Reasoning Procedures II
As already mentioned, for \( \mathcal{ALC} \) with general axioms basic algorithm is non-terminating
Non-Termination

As already mentioned, for $\mathcal{ALC}$ with general axioms basic algorithm is non-terminating

E.g. if $\text{human} \sqsubseteq \exists \text{has-mother} \cdot \text{human} \in \mathcal{T}$, then
$\neg \text{human} \sqcup \exists \text{has-mother} \cdot \text{human}$ added to every node
Non-Termination

- As already mentioned, for $\mathcal{ALC}$ with general axioms basic algorithm is non-terminating

- E.g. if $\text{human} \sqsubseteq \exists \text{has-mother} \cdot \text{human} \in \mathcal{T}$, then $\neg \text{human} \sqcup \exists \text{has-mother} \cdot \text{human}$ added to every node

$$\mathcal{L}(w) = \{\text{human}\}$$
Non-Termination

As already mentioned, for \( \mathcal{ALC} \) with \textbf{general axioms} basic algorithm is \textbf{non-terminating}.

\textbf{E.g.} if human \( \subseteq \exists \text{has-mother.human} \in \mathcal{T} \), then
\[ \neg \text{human} \sqcup \exists \text{has-mother.human} \]
added to every node.

\[ \mathcal{L}(w) = \{ \text{human}, (\neg \text{human} \sqcup \exists \text{has-mother.human}) \} \]
Non-Termination

As already mentioned, for $\mathcal{ALC}$ with general axioms basic algorithm is non-terminating.

E.g. if $\text{human} \subseteq \exists \text{has-mother.human} \in \mathcal{T}$, then $\neg \text{human} \cup \exists \text{has-mother.human}$ added to every node.

$$\mathcal{L}(w) = \{\text{human}, (\neg \text{human} \cup \exists \text{has-mother.human}), \exists \text{has-mother.human}\}$$
Non-Termination

As already mentioned, for $\mathcal{ALC}$ with general axioms basic algorithm is non-terminating.

E.g. if $\text{human} \sqsubseteq \exists \text{has-mother}. \text{human} \in \mathcal{T}$, then $\neg \text{human} \cup \exists \text{has-mother}. \text{human}$ added to every node.

\[ \mathcal{L}(w) = \{ \text{human}, (\neg \text{human} \cup \exists \text{has-mother}. \text{human}), \exists \text{has-mother}. \text{human} \} \]

has-mother

\[ \mathcal{L}(x) = \{ \text{human} \} \]
Non-Termination

As already mentioned, for $\mathcal{ALC}$ with general axioms basic algorithm is non-terminating.

E.g. if $\text{human} \subseteq \exists \text{has-mother}.\text{human} \in \mathcal{T}$, then $\neg \text{human} \cup \exists \text{has-mother}.\text{human}$ added to every node.

\[
\mathcal{L}(w) = \{\text{human}, (\neg \text{human} \cup \exists \text{has-mother}.\text{human}), \exists \text{has-mother}.\text{human}\}
\]

has-mother

\[
\mathcal{L}(x) = \{\text{human}, (\neg \text{human} \cup \exists \text{has-mother}.\text{human})\}
\]
Non-Termination

As already mentioned, for $ALC$ with general axioms basic algorithm is non-terminating

E.g. if human $\subseteq \exists \text{has-mother}.\text{human} \in \mathcal{T}$, then $\neg\text{human} \cup \exists \text{has-mother}.\text{human}$ added to every node

$\mathcal{L}(w) = \{\text{human}, (\neg\text{human} \cup \exists \text{has-mother}.\text{human}), \exists \text{has-mother}.\text{human}\}$

has-mother

$\mathcal{L}(x) = \{\text{human}, (\neg\text{human} \cup \exists \text{has-mother}.\text{human}), \exists \text{has-mother}.\text{human}\}$
As already mentioned, for $\mathcal{ALC}$ with **general axioms** basic algorithm is **non-terminating**.

