First version released on 30 June 2006
This version released on 30 June 2006

Overview
of
A collection of notes on frames

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This segment of my web pages is devoted to various sets of notes on different aspects of the theory of frames. For convenience I call each such set of notes a part. Thus the long term project is to write several parts which as a whole give an account of a decent proportion of frame theory.

At the time of writing the project is not finished. Several parts have been written and released. Other parts are nearly finished and, the devil willing, will be released quite soon. Other parts are still tentative. A list of these parts with their current status is given in Table 1. Other parts may be added in due course.

Each part may, is, and will be written separately without worrying too much about the contents of the other parts. Thus there may be some overlap between certain parts. More importantly, I may have omitted some information, this having slipped through a crack between two parts. To make sure that the whole collection of parts is watertight does need quite a bit of intelligent design, and that would take the best part of a week to organize.

The moral of this story is that you should keep an eye on this Overview. It may change quite a lot as I make more parts available or correct and amend earlier parts.

At the head of each part, and this Overview, you will find a banner.

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This will indicate if any changes have been made (but, of course, won't say what has been changed).

To help with this you can send me any comments you have: typos, bad phrasing, missing information, and so on. I will also welcome comments on the project as a whole; perhaps suggestions for further parts. However, I do ask you not to complain about my sense of humour. I know I don't have one.

The first three parts, labelled (A,B,C) in Table 1, form what might be taken to be a first course in frame theory. Everyone should read (A), if only to see the notation I use throughout the parts. After that (B) and (C) can be read in either order.

Some people might object to these three parts being describe as a first course. For one thing, there is quite a lot of information there, nearly 170 pages in total. Any actual first course would have to select from this material quite severely. Also, there are some fundamental topics not dealt with in these three parts. For instance, I don't deal with separation properties. Thus I would not disagree too strongly with such an objection.

After that the parts are concerned with those topics I have been interested in for most of my career. Mainly centred around the question: When does a frame have a boolean reflection?

Overview

The following parts have been released

(A) The basics of frame theory

(B) The assembly of a frame

(C) The point space of a frame

The following parts should be released fairly soon

(D) The fundamental triangle of a space

(E) Boolean reflections of frames

(F) The higher level CB properties of frames

(G) Examples of higher level assemblies

The following parts are being produced

- The localic approach to boolean reflections

Table 1: The list of parts – including this one

There are some faults which, in an ideal world, should be corrected, and perhaps will be corrected, but only in the long term.

The parts are rather short on historical remarks and citations to the literature. Much of the information in the parts I have known for many years, and have always taken it to be folklore. The two accounts [1, 2] contain quite extensive list of references. Both accounts should be consulted.

On the whole I have tried not to merely repeat material from these two accounts. Often there are several ways of looking at the same material, and where possible I give these alternatives.

At then end of this Overview I have added a list of References. Apart from [1, 2] the first batch are papers that have influenced my thinking on the topics, of which I have been involved in. At the end I give the list of my relevant published papers, then a list of relevant writings that can be found on my web pages. Each one has a number to help you find it. Finally, I add the list of parts in this collection. Of course, these is some overlap between these segments, but no paper is listed twice.

There are other faults. For instance I have included very few exercises (zero I think). I'm a great believer in exercises, so this is something that ought to be corrected. My only excuse is that I have never actually taught this stuff, so I have never had the opportunity to sort out a collection of decent exercises.

I did plan to include several open problems and research topics, but I wasn't very systematic in gathering these. However, this is something I hope I can rectify in the future.

References

[1] Peter T. Johnstone: Stone spaces, Cambridge University Press (1982).

[2] Aleš Pultr: Frames, pages 791-857 of Handbook of Algebra, vol 3, ed M. Hazelwinkel, Elsevier (2003).

[3] R. Beazer and D.S. Macnab: Modal extensions of heyting algebras, Colloquium Math. 41 (1979) 1-12.

