Description Logics—Basics, Applications, and More

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Overview of the Tutorial

- **History and Basics:** Syntax, Semantics, ABoxes, Tboxes, Inference Problems and their interrelationship, and Relationship with other (logical) formalisms
- **Applications of DLs:** ER-diagrams with i.com demo, ontologies, etc. including system demonstration
- **Reasoning Procedures:** simple tableaux and why they work
- **Reasoning Procedures II:** more complex tableaux, non-standard inference problems
- **Complexity issues**
- **Implementing/Optimising DL systems**

Description Logics

- family of logic-based knowledge representation formalisms well-suited for the representation of and reasoning about

➠ **terminological knowledge**

➠ **configurations**

➠ **ontologies**

➠ **database schemata**

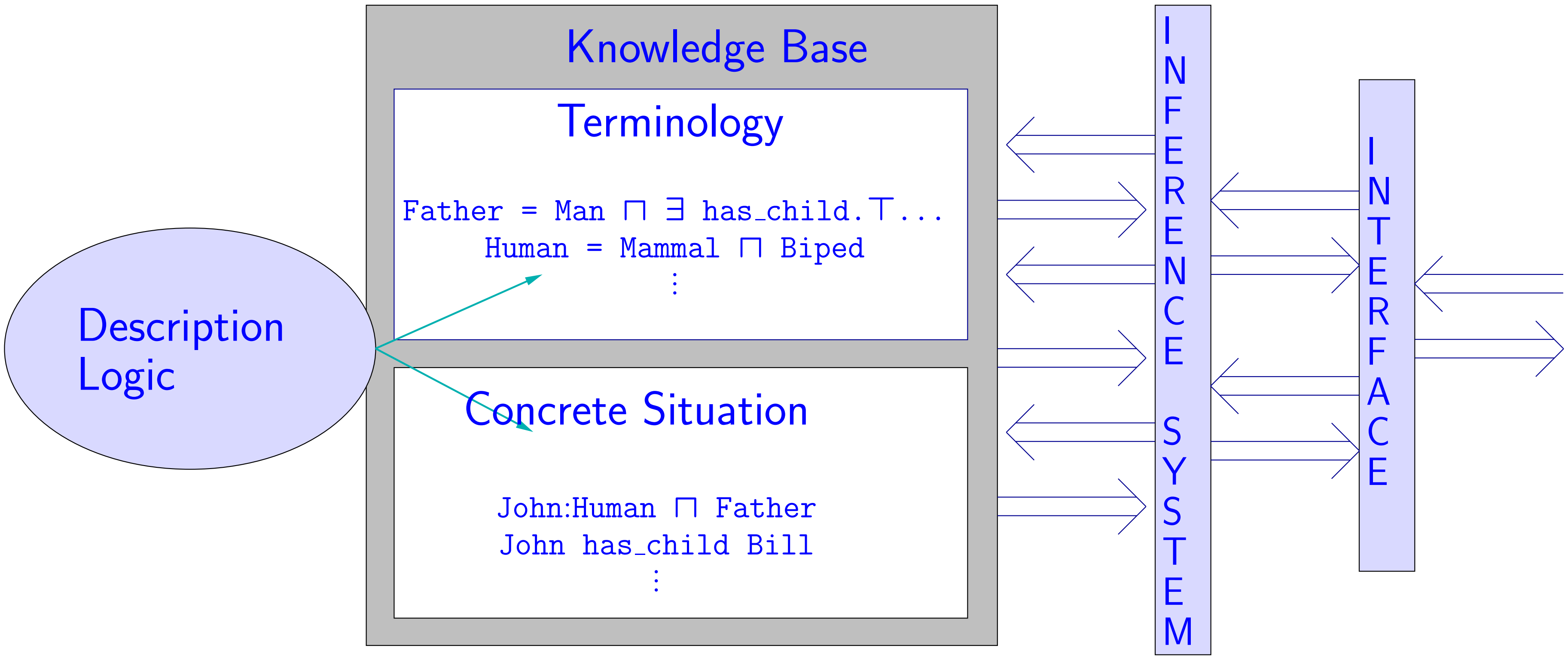
  – schema design, evolution, and query optimisation
  – source integration in heterogeneous databases/data warehouses
  – conceptual modelling of multidimensional aggregation

➠ ...

