# **Description Logic Reasoning**

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- KB consistency reducible to concept consistency via internalisation
  - For logics supporting, e.g., a transitive "top" role

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- Return "C is consistent" iff C is consistent
  - Tree model property

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  - Nodes represent elements of  $\Delta^{\mathcal{I}}$ ; labeled with subconcepts of C
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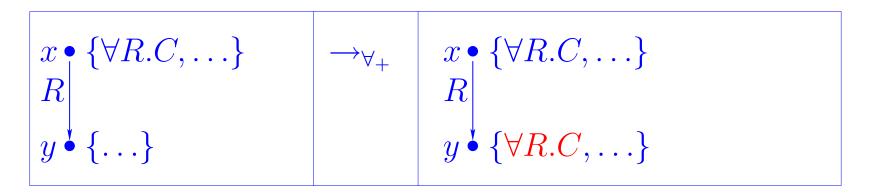
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- $\ \ \, \subset \ \,$  satisfiable iff fully expanded clash free  ${f T}$  found
  - ullet Trivial correspondence between such a  ${f T}$  and a model of C

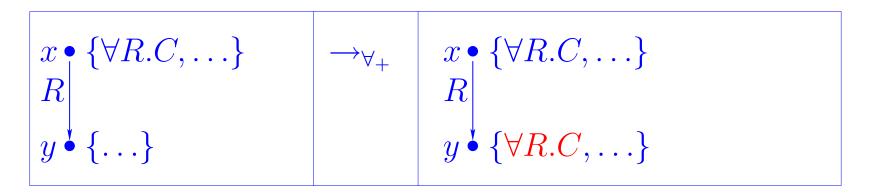
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| $x \bullet \{C_1 \sqcap C_2, \ldots\}$   | $\rightarrow$ $\sqcap$ | $x \bullet \{C_1 \sqcap C_2, C_1, C_2, \ldots\}$                                    |
|--|------------------------|---|
| $x \bullet \{C_1 \sqcup C_2, \ldots\}$   | $\rightarrow$ $\sqcup$ | $x \bullet \{C_1 \sqcap C_2, \textcolor{red}{C}, \ldots\}$ for $C \in \{C_1, C_2\}$ |
| $x \bullet \{\exists R.C, \ldots\}$  | →∃                     | $x \bullet \{\exists R.C, \ldots\}$ $R \bullet \{C\}$                               |
| $\begin{bmatrix} x \bullet \{ \forall R.C, \ldots \} \\ R \\ y \bullet \{ \ldots \} \end{bmatrix}$ | $\rightarrow \forall$  | $x \bullet \{ \forall R.C, \ldots \}$ $R \downarrow$ $y \bullet \{C, \ldots \}$     |

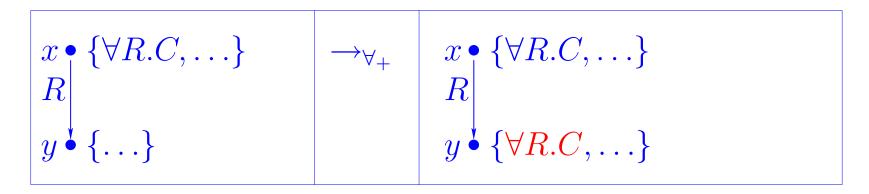


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- ightharpoonup No longer naturally terminating (e.g., if  $C = \exists R. \top$ )
- Need blocking
  - Simple blocking suffices for ALC plus transitive roles
  - I.e., do not expand node label if ancestor has superset label
  - More expressive logics (e.g., with inverse roles) need more sophisticated blocking strategies