[4] C.H. Dowker and Dona Papert: Quotient frame and subspaces, Proc. London Math. Soc (3) 16 (1966) 275-96.

[5] C.H. Dowker and Dona Papert Strauss: Separation axioms for frames, pages 223-240 of Topics in Topology, ed Á. Császár, North Holland (1972).

[6] Insert other stuff of Dowker and DPS

[7] J.R. Isbell: Atomless parts of spaces, Math. Scand. 31 (1972) 5-32.

[8] D.S. Macnab: An algebraic study of modal operators on heyting algebras with applications to topology and sheafification, Ph. D. Thesis, Aberdeen (1976).

[9] D.S. Macnab: Modal operators on heyting algebras, Algebra Universalis 12 (1981) 5-29.

[10] S.B. Niefield and K.I. Rosenthal: Spatial sublocales and essential primes, Topology and its Applications, 26 (1987) 263-269.

[11] S.B. Niefield and K.I. Rosenthal: Constructing locales from quantales, Math. Proc. Cambridge Philos. Soc. 104 (1988) 215-234.

[12] Rosemary A. Sexton: A point-free and point-sensitive analysis of the patch assembly, Ph D Thesis, University of Manchester, 2003. A version of this can be found at **********

The following are my relevant papers. For some a version can be found on my web page under

DOCUMENTS/papersandnotes.html

with the indicated number.

[13] The lattice theoretic part of topological separaton properties, Proc. Edinburgh Math. Soc. 21 (1978) 41-48.

[14] A framework for topology, in *Logic Colloquium 77* (North Holland, 1978) 239-251.

[15] Spaces with boolean assemblies, Colloquium Math. 43 (1980) 23-39.

[16] The Cantor-Bendixson analysis of a frame, Séminaire de mathematique pure, no. 92, Université Catholique de Louvain (1980) 42 pages.

[17] A chapter on some functorial aspects of frame theory, Séminaire de mathematique pure, no. 106, Univesité Catholique de Louvain (1980) 85 pages.

[18] An algebraic version of Cantor-Bendixson analysis, in *Categorical aspects of topology and analysis*, Springer Lecture Notes in Mathematics vol. 915 (1982) 310-323.

[19] Near-discreteness of modules and spaces as measured by Gabriel and Cantor, J. Pure and Applied Alg. 56 (1989) 119-162.

[20] Separating the discrete from the continuous by iterating derivatives, Bull. Soc. Math. Belg. 41 (1989) 417-463.

[21] The extended Cantor-Bendixson analysis of trees, Algebra Universalis 52 (2005) 439-468.
A version can be found on my web page under Number (15P)

[22] (With R. Sexton), Point-sensitive and point-free patch constructions, *to appear* J. Pure and Applied Alg.
A version can be found on my web page under Number (02P)

[23] Regularity, fitness, and the block structure of frames, Applied Categorical Structures 14 (2006) 1-34.
A version can be found on my web page under Number (21P)

[24] A coverage construction of the reals and the irrationals, *submitted*
A version can be found on my web page under Number (17P)

[25] (With R. Sexton), An ordinal indexed hierarchy of separation properties, *submitted*
A version can be found on my web page under Number (16P)

[26] A curious nucleus
Not yet on my web pages

[27] Complemented nuclei on a frame,
Not yet on my web pages

The following are my notes that are relevant but not part of this collection. Each can be found on my web page under

DOCUMENTS/papersandnotes.html

with the indicated number.

[28] The point-free approach to sheafification, Number (06N).

[29] The coverage technique for enriched posets, Number (11N).

[30] The Vietoris modifications of a frame, Number (14N).

The following are the parts of this whole collection.

Frame theory in several parts.

You already know where to find them.

[31] Overview of 'A collections of notes on Frames'.

[32] The basics of frame theory.

[33] The assembly of a frame.

[34] The point space of a frame.

[35] The fundamental triangle of a space.

[36] Boolean reflections of frames.

[37] The higher level CB properties of frames.

[38] Examples of higher level assemblies.