Knowledge Representation and Reasoning in NMR and Extended LP

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I. Introduction

- Some History
- Nonmonotonicity in KR
- How this Lecture is organized

1 Some History

- **1956:** Dartmouth Workshop, ‘‘Artificial Intelligence”
- **1962:** GPS (Newell/Simon)
- **1965:** Resolution (Robinson)
- **1973:** PROLOG (Colmerauer et. al.)
- **1980:** Nonmonotonic Reasoning (Reiter, McCarthy, McDermott/Doyle)
- **1986:** Workshop on DDB and LP (Minker)
- **since 1991:** Series of LPNMR conferences (see http://www.uni-koblenz.de/ag-ki/lpnmr97/)

2 Nonmonotonicity in KR

- **Situation Calculus:** McCarthy/Hayes
- **Frame-Problem:** Describe non-changes efficiently
- **Default Logic**, **Autoepistemic Logic**, **Circumscription**
- **Complexity:** Due to consistency checks: not even r.e. But very compact problem description.

3 How this Lecture is organized

Thursday: Chapter I: Introduction
Chapter II: General NML's

Friday: Chapter III: Automating NML's

Monday: Chapter IV: Nonmon. Semantics of LP: I

Wednesday: Chapter V, VI: Nonmon. Semantics of LP: II

Thursday: Chapter VII: Complexity, Expressibility and Implementation

Friday: Chapter VIII: Applications and Benchmarks

II. General NML's

- Default Logic
- Circumscription
- Rational Closure

1 Default Logic

Definition 1.1 (CWA)

The closed world assumption of a database T is the theory

$$\mathrm{CWA}(T) = \mathrm{Cn}(T \cup \{\neg p :\ T \not\vdash p\})\ ,$$

where p is a ground predicate instance.

In first-order logic, ground terms that are syntactically different can refer to the same individual, e.g., $a = b$ means that the individual referred to by a is the same as the one referred to by b.

$$\text{UNA}: \quad \{t_i \neq t_j \mid t_i \text{ and } t_j \text{ are different ground terms}\} \tag{1}$$

A second assumption is that the number of individuals is finite and that every individual has a name in the underlying language.

$$\text{DCA}: \quad \forall x\,(t_1 \doteq x \vee t_2 \doteq x \vee t_3 \doteq x \vee \cdots \vee t_n \doteq x) \tag{2}$$

Definition 1.2 (Herbrand Models)
Given a theory T in a language $\mathcal{L}$, we define

Herbrand Universe U_T**:** *it is the algebra consisting of all terms that can be built using the constants and function symbols from $\mathcal{L}$, and*

Herbrand Base B_T**:** *it is the set of all atomic formulae of the form $r(t_1,\ldots,t_n)$, where r is a relation symbol of $\mathcal{L}$ and $t_1,\ldots,t_n$ are ground terms from U_T.*

It is well-known that restricting to Herbrand models (as opposed to truth in all models) is a much more complex task — namely Π^1_1-complete (while Π^0_1-complete, i.e. recursively enumerable, for all models).

A default theory is a pair $(\mathcal{D}, W)$. W is a set of first order formulas representing the facts which are known to be true with certainty.
$\mathcal{D}$ is a set of defaults the form

$$\frac{\varphi : \psi_1, \ldots, \psi_n}{\gamma}$$

where φ, ψ_i and γ are classical formulas.

The default has the intuitive reading:

if φ is provable and, for all i $(1 \leq i \leq n)$, $\neg\psi_i$ is not provable, then derive γ.

φ is called the *prerequisite*, ψ_i a *consistency condition* or *justification*, and γ the *consequent* of the default.

Default theories induce so-called *extensions* which represent acceptable belief sets a reasoner may adopt based on the available information.

A formula φ is called a *sceptical* consequence of $(\mathcal{D}, W)$

if and only if

φ is contained in all extensions of $(\mathcal{D}, W)$.

φ is called a *credulous* consequence of $(\mathcal{D}, W)$

if and only if

φ is contained in at least one extension of $(\mathcal{D}, W)$.

Intuitively, E is an extension of $(\mathcal{D}, W)$

if and only if

E is a deductively closed (in the sense of classical logic) superset of W satisfying the following two properties

1. all defaults that are "applicable" with respect to E have been applied,
2. every formula in E has a "derivation" from W and applicable defaults.

Definition 1.3 (Default Proof, $Cn_{\mathrm{Mon}(\mathcal{D})}(W)$)
Let $(\mathcal{D}, W)$ be a default theory, S a set of formulas, and φ a formula. A $(\mathcal{D}, W)$-default proof for φ is a finite sequence $\Delta = (d_1, \ldots, d_n)$ of defaults in $\mathcal{D}$ such that:

1. $W \cup \{\mathrm{cons}(d_1), \ldots, \mathrm{cons}(d_{i-1})\} \vdash \mathrm{pre}(d_i)$, *for* $i \in \{1, \ldots, n\}$,
2. $W \cup \{\mathrm{cons}(d_1), \ldots, \mathrm{cons}(d_n)\} \vdash \varphi$.