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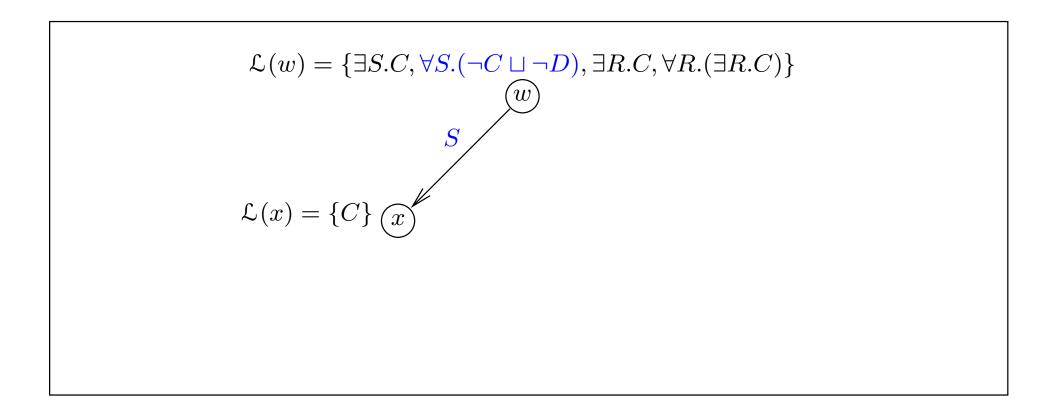
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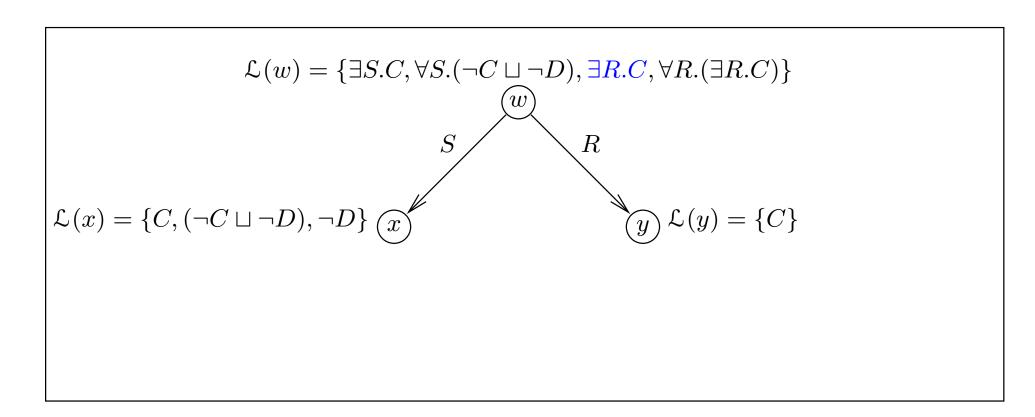
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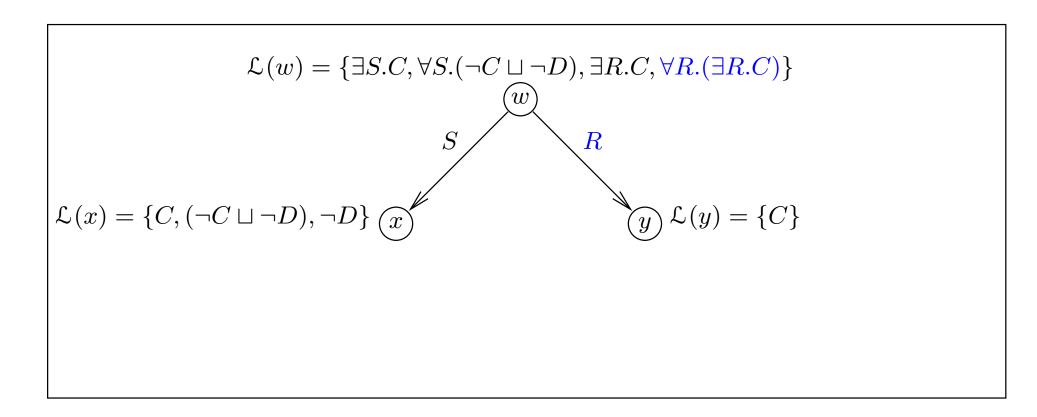
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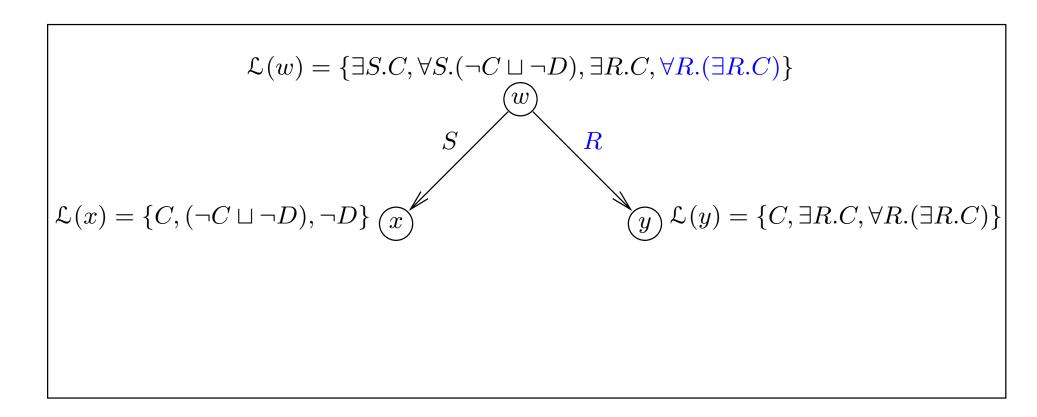