**E.g.** if $\text{human} \sqsubseteq \exists \text{has-mother} \cdot \text{human} \in \mathcal{T}$, then

$\neg \text{human} \sqcup \exists \text{has-mother} \cdot \text{human}$ added to every node

\[
\begin{align*}
\mathcal{L}(w) &= \{\text{human}, (\neg \text{human} \sqcup \exists \text{has-mother} \cdot \text{human}), \exists \text{has-mother} \cdot \text{human}\} \\
\mathcal{L}(x) &= \{\text{human}, (\neg \text{human} \sqcup \exists \text{has-mother} \cdot \text{human}), \exists \text{has-mother} \cdot \text{human}\} \\
\mathcal{L}(y) &= \{\text{human}, (\neg \text{human} \sqcup \exists \text{has-mother} \cdot \text{human}), \exists \text{has-mother} \cdot \text{human}\}
\end{align*}
\]
Blocking

When creating new node, check ancestors for equal (superset) label.
**Blocking**

- When creating a new node, check ancestors for equal (superset) label.
- If such a node is found, the new node is **blocked**.
Blocking

- When creating new node, check ancestors for equal (superset) label
- If such a node is found, new node is blocked

\[
\mathcal{L}(w) = \{ \text{human, } (\neg \text{human} \sqcup \exists \text{has-mother.human}), \exists \text{has-mother.human} \}
\]

\[
\mathcal{L}(x) = \{ \text{human, } (\neg \text{human} \sqcup \exists \text{has-mother.human}) \}
\]
Blocking

- When creating new node, check ancestors for equal (superset) label
- If such a node is found, new node is **blocked**

\[ L(w) = \{ \text{human}, (\neg \text{human} \sqcup \exists \text{has-mother.human}), \exists \text{has-mother.human} \} \]

\[ L(x) = \{ \text{human}, (\neg \text{human} \sqcup \exists \text{has-mother.human}) \} \]
Blocking

- When creating a new node, check ancestors for equal (superset) label.
- If such a node is found, the new node is blocked.

\[ \mathcal{L}(w) = \{ \text{human}, (\neg \text{human} \sqcup \exists \text{has-mother}. \text{human}), \exists \text{has-mother}. \text{human} \} \]

block represents **cyclical** model.
Blocking with More Expressive DLs

Simple subset blocking may not work with more complex logics
Simple subset blocking may not work with more complex logics
E.g., reasoning with inverse roles
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor

Reasoning Procedures II – p. 4/9
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

\[
T = \{ \top \sqsubseteq \forall R^{-}.(\forall S^{-}.\neg C), \top \sqsubseteq \exists R.C \}
\]
Simple subset blocking may not work with more complex logics

E.g., reasoning with inverse roles

- Expanding node label can affect predecessor
- Label of blocking node can affect predecessor
- E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \sqsubseteq \forall R^{-}.(\forall S^{-}.\neg C), \top \sqsubseteq \exists R.C \}$$

$$\bigwedge L(w) = \{ C, \exists S.C \}$$
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

$$\mathcal{T} = \{ \top \subseteq \forall R^{-}.(\forall S^{-}.\neg C), \top \subseteq \exists R.C \}$$

$$\mathcal{L}(w) = \{ C, \exists S.C, \forall R^{-}.(\forall S^{-}.\neg C), \exists R.C \}$$
Simple subset blocking may not work with more complex logics

E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \sqsubseteq \forall R^-. (\forall S^- . \neg C), \top \sqsubseteq \exists R.C \}$$

$$\mathcal{L}(w) = \{ C, \exists S.C, \forall R^- . (\forall S^- . \neg C), \exists R.C \}$$

$$\mathcal{L}(x) = \{ C, \forall R^- . (\forall S^- . \neg C), \exists R.C \}$$
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \subseteq \forall R^{-}.(\forall S^{-}.\neg C), \top \subseteq \exists R.C \}$$

\[
\mathcal{L}(w) = \{ C, \exists S.C, \forall R^{-}.(\forall S^{-}.\neg C), \exists R.C \}
\]

\[
\mathcal{L}(x) = \{ C, \forall R^{-}.(\forall S^{-}.\neg C), \exists R.C \}
\]

\textbf{Blocked}
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
- E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \sqsubseteq \forall R^-.(\forall S^- . \neg C), \top \sqsubseteq \exists R.C \}$$

- $\mathcal{L}(w) = \{ C, \exists S.C, \forall R^-.(\forall S^- . \neg C), \exists R.C \}$
- $\mathcal{L}(y) = \{ C, \forall R^-.(\forall S^- . \neg C), \exists R.C \}$
- $\mathcal{L}(x) = \{ C, \forall R^-.(\forall S^- . \neg C), \exists R.C \}$