Δ *is valid in* S if and only if S *does not contain the negation of a justification of a default in* Δ.

A slightly different, but completely equivalent formulation is to associate to any set of defaults $\mathcal{D}$ a set $\mathrm{Mon}(\mathcal{D})$ of associated *monotonic* inference rules:

$$\mathrm{Mon}(\mathcal{D}) := \{\frac{\mathrm{pre}(d)}{\mathrm{cons}(d)} : d \in \mathcal{D}\}.$$

- Let $\mathrm{Cn}_{\mathrm{Mon}(\mathcal{D})}(W)$ be the closure of W with respect to classical derivability and the rules $\mathrm{Mon}(\mathcal{D})$.
- Formulae having a $(\mathcal{D},W)$-default proof are exactly those contained in $\mathrm{Cn}_{\mathrm{Mon}(\mathcal{D})}(W)$.
- Valid proofs are those that also respect the justifications of the original defaults.

Definition 1.4 (Extension 1)
Let $(\mathcal{D},W)$ be a default theory. E is an extension of $(\mathcal{D},W)$ if and only if E *is a deductively closed superset of W satisfying the conditions*

1. *if $\varphi{:}\psi_1,\ldots,\psi_n/\gamma \in \mathcal{D}$, $\varphi \in E$ and for all i $(1 \leq i \leq n)$ $\neg\psi_i \notin E$, then $\gamma \in E$, and*
2. *$\varphi \in E$ implies there is a $(\mathcal{D},W)$-default proof for φ valid in E. This condition is also called* groundedness.

$\mathcal{D}$	W	Fixed Point(s)
$\frac{bird(x):fly(x)}{fly(x)}$	$bird(tw)$	$\mathrm{Cn}(W \cup \{fly(tw)\})$
$\frac{bird(x):fly(x)}{fly(x)}$	$bird(tw)$ $peng(tw)$ $\forall x(peng(x) \leftarrow \neg fly(x))$	$\mathrm{Cn}(W)$
$\frac{bird(x):fly(x)}{fly(x)}$ $\frac{peng(x):\neg fly(x)}{\neg fly(x)}$	$bird(tw)$ $peng(tw)$	$\mathrm{Cn}(W \cup \{fly(tw)\})$ $\mathrm{Cn}(W \cup \{\neg fly(tw)\})$
$\frac{bird(x):fly(x) \wedge \neg peng(x)}{fly(x)}$ $\frac{peng(x):\neg fly(x)}{\neg fly(x)}$	$bird(tw)$ $peng(tw)$	$\mathrm{Cn}(W \cup \{\neg fly(tw)\})$

Table 1: Some Default Theories and Their Extensions

2 Circumscription

- Is formulated entirely within classical logic.
- Instead of arbitrary models, *minimal* models are considered.
- Can be seen as a better version of CWA.
- Knowledge needs to be formulated using *abnormalities*.

Let $\psi(\underline{p}, \underline{z})$ be a first-order sentence containing the symbols $\underline{p}$ and $\underline{z}$. Circumscription prefers models of $\psi(\underline{p}, \underline{z})$ that are minimal in the predicates $\underline{p}$, assuming that these models have the same interpretation for all symbols not in $\underline{p}$ or $\underline{z}$: such predicates are called *fixed symbols*.

To state this more formally, let $\mathcal{M}_1$ and $\mathcal{M}_2$ be two models of $\psi(\underline{p}, \underline{z})$. $|\mathcal{M}|$ is the universe of model $\mathcal{M}$, and $\mathcal{M}[\![r]\!]$ is the interpretation of the predicate r in $\mathcal{M}$.

Definition 2.1 (Minimal Models)

$$\mathcal{M}_1 \leq^{\underline{p};\underline{z}} \mathcal{M}_2 \quad \Leftrightarrow \quad \begin{cases} 1. \quad |\mathcal{M}_1| = |\mathcal{M}_2|^{\mathrm{a}}. \\ 2. \quad \mathcal{M}_1[\![r]\!] = \mathcal{M}_2[\![r]\!] \textit{ for all } r \textit{ not in } \underline{p}, \underline{z}. \\ 3. \quad \mathcal{M}_1[\![p_i]\!] \subseteq \mathcal{M}_2[\![p_i]\!] \textit{ for all } p_i \in \underline{p}. \end{cases}$$

[a] We also assume that all *terms* get the same interpretation in both models.

Abnormality Theories

McCarthy (1986) originated the concept of an *abnormality theory* as a way to represent default reasoning in circumscription.

$$\forall x\,(bird(x) \wedge \neg ab_1(x) \rightarrow fly(x)). \tag{3}$$

Circumscription minimizes abnormalities, allowing relevant predicates to vary (e.g., fly). Abnormality theories can represent two important types of *defeating* arguments:

- *Type I defeaters*: defeaters that contradict outright a default distinguishes between defeaters that contradict outright a default (Type I defeaters), and
- *Type II defeaters*: those where only the justification for a default is undermined, without cotradicting its conclusion.