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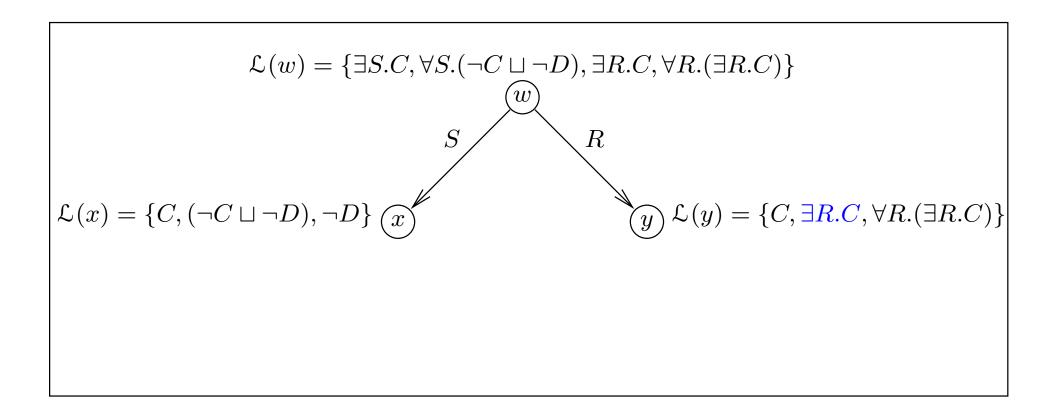
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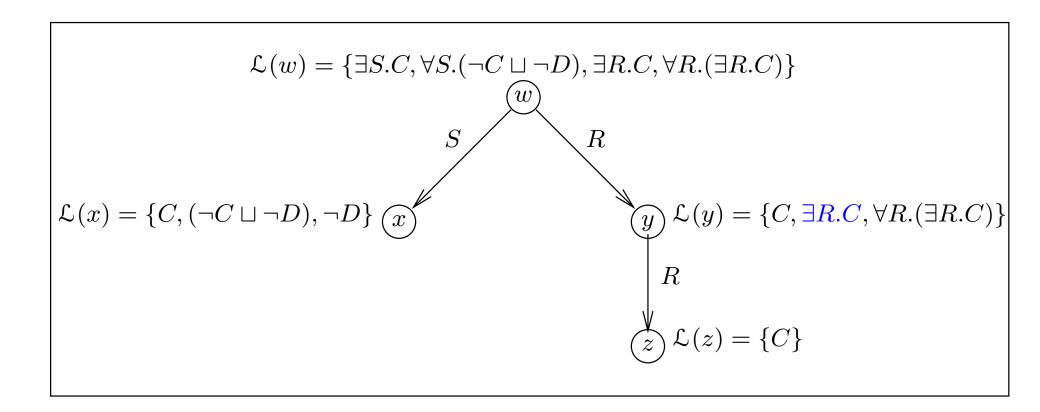
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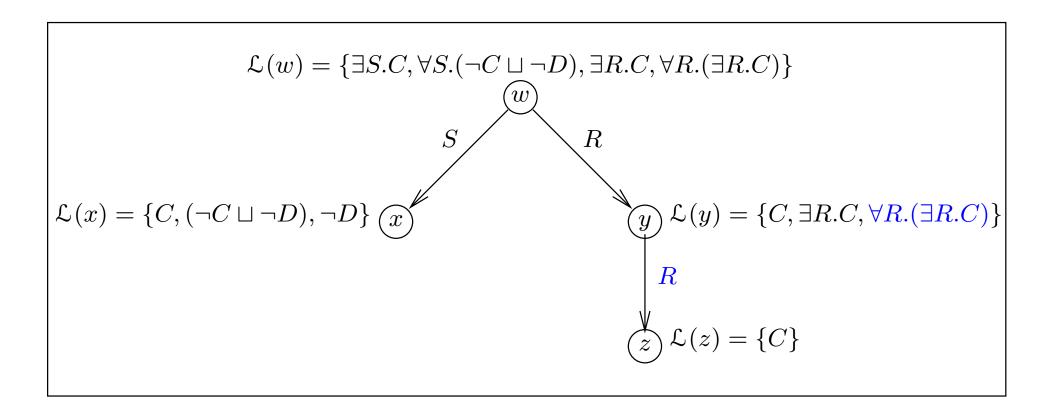


