w : q$
$w : \neg q \vee \Box s$
$w : \Diamond p \vee \Diamond s$
$w : \neg q \vee w : \Box s$
$w : \Diamond p \vee w : \Diamond s$
$(w, f_{\Diamond p}(w)) : R$
$f_{\Diamond p}(w) : p$
$f_{\Diamond p}(w) : s$

$w : \Diamond p$	$w : \Diamond s$
$(w, f_{\Diamond s}(w)) : \neg R$	MC
$(w, f_{\Diamond p}(w)) : R$	$\perp$
$f_{\Diamond p}(w) : p$	

Derivation for:
$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$

fully expanded
open branch

$\Downarrow$

Minimal Model:
$\{w : q, f_{\Diamond p}(w) : p, f_{\Diamond p}(w) : s, (w, f_{\Diamond p}(w)) : R\}$

A derivation example

$w : \Diamond p$
$w : q$
$w : \neg q \vee \Box s$
$w : \Diamond p \vee \Diamond s$
$w : \neg q \vee w : \Box s$
$w : \Diamond p \vee w : \Diamond s$
$(w, f_{\Diamond p}(w)) : R$
$f_{\Diamond p}(w) : p$
$f_{\Diamond p}(w) : s$

/ \

$w : \Diamond p$
$(w, f_{\Diamond s}(w)) : \neg R$
$(w, f_{\Diamond p}(w)) : R$
$f_{\Diamond p}(w) : p$

$w : \Diamond s$
MC
$\perp$

Derivation for:
$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$

Model constraint

Minimal Model:
$\{w : q, f_{\Diamond p}(w) : p,$
$f_{\Diamond p}(w) : s, (w, f_{\Diamond p}(w)) : R\}$

Constraint (MC): $w : \neg q \vee f_{\Diamond p}(w) : \neg p \vee f_{\Diamond p}(w) : \neg s \vee (w, f_{\Diamond p}(w)) : \neg R$

A derivation example

$$
\begin{array}{l}
w : \Diamond p \\
w : q \\
w : \neg q \vee \Box s \\
w : \Diamond p \vee \Diamond s \\
w : \neg q \vee w : \Box s \\
w : \Diamond p \vee w : \Diamond s \\
(w, f_{\Diamond p}(w)) : R \\
f_{\Diamond p}(w) : p \\
f_{\Diamond p}(w) : s
\end{array}
$$

$$
\begin{array}{ll}
w : \Diamond p & w : \Diamond s \\
(w, f_{\Diamond s}(w)) : \neg R & MC \\
(w, f_{\Diamond p}(w)) : R & \bot \\
f_{\Diamond p}(w) : p &
\end{array}
$$

Derivation for:
$\Diamond p \wedge q \wedge (\neg q \vee \Box s) \wedge (\Diamond p \vee \Diamond s)$

$(PUHR)$ rule

Minimal Model:
$\{w : q, f_{\Diamond p}(w) : p,$
$f_{\Diamond p}(w) : s, (w, f_{\Diamond p}(w)) : R\}$

Constraint (MC): $w : \neg q \vee f_{\Diamond p}(w) : \neg p \vee f_{\Diamond p}(w) : \neg s \vee (w, f_{\Diamond p}(w)) : \neg R$

Conclusion

The presented calculus

- is minimal model sound and complete
- terminates
- does not create model duplicate

Most of these properties can be proved by a translation proof using the PUHR approach for the first-order logic