- descendents of semantics networks, frame-based systems, and KL-ONE

- aka terminological KR systems, concept languages, etc.

Architecture of a Standard DL System

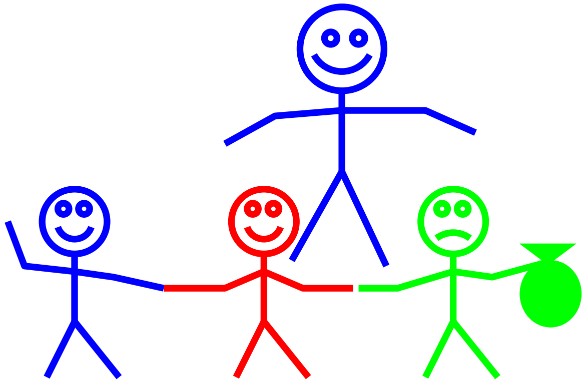


A Description Logic - mainly characterised by a **set of constructors** that allow to build complex **concepts** and **roles** from atomic ones,

concepts correspond to classes / are interpreted as **sets** of objects,

roles correspond to relations / are interpreted as **binary relations** on objects,

Example: Happy Father in the DL $\mathcal{ALC}$



$$\begin{aligned}\texttt{Man} \sqcap\ & (\exists \texttt{has-child.Blue}) \sqcap \\ & (\exists \texttt{has-child.Green}) \sqcap \\ & (\forall \texttt{has-child.Happy} \sqcup \texttt{Rich})\end{aligned}$$

Semantics given by means of an **interpretation** $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$:

Constructor	Syntax	Example	Semantics
atomic concept	A	Human	$A^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
atomic role	R	likes	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
For C, D concepts and R a role name			
conjunction	$C \sqcap D$	Human $\sqcap$ Male	$C^{\mathcal{I}} \cap D^{\mathcal{I}}$
disjunction	$C \sqcup D$	Nice $\sqcup$ Rich	$C^{\mathcal{I}} \cup D^{\mathcal{I}}$
negation	$\neg C$	$\neg$ Meat	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
exists restrict.	$\exists R.C$	$\exists$has-child.Human	$\{x \mid \exists y.\langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\}$
value restrict.	$\forall R.C$	$\forall$has-child.Blond	$\{x \mid \forall y.\langle x, y\rangle \in R^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\}$

Constructor	Syntax	Example	Semantics
number restriction	$(\geq n\ R)$	$(\geq 7$ has-child$)$	$\{x \mid \lvert\{y.\langle x,y\rangle \in R^{\mathcal{I}}\}\rvert \geq n\}$
$(\rightsquigarrow \mathcal{ALCN})$	$(\leq n\ R)$	$(\leq 1$ has-mother$)$	$\{x \mid \lvert\{y.\langle x,y\rangle \in R^{\mathcal{I}}\}\rvert \leq n\}$
inverse role	R^{-}	has-child^{-}	$\{\langle x,y\rangle \mid \langle y,x\rangle \in R^{\mathcal{I}}\}$
trans. role	R^{*}	has-child*	$(R^{\mathcal{I}})^{*}$
concrete domain	$u_1,\ldots,u_n.P$	h-father·age, age. $>$	$\{x \mid \langle u_1^{\mathcal{I}}(x),\ldots,u_n^{\mathcal{I}}(x)\rangle \in P\}$
etc.			