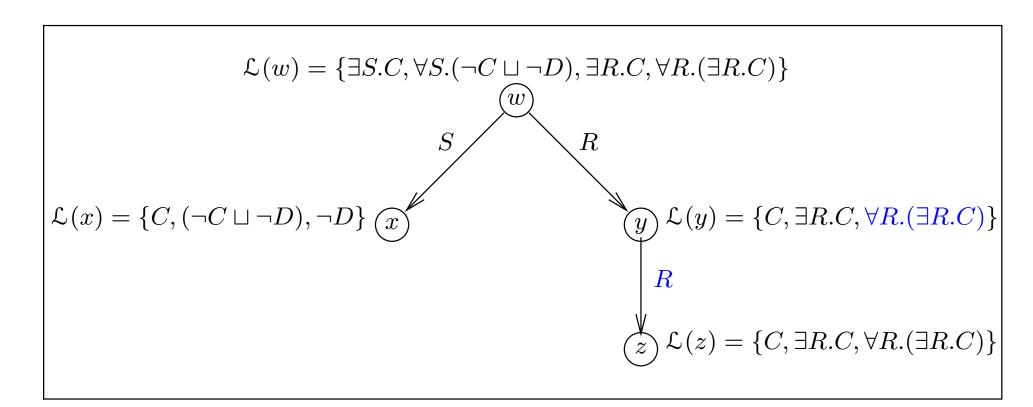


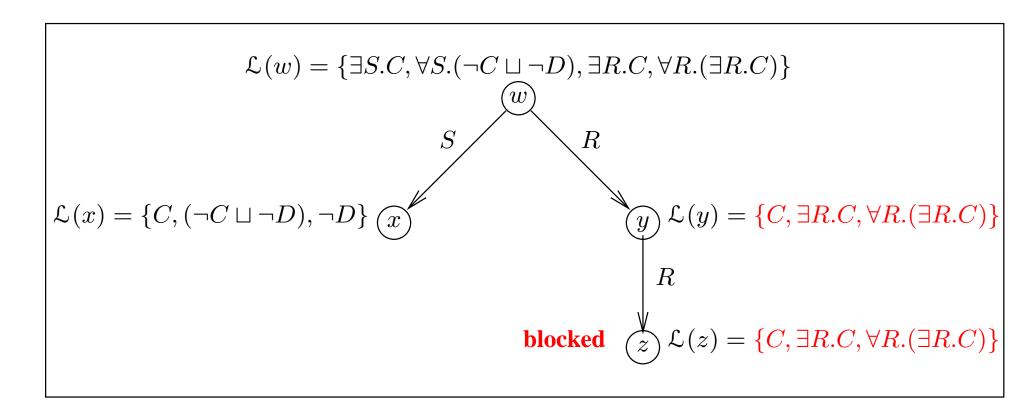




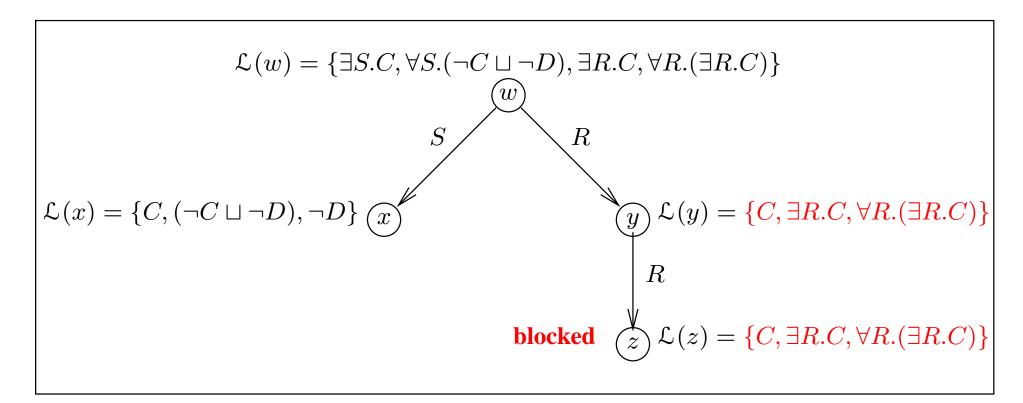






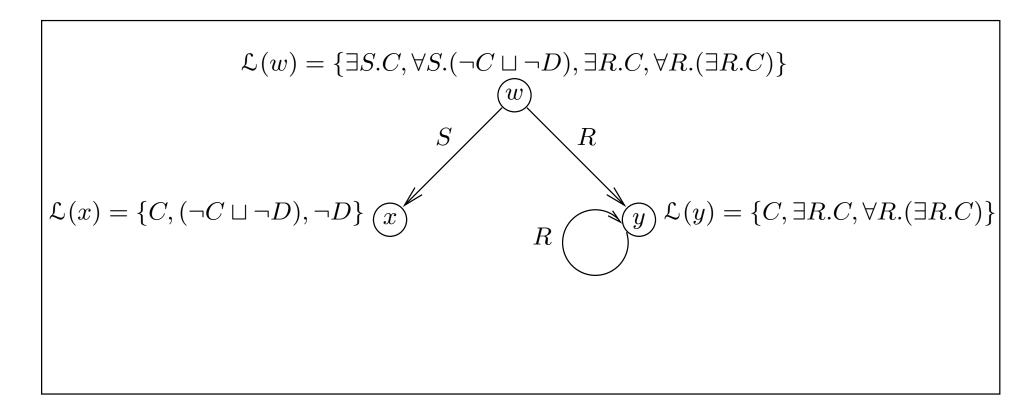


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- Forest instead of Tree (for Aboxes)
  - Root nodes correspond to individuals in Abox

# **Implementing DL Systems**

### **Problems** include:

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  - Mitigated by:
    - Careful choice of algorithm
    - Highly optimised implementation

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- BUT even simple domain encoding is disastrous with large numbers of roles

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- Optimised subsumption testing (search for models)
  - Normalisation and simplification of concepts
  - Absorption (rewriting) of general axioms
  - Davis-Putnam style semantic branching search
  - Dependency directed backtracking
  - Caching of satisfiability results and (partial) models
  - Heuristic ordering of propositional and modal expansion
  - ...

