

# Complexity of DLs

# Complexity of DLs: Overview of the Complexity of Concept Consistency

P	(co-)NP	PSpace	ExpTime	NExpTime
		<i>ALCN</i> (wrt acyc. TBoxes)		

$\mathcal{I}$  inverse roles: h-child<sup>-</sup>  
 $\mathcal{N}$  NRs: ( $\geq n$  h-child)  
 $\mathcal{Q}$  Qual. NRs: ( $\geq n$  h-child Blond)  
 $\mathcal{O}$  nominals: "John" is a concept  
 $\mathcal{F}$  feature chain (dis)agreement  
 $\cdot R^+$  declare roles as transitive  
 $\cdot \neg, \cap, \cup$  Boolean ops on roles

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	$\mathcal{ALUN}$ (NP) without $\exists$ , only $\neg A$	$\mathcal{ALCN}$ (wrt acyc. TBoxes)		

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<i>ACN</i> without $\sqcup$	<i>ACUN</i> (NP) without $\exists$ , only $\neg A$	<i>ALCN</i> (wrt acyc. TBoxes)		

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<p><i>ACN</i> without <math>\sqcup</math></p>	<p><i>ACUN</i> (NP) without <math>\exists</math>, only <math>\neg A</math></p> <p><i>ACE</i> (co-NP) without <math>\sqcup</math> and NRs, only <math>\neg A</math></p>	<p><i>ALCN</i> (wrt acyc. TBoxes)</p>		