Reasoning Procedures II – p. 4/9
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
- E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

$$\mathcal{T} = \{ \top \sqsubseteq \forall R^{-}.(\forall S^{-}.\neg C), \top \sqsubseteq \exists R.C \}$$

\[
\mathcal{L}(w) = \{ C, \exists S.C, \forall R^{-}.(\forall S^{-}.\neg C), \exists R.C \}
\]

\[
\mathcal{L}(y) = \{ C, \forall R^{-}.(\forall S^{-}.\neg C), \exists R.C \}
\]

\[
\mathcal{L}(x) = \{ C, \forall R^{-}.(\forall S^{-}.\neg C), \exists R.C \}
\]
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

$$\mathcal{T} = \{\top \sqsubseteq \forall R^-.(\forall S^-.\neg C), \top \sqsubseteq \exists R.C\}$$

$$\mathcal{L}(w) = \{C, \exists S.C, \forall R^-.(\forall S^-.\neg C), \exists R.C\}$$

**cyclical** model?
Blocking with More Expressive DLs

Simple subset blocking may not work with more complex logics

E.g., reasoning with inverse roles

- Expanding node label can affect predecessor
- Label of blocking node can affect predecessor
- E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

\[ \mathcal{T} = \{ \top \sqsubseteq \forall R^-. (\forall S^- . \neg C), \top \sqsubseteq \exists R.C \} \]

\[ \mathcal{L}(w) = \{ C, \exists S.C, \forall R^-. (\forall S^- . \neg C), \exists R.C \} \]

**cyclical** model?
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

$$\mathcal{T} = \{ \top \sqsubseteq \forall R^-(\forall S^- \neg C), \top \sqsubseteq \exists R.C \}$$

$$\mathcal{L}(w) = \{ C, \exists S.C, \forall R^-(\forall S^- \neg C), \exists R.C, \forall S^- \neg C \}$$

**cyclical** model?
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
- E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \subseteq \forall R^-(\forall S^- \neg C), \top \subseteq \exists R.C \}$$

$$\mathcal{L}(w) = \{ C, \exists S.C, \forall R^-.(\forall S^- \neg C), \exists R.C, \forall S^- \neg C \}$$

Is it a cyclical model?
Simple subset blocking may not work with more complex logics

E.g., reasoning with inverse roles

- Expanding node label can affect predecessor
- Label of blocking node can affect predecessor
- E.g., testing $C \sqcap \exists S. C$ w.r.t. Tbox

$$\mathcal{T} = \{ \top \sqsubseteq \forall R^{-}.(\forall S^{-}.\neg C), \top \sqsubseteq \exists R. C' \}$$

$$\mathcal{L}(w) = \{ C, \exists S. C, \forall R^{-}.(\forall S^{-}.\neg C), \exists R. C, \forall S^{-}.\neg C, \neg C \}$$

Clash
cyclical model?
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
Dynamic Blocking

- Solution (for inverse roles) is **dynamic blocking**
  - Blocks can be established broken and re-established
  - Continue to expand $\forall R.C$ terms in blocked nodes
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[ \mathcal{L}(w) = \{C, \exists S.C\} \]

Reasoning Procedures II – p. 5/9
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**
- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

$$\mathcal{L}(w) = \{C, \exists S.C, \forall R^-.(\forall S^- \neg C), \exists R.C\}$$
Dynamic Blocking

- Solution (for inverse roles) is **dynamic blocking**
  - Blocks can be established broken and re-established
  - Continue to expand $\forall R.C$ terms in blocked nodes
  - Check that cycles satisfy $\forall R.C$ concepts

$$
\mathcal{L}(w) = \{C, \exists S.C, \forall R^-(\forall S^- . \neg C), \exists R.C\}
$$

$$
\mathcal{L}(x) = \{C, \forall R^-(\forall S^- . \neg C), \exists R.C\}
$$
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**
- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[
L(w) = \{C, \exists S.C, \forall R^-.(\forall S^-.\neg C), \exists R.C\}
\]

