

Complexity of DLs

Complexity of DLs: Overview of the Complexity of Concept Consistency

P	(co-)NP	PSpace	ExpTime	NExpTime
		<i>ALCN</i> (wrt acyc. TBoxes)		

\mathcal{I} inverse roles: h-child⁻
 \mathcal{N} NRs: ($\geq n$ h-child)
 \mathcal{Q} Qual. NRs: ($\geq n$ h-child Blond)
 \mathcal{O} nominals: "John" is a concept
 \mathcal{F} feature chain (dis)agreement
 $\cdot R^+$ declare roles as transitive
 $\cdot \neg, \cap, \cup$ Boolean ops on roles

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<i>ACN</i> without \sqcup	<i>ACUN</i> (NP) without \exists , only $\neg A$ <i>ACE</i> (co-NP) without \sqcup and NRs, only $\neg A$	<i>ALCN</i> (wrt acyc. TBoxes)		

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P	(co-)NP	PSpace	ExpTime	NExpTime
<p>\mathcal{ALN} without \sqcup</p> <p>subsumption of \mathcal{FL}_0 \sqcap and \forall only</p>	<p>\mathcal{ALUN} (NP) without \exists, only $\neg A$</p> <p>\mathcal{ALCE} (co-NP) without \sqcup and NRs, only $\neg A$</p>	<p>\mathcal{ALCN} (wrt acyc. TBoxes)</p>		

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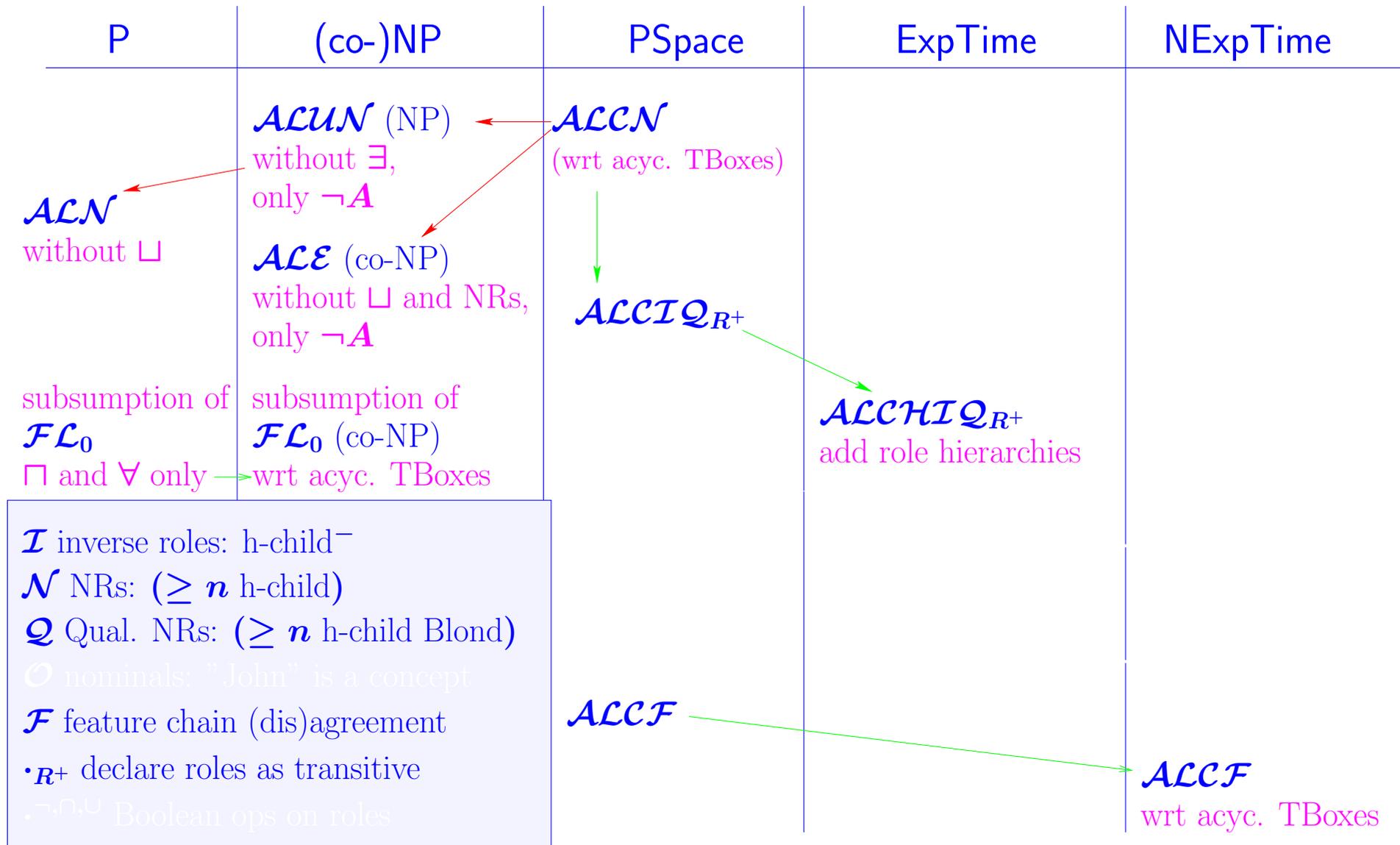
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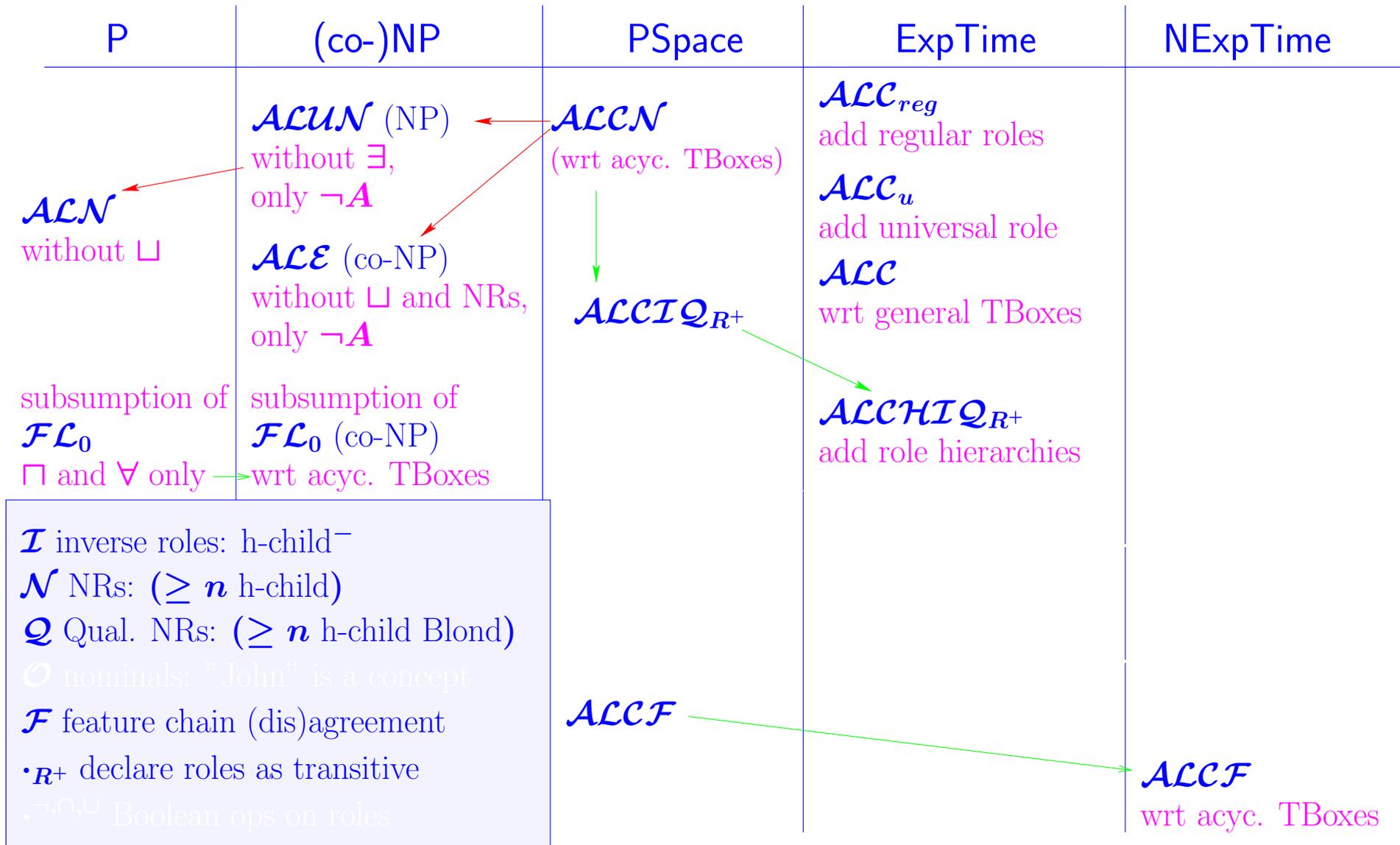