3 Rational Closure

- How much *nonmonotonicity* do logics like DL, CIRC contain?
- Ask for the abstract properties of the induced consequence-relation $\mid\!\sim$ of a given system.
- We focus on the induced *sceptical* consequence relation.

We consider the following set-up:

Default-base $\mathcal{K}$: A given situation is modeled by a (finite) set of *conditional assertions* of the form "$\alpha \mid\sim \beta$", where α, β are expressions in propositional logic.

We also allow *hard* knowledge, which is true in any case and not subject to later revision: T_{hard}.

Closure of $\mathcal{K}$: Starting from $\mathcal{K}$ one attempts to derive additional conditional assertions. The set of all conditional assertions derived from $\mathcal{K}$ is called the closure of $\mathcal{K}$. We will consider several different closure operators.

Query ?-$c \mid\sim d$: Finally we can ask queries about $\mathcal{K}$. These are again of the form "$\gamma \mid\sim \delta$" and will be answered by "yes" or "no", depending on wether "$\gamma \mid\sim \delta$" is in the closure of $\mathcal{K}$ or not.

$$\frac{\alpha \mid\!\sim \beta,\ \alpha \mid\!\sim \gamma}{\alpha \wedge \beta \mid\!\sim \gamma} \qquad \textit{(Cautious Monotonicity)}$$

Definition 3.1 (Preferential Closure $\mathcal{K}^{\mathrm{pref}}$)
The preferential closure $\mathcal{K}^{\mathrm{pref}}$ of a knowledge base is defined by closing the given knowledge base by the rules: Reflexivity, Left Logical Equivalence, Right Weakening, Cut, Cautious Monotonicity and the following rule

$$\frac{\alpha \mid\!\sim \gamma,\ \beta \mid\!\sim \gamma}{\alpha \vee \beta \mid\!\sim \gamma} \qquad \text{(Or).}$$

Example 3.2 (Weakness of $\mathcal{K}^{\mathrm{pref}}$)
We consider the knowledge base

$$\mathcal{K} := \{ quaker \mathrel{|\!\sim} pacifist, republican \mathrel{|\!\sim} \neg pacifist \}.$$

Although both "$quaker \land \neg republican \mathrel{|\!\sim} pacifist$" and "$republican \land \neg quaker \mathrel{|\!\sim} \neg pacifist$" are expected to be contained in the preferential closure, they are not.

$$\frac{\alpha \mathrel{|\!\not\sim} \neg\beta,\ \alpha \mathrel{|\!\sim} \gamma}{\alpha \land \beta \mathrel{|\!\sim} \gamma} \qquad \textit{(Rationality)}$$

<u>$\mathcal{K} \rightsquigarrow \mathcal{K}^{\mathrm{cum}}$</u>**:** This closure operation, as well as $\mathcal{K} \rightsquigarrow \mathcal{K}^{\mathrm{pref}}$ is *monotone*, i.e.

$$\mathcal{K} \subseteq \mathcal{K}' \implies \mathcal{K}^{\mathrm{cum}} \subseteq (\mathcal{K}')^{\mathrm{cum}}.$$

The reason for this is that both operations are defined by *inference rules* of the form:

$$\text{“if } \alpha \mid\!\sim \beta \text{ and } \gamma \mid\!\sim \delta \text{ then } \zeta \mid\!\sim \eta\text{”.}$$

<u>$\mathcal{K} \rightsquigarrow \mathcal{K}^{\mathrm{rat}}$</u>**:** This operation is *nonmonotonic* and not so easy to describe. But Lehmann and Magidor give in (Lehmann and Magidor 1992) a very effective and intuitive algorithm to compute $\mathcal{K}^{\mathrm{rat}}$. We will restrict ourselves to the explanation of this algorithm.

Two important concepts have to be introduced before we can give the definition of the algorithm that computes $\mathcal{K}^{\mathrm{rat}}$:

1. the notion of an *exceptional formula*,
2. the notion of *rank* for formulas.

Definition 3.3 (Exceptional Formulas)

A formula α *is called* exceptional *for* $\mathcal{K}$*, if* $\mathbf{t} \mathrel{|\!\!\sim} \neg\alpha \in \mathcal{K}^{\mathrm{pref}}$. *A conditional assertion* $\alpha \mathrel{|\!\!\sim} \beta$ *is called* exceptional *for* $\mathcal{K}$*, if* α *is exceptional for* $\mathcal{K}$.

$$\alpha \mid\!\sim \beta \in \mathcal{K}^{\mathrm{pref}} \textit{ if and only if } \alpha \mid\!\sim \mathbf{f} \in (\mathcal{K} \cup \{\alpha \mid\!\sim \neg\beta\})^{\mathrm{rat}}.$$

This means that once we have an efficient method for computing $\mathcal{K}^{\mathrm{rat}}$ we can use it for the computation of $\mathcal{K}^{\mathrm{pref}}$.

In (Dix and Schmitt 1993) many of the benchmark problems of (Lifschitz 1989) are discussed and it is investigated how they are handled by the various closures introduced above.

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