\[
L(x) = \{C, \forall R^-.(\forall S^-.\neg C), \exists R.C\}
\]
Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[
\begin{align*}
\mathcal{L}(w) &= \{C, \exists S.C, \forall R^{-}.(\forall S^{-}.\neg C), \exists R.C\} \\
\mathcal{L}(y) &= \{C, \forall R^{-}.(\forall S^{-}.\neg C), \exists R.C\} \\
\mathcal{L}(x) &= \{C, \forall R^{-}.(\forall S^{-}.\neg C), \exists R.C\}
\end{align*}
\]
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[
\mathcal{L}(y) = \{C, \forall R^{-}.(\forall S^{-} . \neg C), \exists R.C\}
\]
\[
\mathcal{L}(w) = \{C, \exists S.C, \forall R^{-}.(\forall S^{-} . \neg C), \exists R.C, \forall S^{-} . \neg C\}
\]
\[
\mathcal{L}(x) = \{C, \forall R^{-}.(\forall S^{-} . \neg C), \exists R.C\}
\]
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[
\mathcal{L}(w) = \{C, \exists S.C, \forall R^{-}(\forall S^{-}.\neg C),
\exists R.C, \forall S^{-}.\neg C\}
\]

\[
\mathcal{L}(y) = \{C, \forall R^{-}.(\forall S^{-}.\neg C),
\exists R.C\}
\]

\[
\mathcal{L}(x) = \{C, \forall R^{-}.(\forall S^{-}.\neg C),
\exists R.C\}
\]
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[
\begin{align*}
\mathcal{L}(w) &= \{C, \exists S.C, \forall R^-.(\forall S^- \cdots \neg C), \exists R.C, \forall S^- \cdots \neg C\} \\
\mathcal{L}(y) &= \{C, \forall R^-.(\forall S^- \cdots \neg C), \exists R.C\} \\
\mathcal{L}(x) &= \{C, \forall R^-.(\forall S^- \cdots \neg C), \exists R.C\} \\
\mathcal{L}(z) &= \{C, \forall R^-.(\forall S^- \cdots \neg C), \exists R.C\}
\end{align*}
\]
Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[
\mathcal{L}(w) = \{C, \exists S.C, \forall R^-. (\forall S^- . \neg C), \\
\quad \exists R.C, \forall S^- . \neg C\}
\]

\[
\mathcal{L}(y) = \{C, \forall R^- . (\forall S^- . \neg C), \\
\quad \exists R.C\}
\]

\[
\mathcal{L}(x) = \{C, \forall R^- . (\forall S^- . \neg C), \\
\quad \exists R.C, \forall S^- . \neg C\}
\]

\[
\mathcal{L}(z) = \{C, \forall R^- . (\forall S^- . \neg C), \\
\quad \exists R.C\}
\]
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[ \mathcal{L}(w) = \{ C, \exists S.C, \forall R^-.(\forall S^-.\neg C), \exists R.C, \forall S^-.-C, -C \} \]

\[ \mathcal{L}(y) = \{ C, \forall R^-.(\forall S^-.-C), \exists R.C \} \]

\[ \mathcal{L}(x) = \{ C, \forall R^-.(\forall S^-.-C), \exists R.C, \forall S^-.-C \} \]

\[ \mathcal{L}(z) = \{ C, \forall R^-.(\forall S^-.-C), \exists R.C \} \]

**Clash**
Non-finite Models

With number restrictions some satisfiable concepts have only non-finite models
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R.C, \top \subseteq \leq 1 R^- \}$
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R.C, \top \subseteq \leq 1 R^- \}$

\[(w) \mathcal{L}(w) = \{ \neg C \}\]
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing \( \neg C \) w.r.t. \( T = \{ \top \subseteq \exists R.C, \top \subseteq \leq 1R^- \} \)

\[
\overline{w} \mathcal{L}(w) = \{ \neg C, \exists R.C, \leq 1R^- \}
\]
With number restrictions some satisfiable concepts have only non-finite models

E.g., testing $\neg C$ w.r.t. $T = \{ \top \sqsubseteq \exists R.C, \top \sqsubseteq \leq 1 R^- \}$
Non-finite Models

With number restrictions some satisfiable concepts have only non-finite models.