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P	(co-)NP	PSpace	ExpTime	NExpTime
<p>ACN without \sqcup</p> <p>subsumption of FL₀ \sqcap and \forall only</p>	<p>ACUN (NP) without \exists, only $\neg A$</p> <p>ACE (co-NP) without \sqcup and NRs, only $\neg A$</p> <p>subsumption of FL₀ (co-NP) wrt acyc. TBoxes</p>	<p>ALCN (wrt acyc. TBoxes)</p>		
<div style="border: 1px solid black; padding: 5px;"> <p>\mathcal{I} inverse roles: h-child⁻</p> <p>\mathcal{N} NRs: ($\geq n$ h-child)</p> <p>\mathcal{Q} Qual. NRs: ($\geq n$ h-child Blond)</p> <p>\mathcal{O} nominals: "John" is a concept</p> <p>\mathcal{F} feature chain (dis)agreement</p> <p>• R^+ declare roles as transitive</p> <p>• \neg, \cap, \cup Boolean ops on roles</p> </div>		<p>ALCF</p>		<p>ALCF wrt acyc. TBoxes</p>

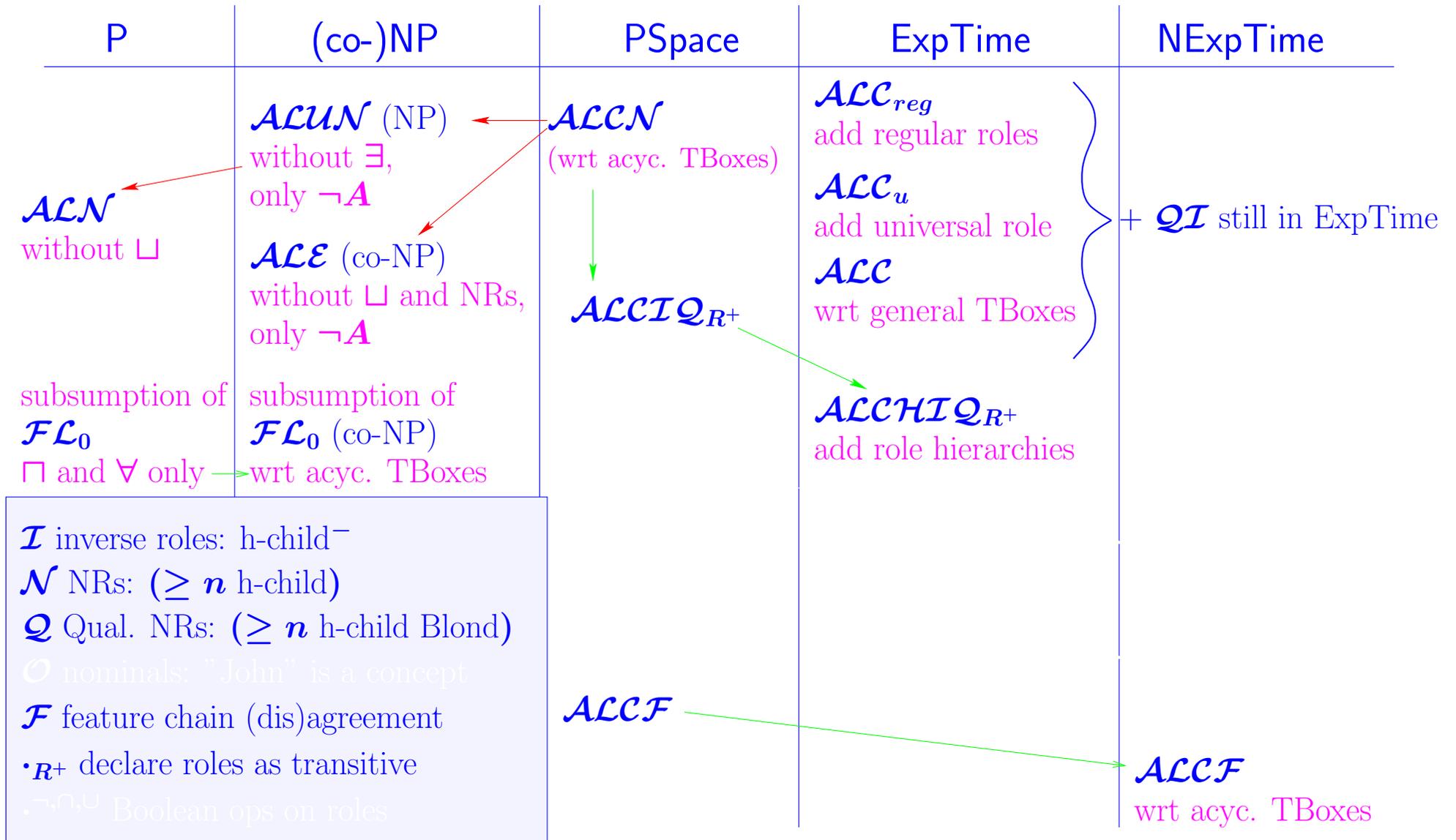