Many other different DLs/DL constructors have been investigated

For terminological knowledge: **TBox** contains

Concept definitions $A \doteq C$ (A a concept name, C a complex concept)

Father $\doteq$ Man $\sqcap$ $\exists$has-child.Human

Human $\doteq$ Mammal $\sqcap$ $\forall$has-child^{-}.Human

$\leadsto$ introduce macros/names for concepts, can be (a)cyclic

Axioms $C_1 \sqsubseteq C_2$ (C_i complex concepts)

$\exists$favourite.Brewery $\sqsubseteq$ $\exists$drinks.Beer

$\leadsto$ restrict your models

An interpretation $\mathcal{I}$ **satisfies**

a **concept definition** $A \doteq C$ iff $A^{\mathcal{I}} = C^{\mathcal{I}}$

an **axiom** $C_1 \sqsubseteq C_2$ iff $C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$

a **TBox** $\mathcal{T}$ iff $\mathcal{I}$ satisfies all definitions and axioms in $\mathcal{T}$

$\leadsto$ $\mathcal{I}$ is a **model** of $\mathcal{T}$

For assertional knowledge: **ABox** contains

Concept assertions	a : C	(a an individual name, C a complex concept)
	John : Man $\sqcap$ $\forall$has-child.(Male $\sqcap$ Happy)	
Role assertions	$\langle a_1, a_2 \rangle$: R	(a_i individual names, R a role)
	$\langle$John, Bill$\rangle$: has-child	

An interpretation $\mathcal{I}$ **satisfies**

a **concept assertion**	$a : C$	iff	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
a **role assertion**	$\langle a_1, a_2 \rangle : R$	iff	$\langle a_1^{\mathcal{I}}, a_2^{\mathcal{I}} \rangle \in R^{\mathcal{I}}$
an **ABox**	$\mathcal{A}$	iff	$\mathcal{I}$ satisfies all assertions in $\mathcal{A}$
		$\leadsto$	$\mathcal{I}$ is a **model** of $\mathcal{A}$

Subsumption: $C \sqsubseteq D$ Is $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ in all interpretations $\mathcal{I}$?

w.r.t. TBox $\mathcal{T}$: $C \sqsubseteq_{\mathcal{T}} D$ Is $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$ in all models $\mathcal{I}$ of $\mathcal{T}$?

$\leadsto$ structure your knowledge, compute taxonomy

Consistency: Is C consistent w.r.t. $\mathcal{T}$? Is there a model $\mathcal{I}$ of $\mathcal{T}$ with $C^{\mathcal{I}} \neq \emptyset$?

of ABox $\mathcal{A}$: Is $\mathcal{A}$ consistent? Is there a model of $\mathcal{A}$?

of KB ($\mathcal{T}$,$\mathcal{A}$): Is ($\mathcal{T}$,$\mathcal{A}$) consistent? Is there a model of both $\mathcal{T}$ and $\mathcal{A}$?

Inference Problems are **closely related:**

$C \sqsubseteq_{\mathcal{T}} D$ iff $C \sqcap \neg D$ is **in**consistent w.r.t. $\mathcal{T}$,
(no model of $\mathcal{I}$ has an instance of $C \sqcap \neg D$)

C is consistent w.r.t. $\mathcal{T}$ iff **not** $C \sqsubseteq_{\mathcal{T}} A \sqcap \neg A$

$\leadsto$ **Decision Procdures for consistency (w.r.t. TBoxes) suffice**

Introduction to DL: Basic Inference Problems II

For most DLs, the basic inference problems are **decidable**,
with complexities between **P** and **ExpTime**.

Why is decidability important? Why does semi-decidability not suffice?