E.g., testing $\neg C$ w.r.t. $T = \{\top \subseteq \exists R.C, \top \subseteq \leq 1R^-\}$

\[
\begin{array}{c}
\circlearrowleft \  L(w) = \{\neg C, \exists R.C, \leq 1R^-\} \\
R \\
\circlearrowright \  L(x) = \{C, \exists R.C, \leq 1R^-\}
\end{array}
\]
With number restrictions some satisfiable concepts have only non-finite models

E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{ \top \sqsubseteq \exists R.C, \top \sqsubseteq \leq 1 R^- \}$

\[ \mathcal{L}(w) = \{ \neg C, \exists R.C, \leq 1 R^- \} \]

\[ \mathcal{L}(x) = \{ C, \exists R.C, \leq 1 R^- \} \]

\[ \mathcal{L}(y) = \{ C, \exists R.C, \leq 1 R^- \} \]
With number restrictions some satisfiable concepts have only non-finite models.

E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{\top \sqsubseteq \exists R.C, \top \sqsubseteq \leq 1R^-\}$

\[
\begin{align*}
\mathcal{L}(w) &= \{\neg C, \exists R.C, \leq 1R^-\} \\
\mathcal{L}(x) &= \{C, \exists R.C, \leq 1R^-\} \\
\mathcal{L}(y) &= \{C, \exists R.C, \leq 1R^-\}
\end{align*}
\]
With number restrictions some satisfiable concepts have only non-finite models

E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R.C, \top \subseteq \leq 1R^- \}$

Cyclical model?
With number restrictions some satisfiable concepts have only non-finite models

E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R.C, \top \subseteq \leq 1R^- \}$
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R.C, \top \subseteq \leq 1 R^- \}$

$\mathcal{L}(w) = \{ \neg C, \exists R.C, \leq 1 R^- \}$

$\mathcal{L}(x) = \{ C, \exists R.C, \leq 1 R^- \}$

$\Rightarrow w = x$

Cyclical model?
With number restrictions some satisfiable concepts have only non-finite models

E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R. C, \top \subseteq \leq 1R^- \}$

$\mathcal{L}(w) = \{ \neg C, \exists R. C, \leq 1R^-, C \}$

Clash

Cyclical model?
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $T = \{ \top \sqsubseteq \exists R.C, \top \sqsubseteq \leq 1R^- \}$

\[
\begin{align*}
\mathcal{L}(w) &= \{ \neg C, \exists R.C, \leq 1R^- \} \\
\mathcal{L}(x) &= \{ C, \exists R.C, \leq 1R^- \} \\
\mathcal{L}(y) &= \{ C, \exists R.C, \leq 1R^- \}
\end{align*}
\]

model must be non-finite
Inadequacy of Dynamic Blocking

With non-finite models, even dynamic blocking not enough
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing \( \neg C \) w.r.t. \( T = \{ \top \subseteq \exists R.(C \cap \exists R^- \neg C), \top \subseteq \leq 1R^- \} \)
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough

- E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R. (C \land \exists R^- \cdot \neg C'), \top \subseteq \leq R^- \}$

$$\{w \in \mathcal{L}(w) = \{ \neg C' \}$$
Inadequacy of Dynamic Blocking

With non-finite models, even dynamic blocking not enough

E.g., testing $\neg C$ w.r.t. $T = \{ \top \sqsubseteq \exists R. (C \cap \exists R^- \neg C), \top \sqsubseteq \leq 1R^- \}$

\[ \mathcal{L}(w) = \{ \neg C, \exists R. (C \cap \exists R^- \neg C), \leq 1R^- \} \]
Inadequacy of Dynamic Blocking

With non-finite models, even dynamic blocking not enough

E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R.(C \cap \exists R^-.\neg C), \top \subseteq \leq 1R^- \}$

\[
\begin{align*}
\circ w & \quad \mathcal{L}(w) = \{ \neg C, \exists R.(C \cap \exists R^- . \neg C), \leq 1R^- \} \\
\downarrow & \\
R & \\
\circ x & \quad \mathcal{L}(x) = \{ (C \cap \exists R^- . \neg C) \}
\end{align*}
\]
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing $\neg C$ w.r.t. $T = \{\top \subseteq \exists R.(C \land \exists R^- \cdot \neg C), \top \subseteq \leq 1 R^-\}$

\[
\begin{align*}
\mathcal{L}(w) &= \{\neg C, \exists R.(C \land \exists R^- \cdot \neg C), \leq 1 R^-\} \\
\mathcal{L}(x) &= \{(C \land \exists R^- \cdot \neg C), \exists R.(C \land \exists R^- \cdot \neg C), \leq 1 R^-, C, \exists R^- \cdot \neg C\}
\end{align*}
\]

