# Logics with counting: Very Challenging Presentation Topics

Ian Pratt-Hartmann

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Here is a list of very challenging topics which you might like to give a presentation on for the course *Logics with Counting*. They are in no particular order. All of the topics can be done (more or less successfully) by anyone who has followed the course; and none of them lacks the potential for doing serious and challenging work. Note also that these are only suggestions. If you have a better idea, then I'd be happy to hear about it.

I have provided a few bibliographic suggestions to get you started. These are not intended to be definitive.

If this is too daunting, don't worry! It is perfectly acceptable to pick a topic that I have discussed in class, and give your own presentation on it. I realize that you have other courses to take and lives to get on with ....

1. Suppose we are given a finite set of formulas of propositional logic, together with a probability assignment to each formula, for example: $P(p_1 \wedge \neg p_2) = 0.6$, $P(p_2 \rightarrow (\neg p_4 \vee p_1)) = 0.72$, .... How do we decide whether such an assignment is coherent, according to the usual laws of probability theory? [11, 12, 4]

2. In class we encountered the numerical syllogistic: sets of formulas of the forms $\exists_{\bowtie C} x(\pm p(x) \wedge \pm q(x))$, where $\bowtie$ is either $\leq$ or $\geq$. We saw in particular that this fragment is NPTIME-complete, even when all numerical subscripts are limited to 0 and 1. But how hard is it in practice? What happens if you generate random problems with (say) $m$ formulas and $n$ proposition letters? What proportion of generated problems is satisfiable? What happens to this probability as the ratio $m/n$ varies? Investigate. [10, 13]

3. In class we encountered the numerical syllogistic: sets of formulas of the forms $\exists_{\bowtie C} x(\pm p(x) \wedge \pm q(x))$, where $\bowtie$ is either $\leq$ or $\geq$. This can be naturally extended to the numerically definite *relational* syllogistic: sets of formulas of the forms $\exists_{\bowtie C} x(\pm p(x) \wedge \exists_{\bowtie C} x(\pm q(x) \wedge \pm r(x, y)))$. This fragment is certainly in NEXPTIME, since it is contained in the two-variable fragment with counting. It was shown in [13] that it is also NEXPTIME-hard, but only if numerical subscripts are encoded in binary. Is it still NEXPTIME-hard under unary coding?

4. Graded modal logic is the extension of propositional modal logic in which the modality $\Diamond$ is equipped with numerical constraints. Thus: $\Diamond_{\leq C}\varphi$ is to be read as "There are at most $C$ accessible possible worlds in which $\varphi$ is true". Outline the history of these systems, and explain the key results. What open problems remain? [3, 6]

5. Counting quantifiers often arise in so-called *description logics* (earlier known as *terminological logics*. Give a report on the progress that has been made in implementing decision-procedures for such logics? What syntactic restrictions are typically imposed? What techniques have been employed? How well do they work in practice? [1, 5]

6. Discuss the problems posed by counting quantifiers in the general area of querying *open-world databases*. [2]

7. What would happen to the theorems presented on class concerning the two-variable fragment with counting quantifiers is we added additional sorts of quantifiers? For example, "there exist infinitely many $x$ such that ...", "there exists an even number of $x$ such that ...". You may find that you need to limit the contexts in which some of these quantifiers can occur in order to retain decidability. Are there any cases for which you can prove undecidability? The best source is probably Ch. 4 of the class notes. (Good luck!)

8. We saw in class that graded model logic remains decidable even when additional restrictions, such as transitivity, symmetry, etc. are imposed on the accessibility relation. Many (but not all!) of the logics considered in this course lack this feature: decidability is destroyed when it is assumed that certain predicates denote transitive relations or equivalence relations. Summarize what is known in this area. [7, 15].

9. An important class of problems concerns the interpretation of various logics over particular sorts of data-structures: most notably, strings and trees. Various results exist concerning the two-variable logic (without counting) in these cases. What are they? Do you think these results could be extended to logics with counting quantifiers? [8, 9].

10. It was shown in [14] that no finite set of 'syllogism-like' rules can be sound and complete for the numerical syllogistic. Can you relax the restrictions on the kinds of proof systems considered so that you *do* get a sound and complete system? Can you at least suggest some inferential principles that any such system might have to incorporate?

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