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P	(co-)NP	PSpace	ExpTime	NExpTime
<p><math>\mathcal{ALN}</math> without <math>\sqcup</math></p> <p>subsumption of <math>\mathcal{FL}_0</math> <math>\sqcap</math> and <math>\forall</math> only</p>	<p><math>\mathcal{ALUN}</math> (NP) without <math>\exists</math>, only <math>\neg A</math></p> <p><math>\mathcal{ALCE}</math> (co-NP) without <math>\sqcup</math> and NRs, only <math>\neg A</math></p>	<p><math>\mathcal{ALCN}</math> (wrt acyc. TBoxes)</p>		

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P	(co-)NP	PSPACE	ExpTime	NExpTime
<p><math>\mathcal{ALN}</math> without <math>\sqcup</math></p> <p>subsumption of <math>\mathcal{FL}_0</math> <math>\sqcap</math> and <math>\forall</math> only</p>	<p><math>\mathcal{ALUN}</math> (NP) without <math>\exists</math>, only <math>\neg A</math></p> <p><math>\mathcal{ALC}</math> (co-NP) without <math>\sqcup</math> and NRs, only <math>\neg A</math></p> <p>subsumption of <math>\mathcal{FL}_0</math> (co-NP) wrt acyc. TBoxes</p>	<p><math>\mathcal{ALCN}</math> (wrt acyc. TBoxes)</p>		