If subsumption (and hence consistency) is undecidable, and

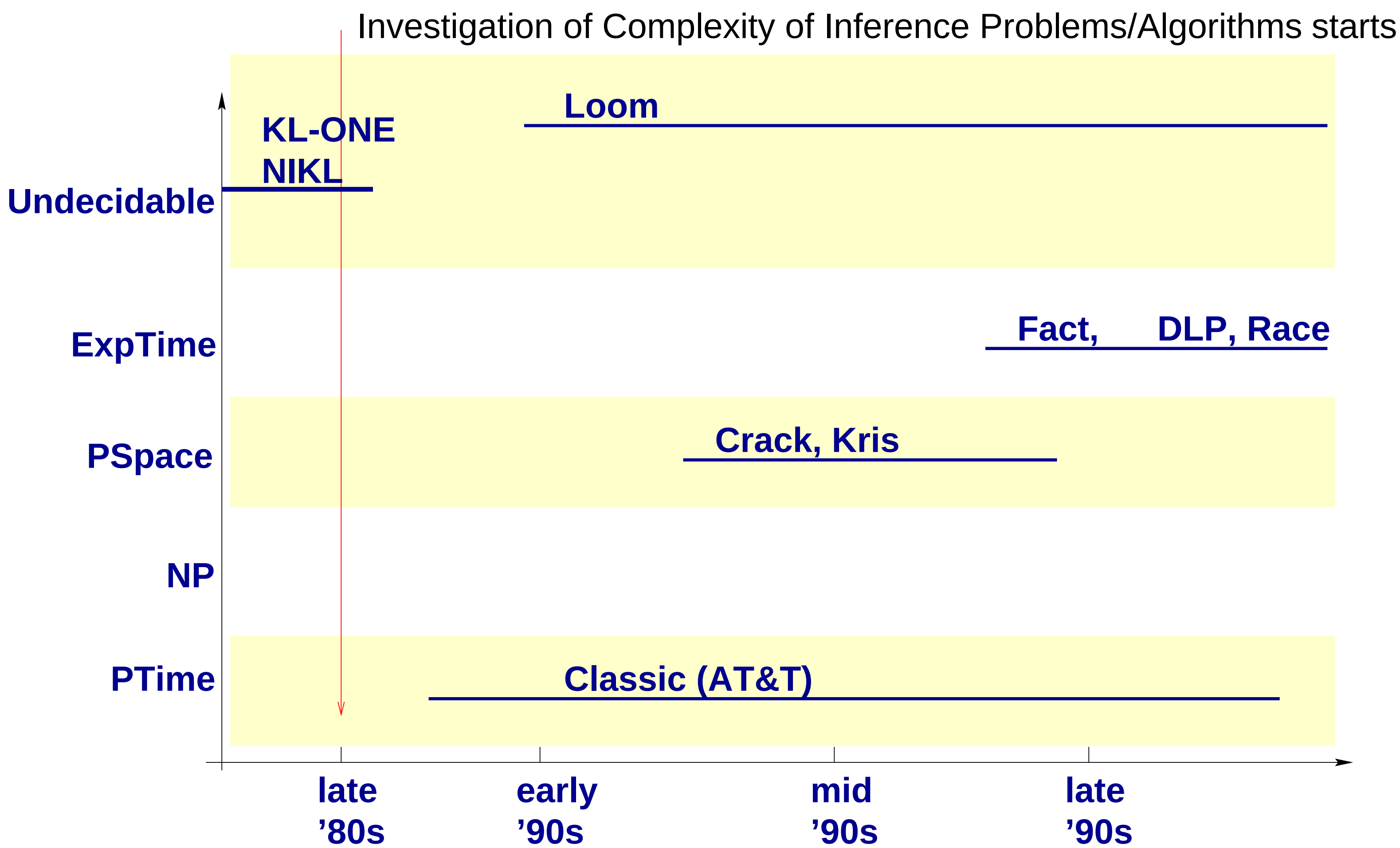
➠ subsumption is semi-decidable, then consistency is **not** semi-decidable

➠ consistency is semi-decidable, then subsumption is **not** semi-decidable

➠ Quest for a "highly expressive" DL with decidable/"practicable" inference problems

where **expressiveness** depends on the application
practicability changed over the time

Complexity of Inferences provided by DL systems over the time



In the last 5 years, DL-based systems were built that

✔ can handle DLs far more expressive than $\mathcal{ALC}$ (close relatives of converse-DPDL)

- Number restrictions: "people having at most 2 children"
- Complex roles: inverse ("has-child" — "child-of"),
  transitive closure ("offspring" — "has-child"),
  role inclusion ("has-daughter" — "has-child"), etc.