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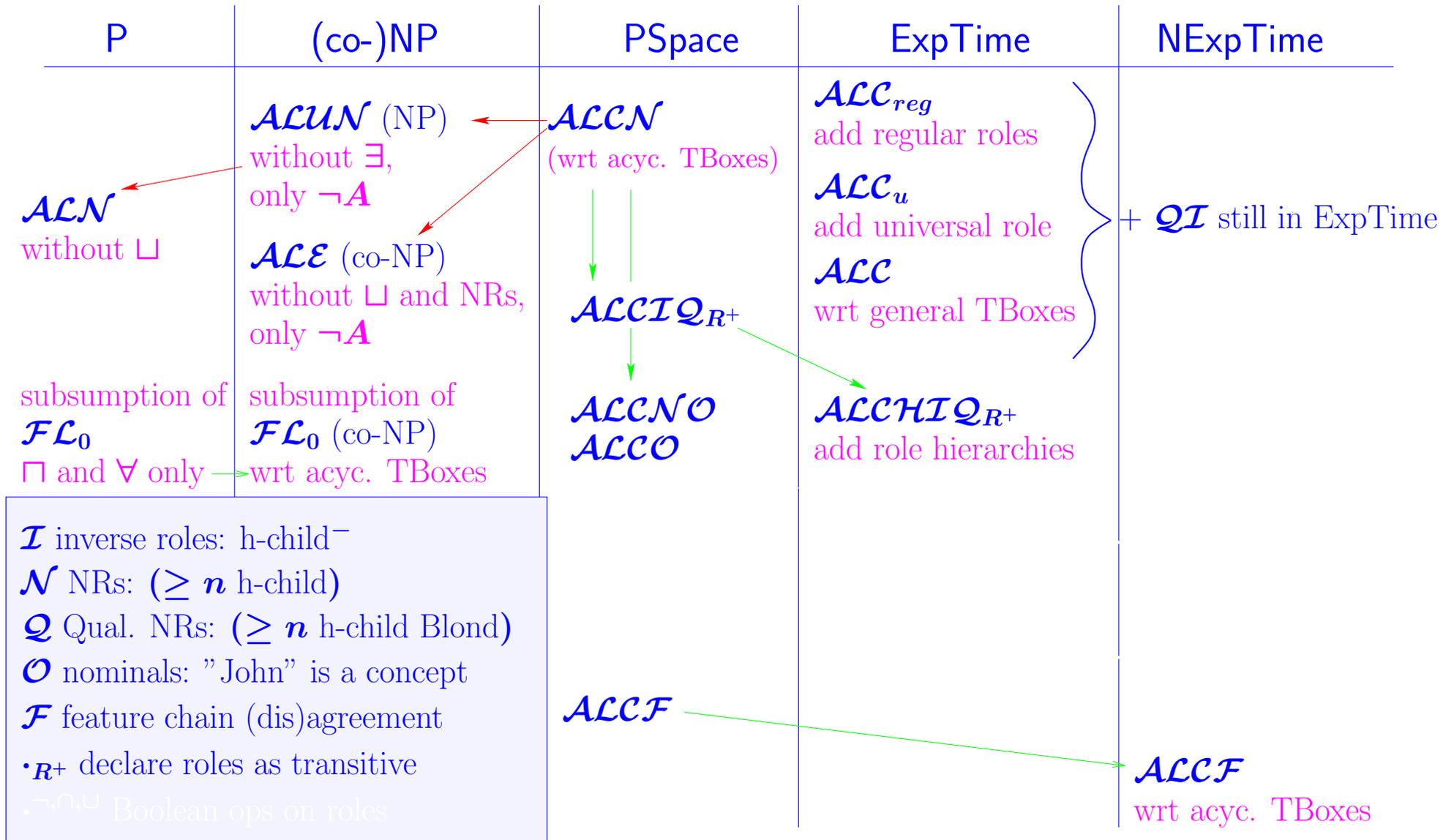
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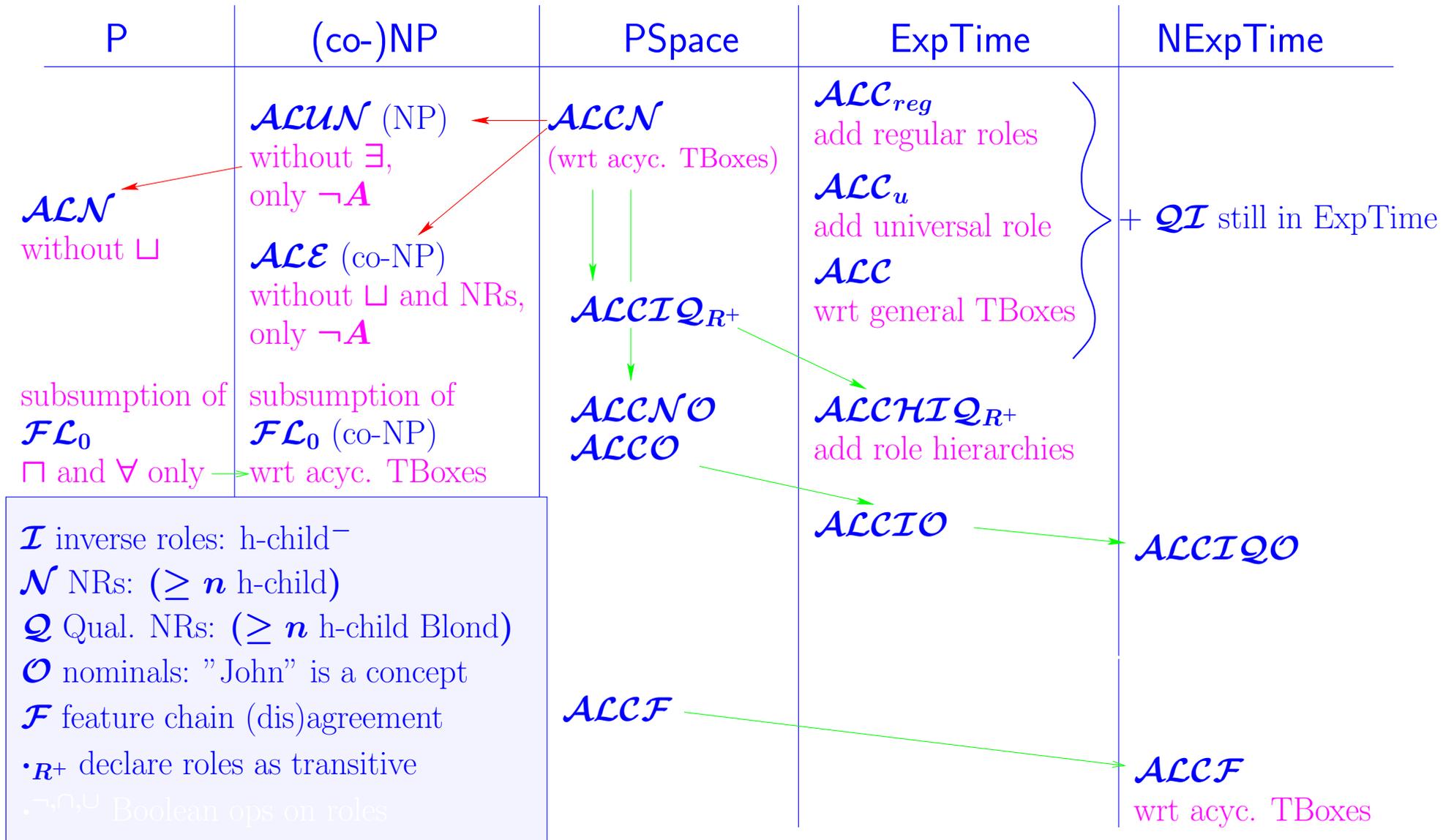
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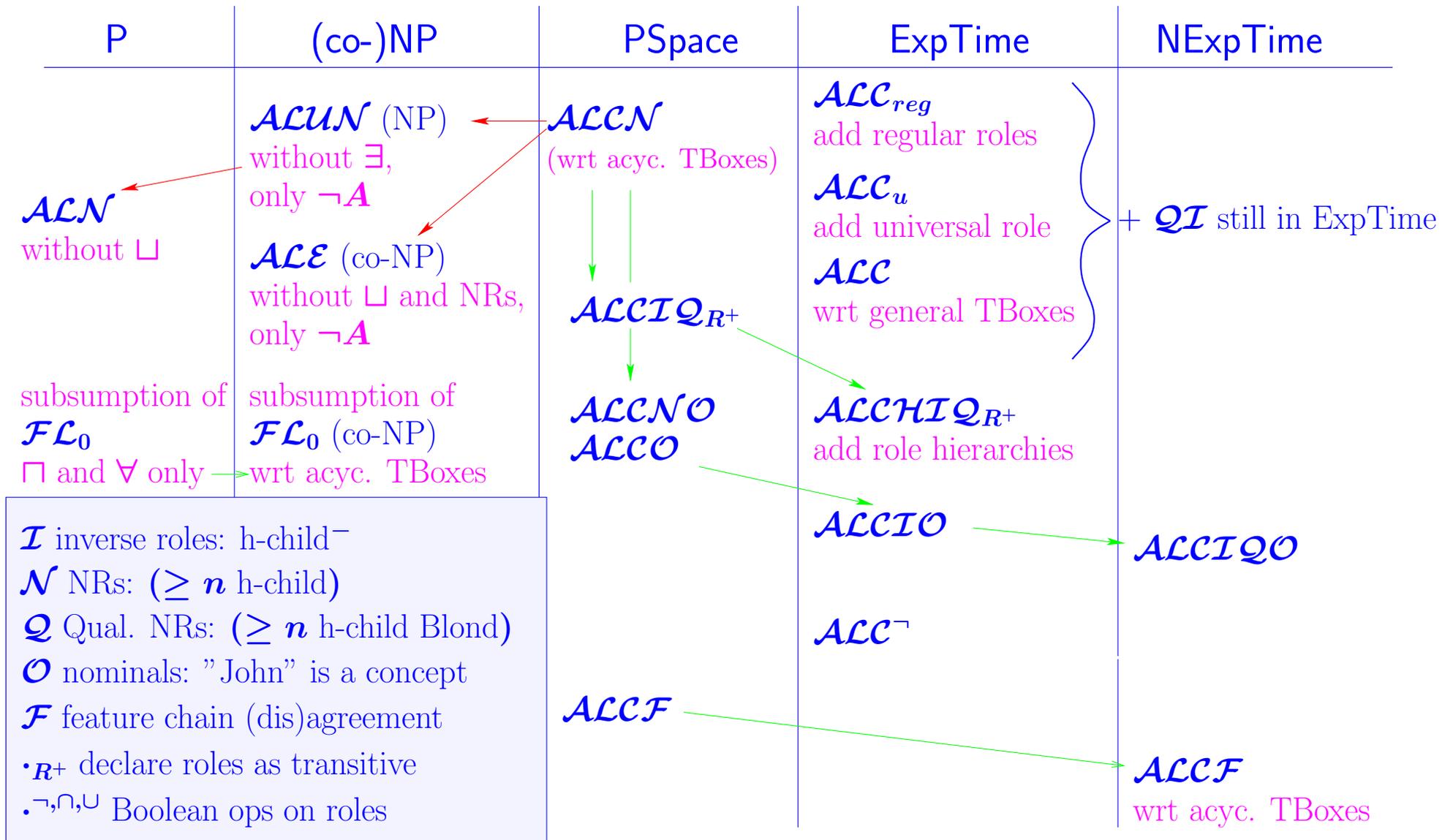
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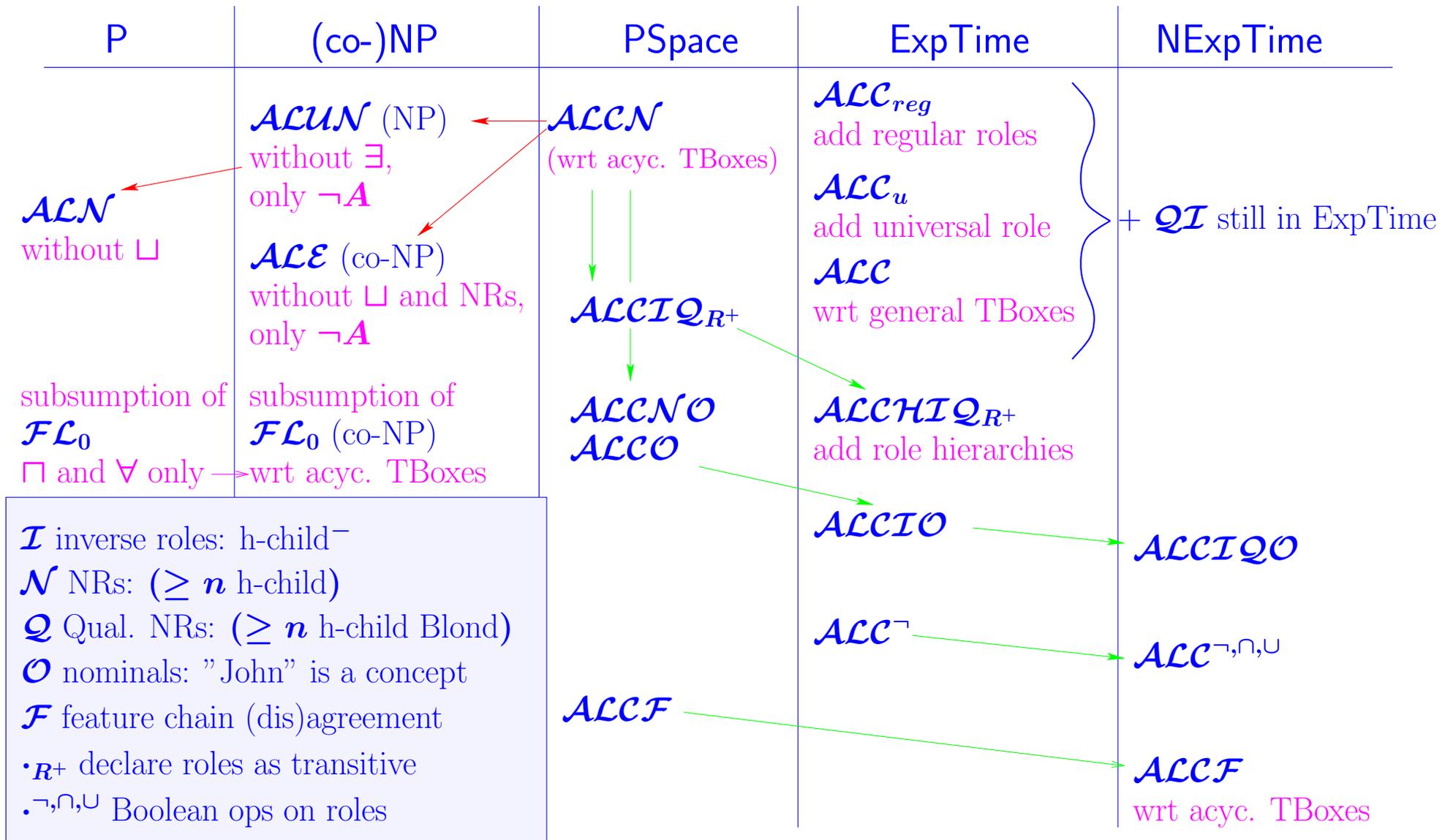
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We left out a variety of complexity results for

- ⇒ **concept consistency of other DLs**
(e.g., those with “concrete domains”)
- ⇒ **other standard inferences**
(e.g., deciding consistency of ABoxes w.r.t. TBoxes)
- ⇒ **“non-standard” inferences such as**
 - matching and unification of concepts
 - rewriting concepts
 - least common subsumer (of a set of concepts)
 - most specific concept (of an ABox individual)