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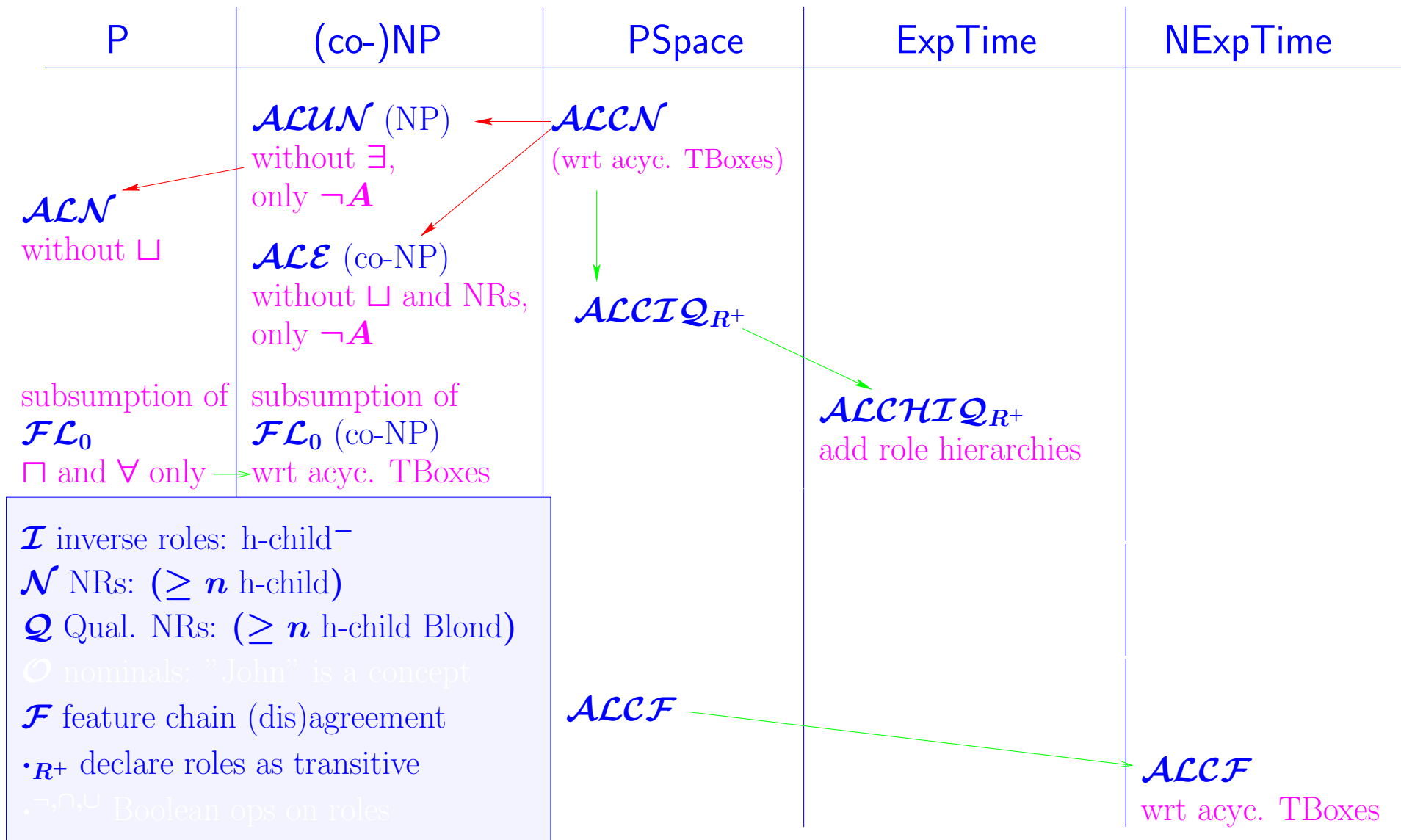
•  $R^+$  declare roles as transitive

•  $\neg, \cap, \cup$  Boolean ops on roles

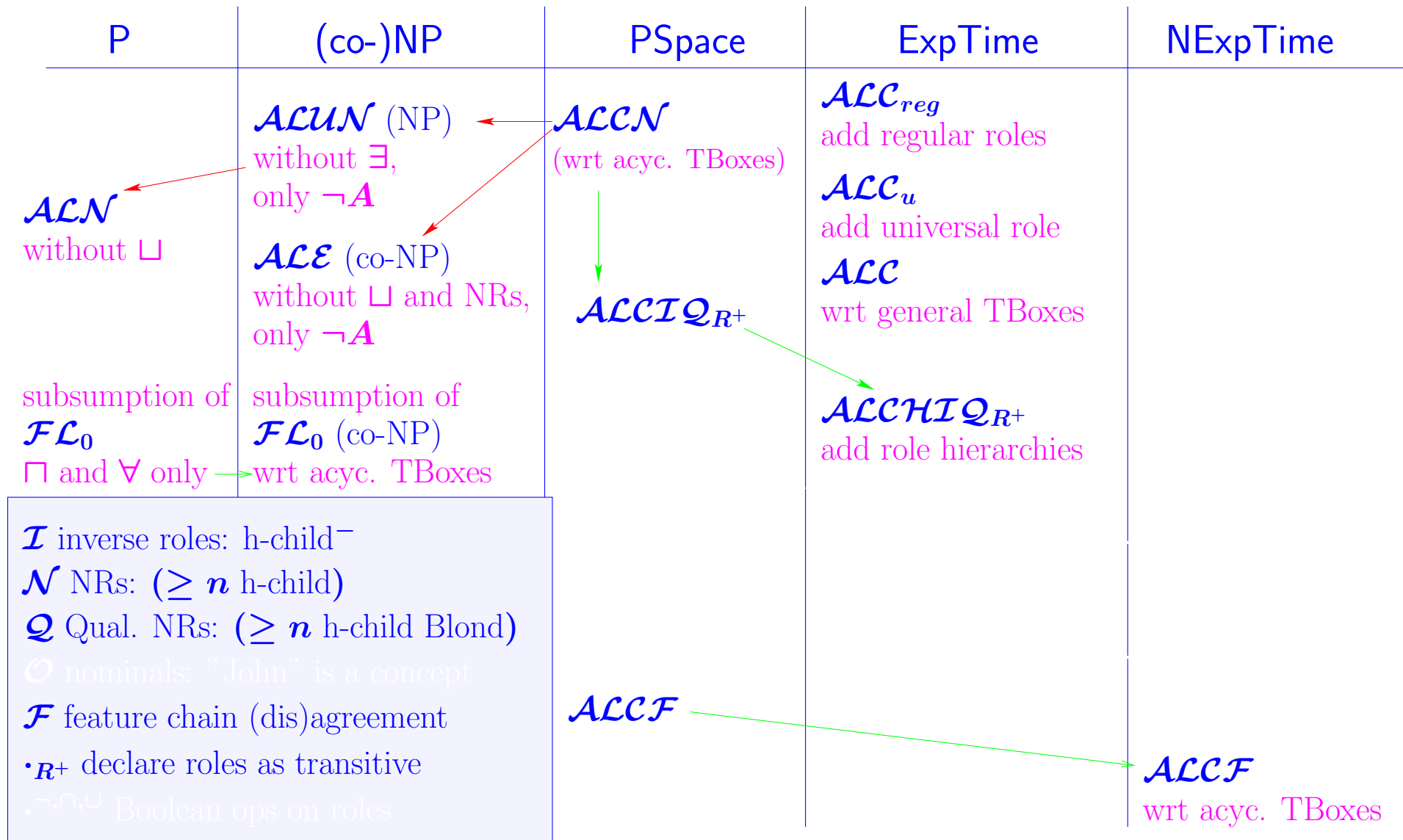
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P	(co-)NP	PSpace	ExpTime	NExpTime
<p><b>ACN</b> without <math>\sqcup</math></p> <p>subsumption of <b>FL<sub>0</sub></b> <math>\sqcap</math> and <math>\forall</math> only</p>	<p><b>ACUN</b> (NP) without <math>\exists</math>, only <math>\neg A</math></p> <p><b>ACE</b> (co-NP) without <math>\sqcup</math> and NRs, only <math>\neg A</math></p> <p>subsumption of <b>FL<sub>0</sub></b> (co-NP) wrt acyc. TBoxes</p>	<p><b>ALCN</b> (wrt acyc. TBoxes)</p>		
<div style="border: 1px solid black; padding: 5px;"> <p><math>\mathcal{I}</math> inverse roles: h-child<sup>-</sup></p> <p><math>\mathcal{N}</math> NRs: (<math>\geq n</math> h-child)</p> <p><math>\mathcal{Q}</math> Qual. NRs: (<math>\geq n</math> h-child Blond)</p> <p><math>\mathcal{O}</math> nominals: "John" is a concept</p> <p><math>\mathcal{F}</math> feature chain (dis)agreement</p> <p>• <math>R^+</math> declare roles as transitive</p> <p>• <math>\neg, \cap, \cup</math> Boolean ops on roles</p> </div>		<p><b>ALCF</b></p>		<p><b>ALCF</b> wrt acyc. TBoxes</p>

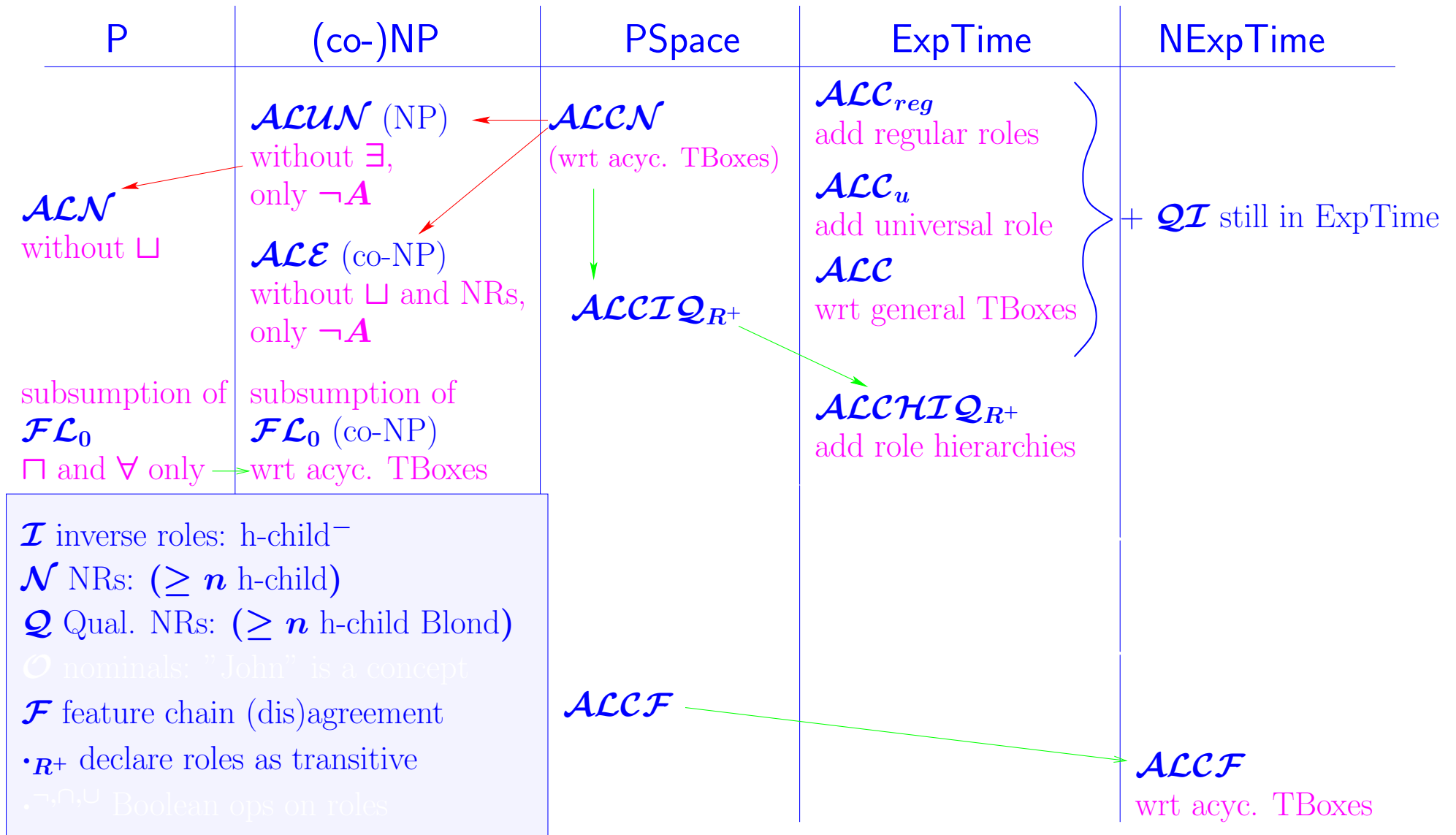
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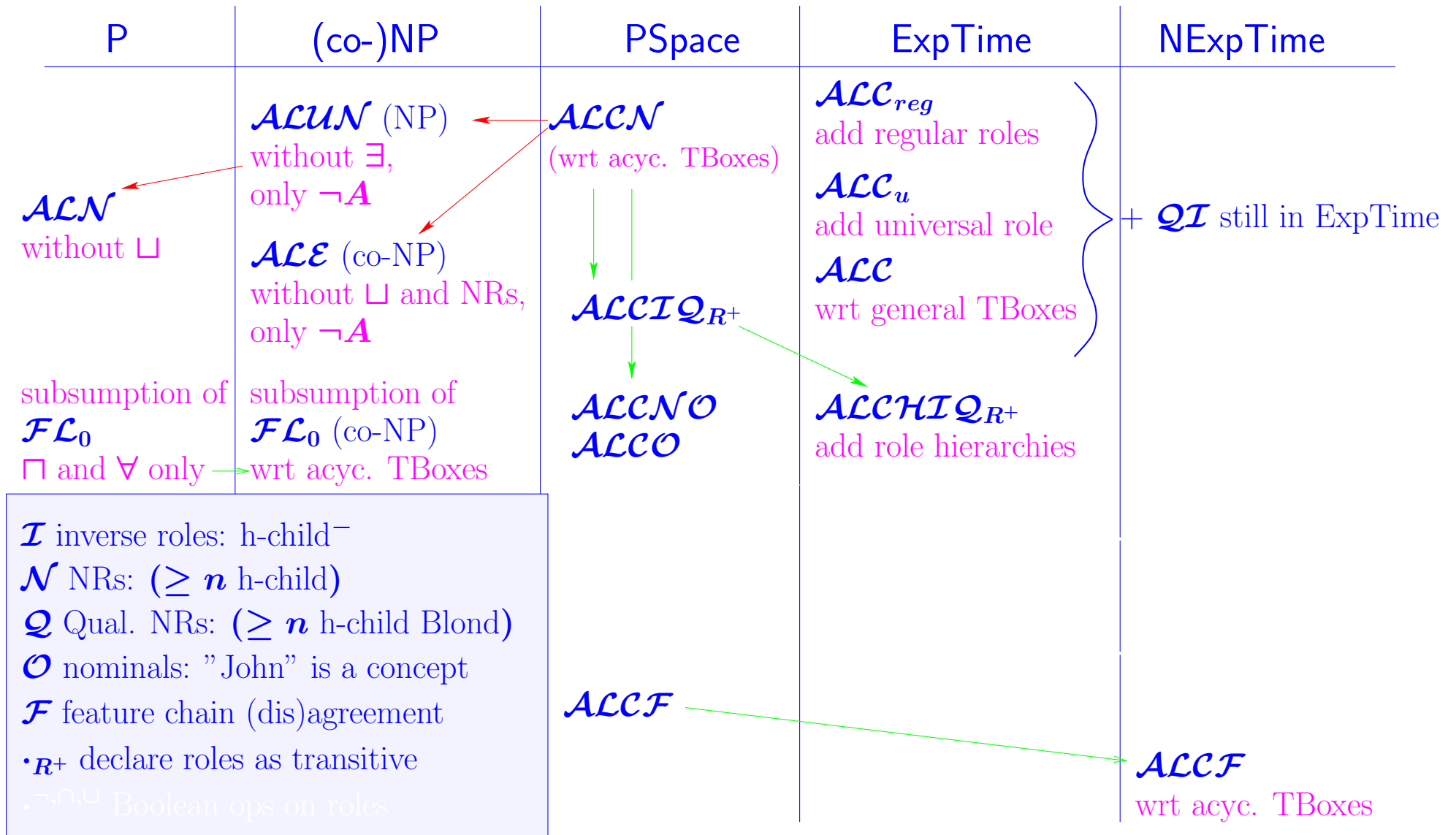
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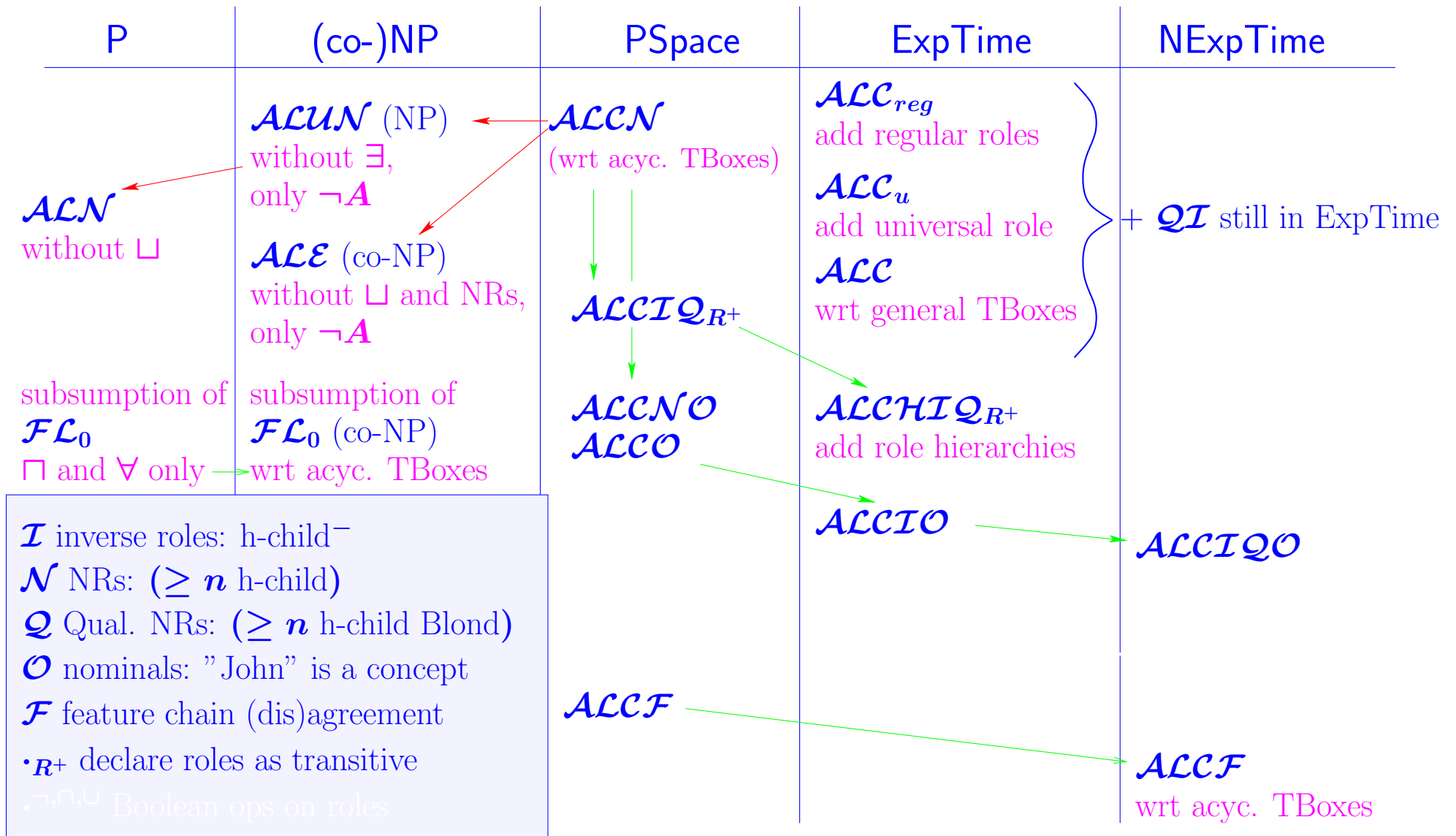
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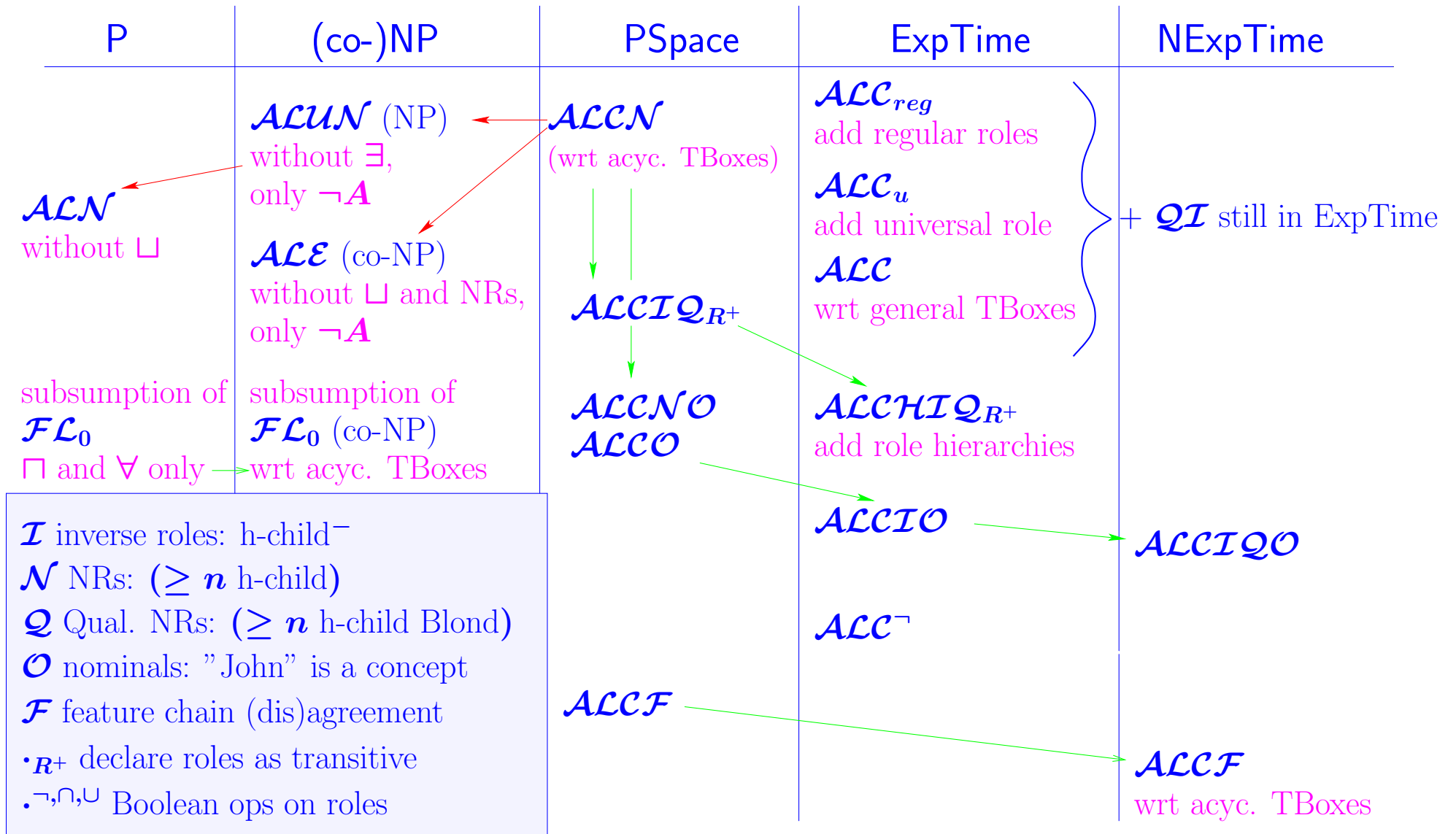