✔ implement provably sound and complete inference algorithms
(for ExpTime-complete problems)

✔ can handle large knowledge bases
(e.g., Galen medical terminology ontology: 2,740 concepts, 413 roles, 1,214 axioms)

✔ are highly optimised versions of **tableau-based** algorithms

✔ perform (surprisingly well) on benchmarks for modal logic reasoners
(Tableaux'98, Tableaux'99)

Relationship with Other Logical Formalisms: First Order Predicate Logic

Most DLs are decidable fragments of FOL: Introduce

a unary predicate ϕ_A for a concept name A
a binary relation ρ_R for a role name R

Translate complex concepts C, D as follows:

$$
\begin{aligned}
t_x(A) &= \phi_A(x), & t_y(A) &= \phi_A(y),\\
t_x(C \sqcap D) &= t_x(C) \land t_x(D), & t_y(C \sqcap D) &= t_y(C) \land t_y(D),\\
t_x(C \sqcup D) &= t_x(C) \lor t_x(D), & t_y(C \sqcup D) &= t_y(C) \lor t_y(D),\\
t_x(\exists R.C) &= \exists y.\rho_R(x,y) \land t_y(C), & t_y(\exists R.C) &= \exists x.\rho_R(y,x) \land t_x(C),\\
t_x(\forall R.C) &= \forall y.\rho_R(x,y) \Rightarrow t_y(C), & t_y(\forall R.C) &= \forall x.\rho_R(y,x) \Rightarrow t_x(C).
\end{aligned}
$$

A TBox $\mathcal{T} = \{C_i \sqsubseteq D_i\}$ is translated as

$$\Phi_{\mathcal{T}} = \forall x. \bigwedge_{1 \le i \le n} t_x(C_i) \Rightarrow t_x(D_i)$$

Relationship with Other Logical Formalisms: First Order Predicate Logic II

$$\begin{aligned}
C \text{ is consistent} &\text{ iff its translation } t_x(C) \text{ is satisfiable,}\\
C \text{ is consistent w.r.t. } \mathcal{T} &\text{ iff its translation } t_x(C) \wedge \Phi_{\mathcal{T}} \text{ is satisfiable,}\\
C \sqsubseteq D &\text{ iff } t_x(C) \Rightarrow t_x(D) \text{ is valid}\\
C \sqsubseteq_{\mathcal{T}} D &\text{ iff } \Phi_t \Rightarrow \forall x.(t_x(C) \Rightarrow t_x(D)) \text{ is valid.}
\end{aligned}$$

$\leadsto$ $\mathcal{ALC}$ is a fragment of FOL with 2 variables (L2), known to be decidable

$\leadsto$ $\mathcal{ALC}$ with inverse roles and Boolean operators on roles is a fragment of L2

$\leadsto$ further adding number restrictions yields a fragment of C2 (L2 with "counting quantifiers"), known to be decidable

- in contrast to most DLs, adding transitive roles/transitive closure operator to L2 leads to **undecidability**
- many DLs (like many modal logics) are fragments of the **Guarded Fragment**
- most DLs are less complex than L2: L2 is NExpTime-complete, most DLs are in ExpTime

DLs and Modal Logics are closely related:

$\mathcal{ALC} \rightleftarrows$ multi-modal $\mathbf{K}$:

$$C \sqcap D \rightleftarrows C \land D, \qquad C \sqcup D \rightleftarrows C \lor D$$
$$\neg C \rightleftarrows \neg C \quad ,$$
$$\exists R.C \rightleftarrows \langle R \rangle C \quad , \qquad \forall R.C \rightleftarrows [R]C$$

transitive roles $\stackrel{\bullet}{\rightleftarrows}$ transitive frames (e.g., in $\mathbf{K4}$)

regular expressions on roles $\stackrel{\bullet}{\rightleftarrows}$ regular expressions on programs (e.g., in PDL)

inverse roles $\stackrel{\bullet}{\rightleftarrows}$ converse programs (e.g., in C-PDL)

number restrictions $\stackrel{\bullet}{\rightleftarrows}$ deterministic programs (e.g., in D-PDL)

➪ no TBoxes available in modal logics
  $\leadsto$ "internalise" axioms using a universal role u: $C \doteq D \rightleftarrows [u](C \Leftrightarrow D)$

➪ no ABox available in modal logics $\leadsto$ use nominals