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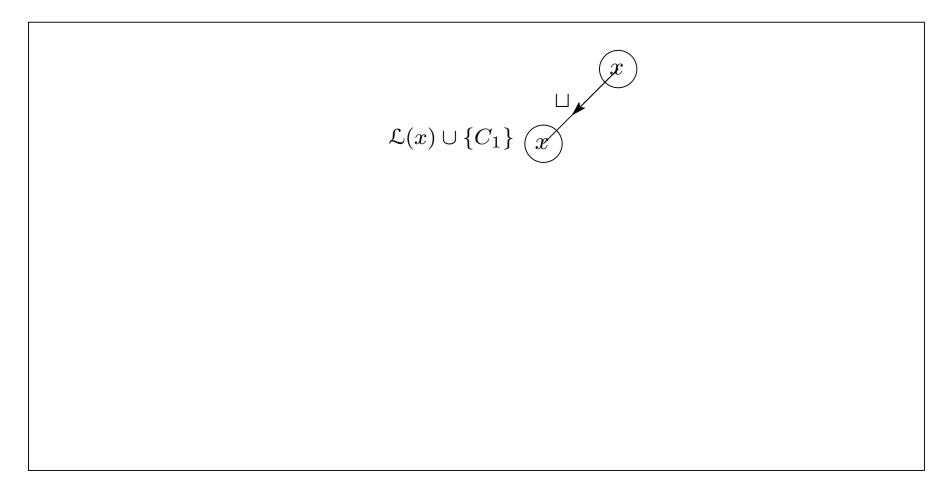
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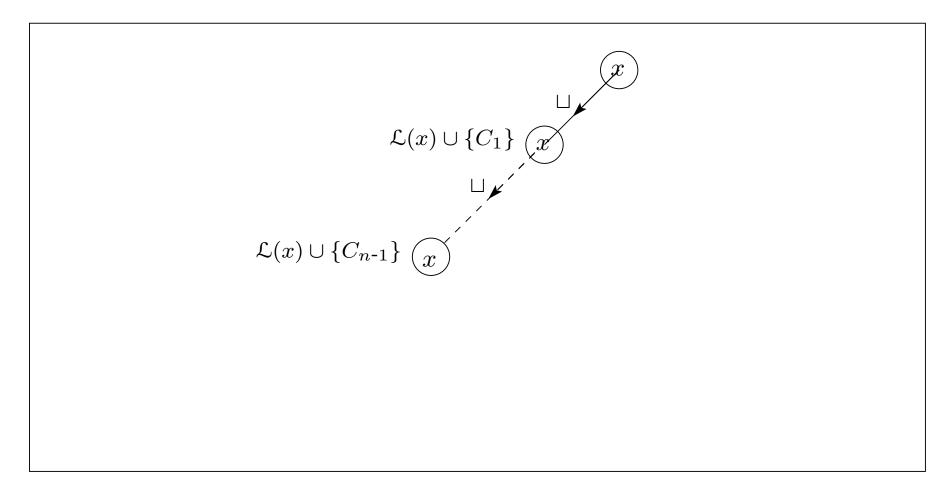
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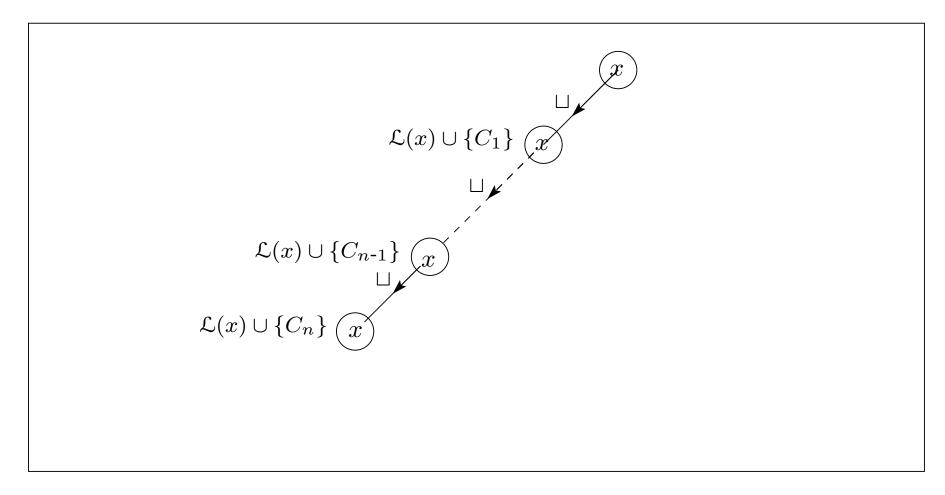
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  - Effect is to prune away part of the search space
- Highly effective essential for usable system
  - E.g., GALEN KB, 30s (with) → months++ (without)