Reasoning Procedures II – p. 7/9
Inadequacy of Dynamic Blocking

With non-finite models, even dynamic blocking not enough

E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R.(C \cap \exists R^- \neg C'), \top \subseteq \leq 1R^- \}$

Diagram:

```
\( \mathcal{L}(w) = \{ \neg C, \exists R.(C \cap \exists R^- \neg C'), \leq 1R^- \} \)
\( \mathcal{L}(x) = \{ (C \cap \exists R^- \neg C'), \exists R.(C \cap \exists R^- \neg C'), \leq 1R^-, C, \exists R^- \neg C' \} \)
\( \mathcal{L}(y) = \{ (C \cap \exists R^- \neg C'), \exists R.(C \cap \exists R^- \neg C'), \leq 1R^-, C, \exists R^- \neg C' \} \)
```
Inadequacy of Dynamic Blocking

With non-finite models, even dynamic blocking not enough

E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R. (C \cap \exists R^-. \neg C), \top \subseteq \leq 1R^- \}$

Diagram:

- $w$: $L(w) = \{ \neg C, \exists R. (C \cap \exists R^- \neg C), \leq 1R^- \}$
- $x$: $L(x) = \{ (C \cap \exists R^- \neg C), \exists R. (C \cap \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C \}$
- $y$: $L(y) = \{ (C \cap \exists R^- \neg C), \exists R. (C \cap \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C \}$

Blocked
Inadequacy of Dynamic Blocking

With non-finite models, even dynamic blocking not enough

E.g., testing \( \neg C \) w.r.t. \( T = \{ \top \subseteq \exists R.(C \cap \exists R^- \neg C), \top \subseteq \leq 1R^- \} \)

\[
\begin{align*}
\mathcal{L}(w) &= \{-C, \exists R.(C \cap \exists R^- \neg C), \leq 1R^- \} \\
\mathcal{L}(x) &= \{(C \cap \exists R^- \neg C), \exists R.(C \cap \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C \} \\
\mathcal{L}(y) &= \{(C \cap \exists R^- \neg C), \exists R.(C \cap \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C \} \\
\end{align*}
\]

**But** \( \exists R^- \neg C \in \mathcal{L}(y) \) **not satisfied**
Inadequacy of Dynamic Blocking

With non-finite models, even dynamic blocking not enough
E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{\top \subseteq \exists R.(C \cap \exists R^- \neg C'), \top \subseteq \leq 1R^-\}$

\[
\begin{align*}
\circled{w} & \quad \mathcal{L}(w) = \{\neg C, \exists R.(C \cap \exists R^- \neg C'), \leq 1R^-\} \\
R & \\
\circled{x} & \quad \mathcal{L}(x) = \{(C \cap \exists R^- \neg C'), \exists R.(C \cap \exists R^- \neg C'), \leq 1R^-, C, \exists R^- \neg C\} \\
R^- & \quad \text{Blocked} \\
\circled{y} & \quad \mathcal{L}(y) = \{(C \cap \exists R^- \neg C'), \exists R.(C \cap \exists R^- \neg C'), \leq 1R^-, C, \exists R^- \neg C\}
\end{align*}
\]

But $\exists R^- \neg C \in \mathcal{L}(y)$ not satisfied

Inconsistency due to $\leq 1R^- \in \mathcal{L}(y)$ and $C \in \mathcal{L}(x)$
Problem due to $\exists R^- . \neg C$ term only satisfied in predecessor of blocking node

$$L(w) = \{ \neg C, \exists R. (C \land \exists R^- . \neg C), \leq 1R^- \}$$

$$R$$

$$L(x) = \{(C \land \exists R^- . \neg C), \exists R. (C \land \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C\}$$
Double Blocking I

Problem due to $\exists R^-. \neg C$ term **only** satisfied in **predecessor** of blocking node

$$\mathcal{L}(w) = \{ \neg C, \exists R. (C \cap \exists R^- . \neg C), \leq 1 R^- \}$$

$$\mathcal{L}(x) = \{(C \cap \exists R^- . \neg C), \exists R. (C \cap \exists R^- . \neg C), \leq 1 R^-, C, \exists R^- . \neg C\}$$