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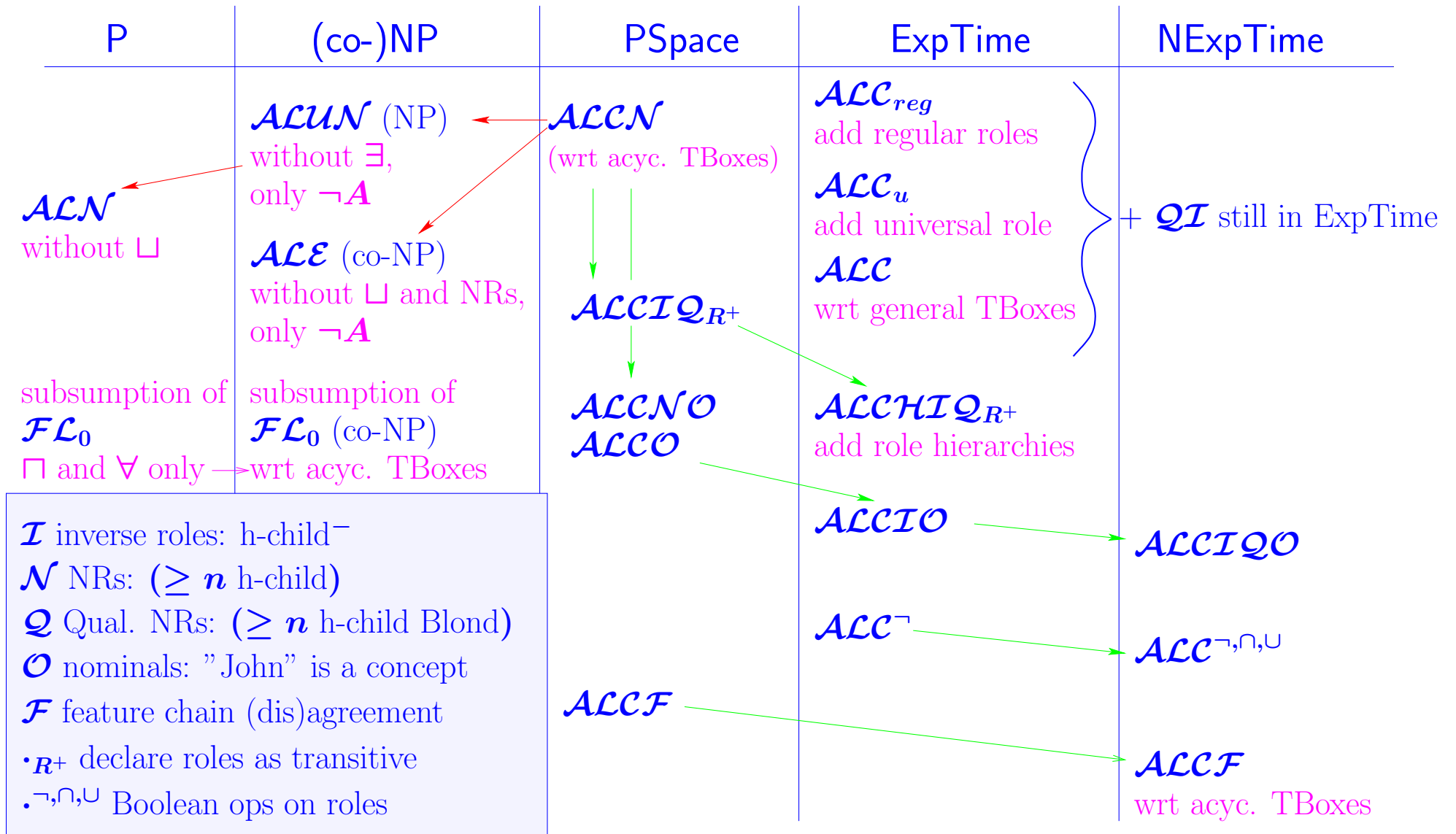
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We left out a variety of complexity results for

- ⇒ **concept consistency of other DLs**  
(e.g., those with “concrete domains”)
- ⇒ **other standard inferences**  
(e.g., deciding consistency of ABoxes w.r.t. TBoxes)
- ⇒ **“non-standard” inferences such as**
  - matching and unification of concepts
  - rewriting concepts
  - least common subsumer (of a set of concepts)
  - most specific concept (of an ABox individual)