E.g., if 
$$\exists R. \neg A \sqcap \forall R. (A \sqcap B) \sqcap (C_1 \sqcup D_1) \sqcap \ldots \sqcap (C_n \sqcup D_n) \subseteq \mathcal{L}(x)$$







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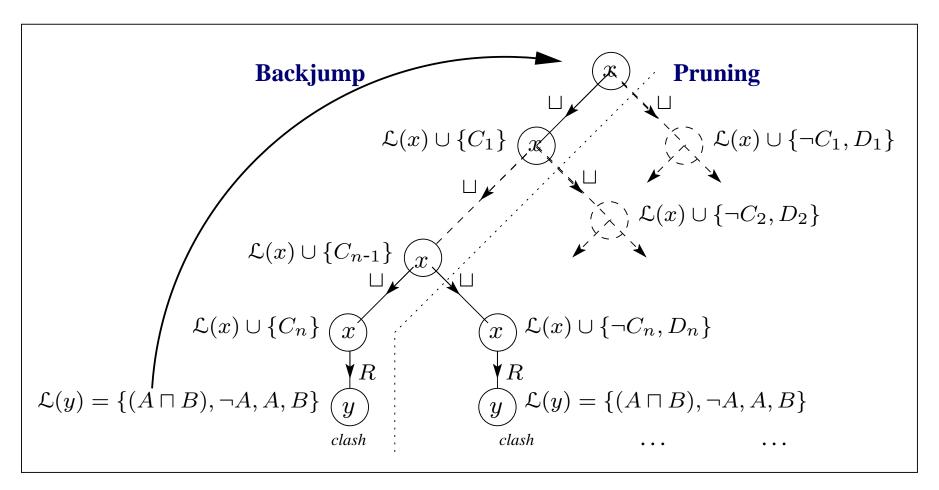
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#### Tools and Infrastructure

Support for large scale ontological engineering and deployment

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- Already seeing some (partial) implementations
  - Cerebra system (Network Inference), Racer system (Hamburg)

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- Standard solution is weaker semantics for nominals
  - Treat nominals as (disjoint) primitive classes
  - Loss of completeness/soundness

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- How can reasoners be developed/adapted for extended languages
  - Some existing work on language fusions and hybrid reasoners

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- Reasoning with individuals
  - Deployment of web ontologies will mean reasoning with (possibly very large numbers of) individuals/tuples
  - Unlikely that standard Abox techniques will be able to cope

Excessive memory usage

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- Reasoning with very large KBs
  - DL systems shown to work with ≈100k concept KB [Haarslev & Möller]
  - But KB only exploited small part of DL language

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- To support ontology design
- Justifications and proofs (e.g., of query results)
- "Non-Standard Inferences", e.g., LCS, matching
  - To support ontology integration
  - To support "bottom up" design of ontologies

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- Highly Optimised implementations used in DL systems
- Challenges remain
  - Reasoning with full OWL language
  - (Convincing) demonstration(s) of scalability
  - New reasoning tasks
  - Development of (high quality) tools and infrastructure

Members of the OIL, DAML+OIL and OWL development teams, in particular Dieter Fensel (DERI), Frank van Harmelen (Amsterdam) and Peter Patel-Schneider (Bell Labs)







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- Uli Sattler, Carole Goble and other Members of the Information Management, Medical Informatics and Formal Methods Groups at the University of Manchester





#### Resources

```
Slides from this talk
 http://www.cs.man.ac.uk/~horrocks/Slides/Innsbruck-tutorial/
FaCT system (open source)
 http://www.cs.man.ac.uk/FaCT/
OilEd (open source)
 http://oiled.man.ac.uk/
OIL
 http://www.ontoknowledge.org/oil/
W3C Web-Ontology (WebOnt) working group (OWL)
 http://www.w3.org/2001/sw/WebOnt/
DL Handbook, Cambridge University Press
 http://books.cambridge.org/0521781760.htm
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