Solution is **Double Blocking** (pairwise blocking)
Double Blocking I

Problem due to $\exists R^-. \neg C$ term only satisfied in predecessor of blocking node

\[ L(w) = \{ \neg C, \exists R. (C \cap \exists R^- \neg C), \leq 1R^- \} \]

\[ L(x) = \{(C \cap \exists R^- \neg C), \exists R. (C \cap \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C \} \]

Solution is Double Blocking (pairwise blocking)

- Predecessors of blocked and blocking nodes also considered
Double Blocking I

Problem due to $\exists R^-. \neg C$ term **only** satisfied in **predecessor** of blocking node

\[ \mathcal{L}(w) = \{ \neg C, \exists R. (C \cap \exists R^-. \neg C), \leq 1R^- \} \]

\[ \mathcal{L}(x) = \{(C \cap \exists R^-. \neg C), \exists R. (C \cap \exists R^-. \neg C), \leq 1R^-, C, \exists R^- . \neg C \} \]

Solution is **Double Blocking** (pairwise blocking)

- Predecessors of blocked and blocking nodes also considered
- In particular, $\exists R. C$ terms satisfied in predecessor of blocking node must also be satisfied in predecessor of blocked node
  
  $\neg C \in \mathcal{L}(w)$
Due to pairwise condition, block no longer holds

\[ \mathcal{L}(w) = \{ \neg C, \exists R. (C \cap \exists R^-. \neg C), \leq 1R^- \} \]

\[ \mathcal{L}(x) = \{(C \cap \exists R^- . \neg C), \exists R. (C \cap \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C\} \]

\[ \mathcal{L}(y) = \{(C \cap \exists R^- . \neg C), \exists R. (C \cap \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C\} \]
Due to pairwise condition, block no longer holds
Expansion continues and contradiction discovered

\[ \mathcal{L}(w) = \{ \neg C, \exists R. (C \cap \exists R^- . \neg C), \leq 1R^- \} \]

\[ \mathcal{L}(x) = \{ (C \cap \exists R^- . \neg C), \exists R. (C \cap \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C \} \]

\[ \mathcal{L}(y) = \{ (C \cap \exists R^- . \neg C), \exists R. (C \cap \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C \} \]
Due to pairwise condition, block no longer holds
Expansion continues and contradiction discovered

\[
\begin{align*}
\mathcal{L}(w) = \{ \neg C, \exists R. (C \cap \exists R^- \neg C), \leq 1 R^- \} \\
\mathcal{L}(x) = \{ (C \cap \exists R^- \neg C), \exists R. (C \cap \exists R^- \neg C), \leq 1 R^-, C, \exists R^- \neg C \} \\
\mathcal{L}(y) = \{ (C \cap \exists R^- \neg C), \exists R. (C \cap \exists R^- \neg C), \leq 1 R^-, C, \exists R^- \neg C \} \\
\mathcal{L}(z) = \{ \neg C \}
\end{align*}
\]
Due to pairwise condition, block no longer holds

Expansion continues and contradiction discovered

\[
\begin{align*}
\wedge (w) & \mathcal{L}(w) = \{ \neg C, \exists R. (C \cap \exists R^- \neg C), \leq 1R^- \} \\
\downarrow & \leftarrow R \\
\wedge (x) & \mathcal{L}(x) = \{ (C \cap \exists R^- \neg C), \exists R. (C \cap \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C \} \\
\downarrow & \leftarrow R^- \\
\wedge (y) & \mathcal{L}(y) = \{ (C \cap \exists R^- \neg C), \exists R. (C \cap \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C \} \\
\downarrow & \leftarrow R^- \\
\wedge (z) & \mathcal{L}(z) = \{ \neg C \}
\end{align*}
\]
Due to pairwise condition, block no longer holds

Expansion continues and contradiction discovered

\[ \mathcal{L}(w) = \{ \neg C, \exists R.(C \cap \exists R^- \neg C), \leq 1R^- \} \]

\[ \mathcal{L}(x) = \{(C \cap \exists R^- \neg C), \exists R.(C \cap \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C, \neg C \} \]

\[ \mathcal{L}(y) = \{(C \cap \exists R^- \neg C), \exists R.(C \cap \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C \} \]

Clash