Reasoning Procedures II
As already mentioned, for $\mathcal{ALC}$ with general axioms basic algorithm is non-terminating.
Non-Termination

As already mentioned, for $\mathcal{ALC}$ with \textbf{general axioms} basic algorithm is \textbf{non-terminating}

\textbf{E.g.} if human $\sqsubseteq \exists \text{has-mother}.\text{human} \in \mathcal{T}$, then  
$\neg \text{human} \sqcup \exists \text{has-mother}.\text{human}$ added to every node
Non-Termination

- As already mentioned, for ALC with \textbf{general axioms} basic algorithm is \textbf{non-terminating}

\textbf{E.g.} if human \sqsubseteq \exists \text{has-mother}.human \in \mathcal{T}, then
\neg \text{human} \sqcup \exists \text{has-mother}.human added to every node

\[ \mathcal{L}(w) = \{ \text{human} \} \]
Non-Termination

As already mentioned, for ALC with general axioms basic algorithm is **non-terminating**

**E.g.** if \( \text{human} \sqsubseteq \exists \text{has-mother}. \text{human} \in \mathcal{T} \), then
\[
\neg \text{human} \sqcup \exists \text{has-mother}. \text{human}
\]
added to every node

\[
\mathcal{L}(w) = \{ \text{human}, (\neg \text{human} \sqcup \exists \text{has-mother}. \text{human}) \}
\]
Non-Termination

- As already mentioned, for $\mathcal{ALC}$ with general axioms basic algorithm is non-terminating

- E.g. if $\text{human} \sqsubseteq \exists \text{has-mother}. \text{human} \in \mathcal{T}$, then $\neg \text{human} \sqcup \exists \text{has-mother}. \text{human}$ added to every node

$$w L(w) = \{ \text{human}, (\neg \text{human} \sqcup \exists \text{has-mother}. \text{human}), \exists \text{has-mother}. \text{human} \}$$
Non-Termination

- As already mentioned, for \( \mathcal{ALC} \) with \textbf{general axioms} basic algorithm is \textbf{non-terminating}

- \textbf{E.g.} if \( \text{human} \sqsubseteq \exists \text{has-mother}. \text{human} \in \mathcal{T} \), then
  \( \neg \text{human} \sqcup \exists \text{has-mother}. \text{human} \) added to every node

\[
L(w) = \{ \text{human}, (\neg \text{human} \sqcup \exists \text{has-mother}. \text{human}), \exists \text{has-mother}. \text{human} \}
\]

\[
L(x) = \{ \text{human} \}
\]
Non-Termination

As already mentioned, for \( ALC \) with \textbf{general axioms} basic algorithm is \textbf{non-terminating}

\textbf{E.g.} if \( \text{human} \sqsubseteq \exists \text{has-mother}. \text{human} \in \mathcal{T} \), then
\[ \lnot \text{human} \sqcup \exists \text{has-mother}. \text{human} \]
added to every node

\[ L(w) = \{ \text{human}, (\lnot \text{human} \sqcup \exists \text{has-mother}. \text{human}), \exists \text{has-mother}. \text{human} \} \]

\[ L(x) = \{ \text{human}, (\lnot \text{human} \sqcup \exists \text{has-mother}. \text{human}) \} \]
Non-Termination

☞ As already mentioned, for \textit{ALC} with \textbf{general axioms} basic algorithm is \textbf{non-terminating}

☞ \textbf{E.g.} if \textit{human} $\sqsubseteq \exists \text{has-mother} \cdot \text{human} \in \mathcal{T}$, then
\[ \neg \text{human} \sqcup \exists \text{has-mother} \cdot \text{human} \text{ added to every node} \]

\begin{align*}
\mathcal{L}(w) &= \{ \text{human}, (\neg \text{human} \sqcup \exists \text{has-mother} \cdot \text{human}), \exists \text{has-mother} \cdot \text{human} \} \\
\text{has-mother} \\
\mathcal{L}(x) &= \{ \text{human}, (\neg \text{human} \sqcup \exists \text{has-mother} \cdot \text{human}), \exists \text{has-mother} \cdot \text{human} \} 
\end{align*}
As already mentioned, for ALC with general axioms basic algorithm is non-terminating

E.g. if human ∈ T, then ~human ∪ ∃has-mother.human added to every node

\[
\begin{align*}
L(w) &= \{ \text{human}, (~\text{human} \cup \exists \text{has-mother}.\text{human}), \exists \text{has-mother}.\text{human} \} \\
\text{has-mother} \\
L(x) &= \{ \text{human}, (~\text{human} \cup \exists \text{has-mother}.\text{human}), \exists \text{has-mother}.\text{human} \} \\
\text{has-mother} \\
L(y) &= \{ \text{human}, (~\text{human} \cup \exists \text{has-mother}.\text{human}), \exists \text{has-mother}.\text{human} \}
\end{align*}
\]
Blocking

- When creating new node, check ancestors for equal (superset) label
Blocking

- When creating new node, check ancestors for equal (superset) label
- If such a node is found, new node is **blocked**
Blocking

- When creating a new node, check ancestors for equal (superset) label
- If such a node is found, the new node is **blocked**

\[
\begin{align*}
\mathcal{L}(w) &= \{\text{human}, (\neg \text{human} \sqcup \exists \text{has-mother}.\text{human}), \exists \text{has-mother}.\text{human}\} \\
\text{has-mother} \\
\mathcal{L}(x) &= \{\text{human}, (\neg \text{human} \sqcup \exists \text{has-mother}.\text{human})\}
\end{align*}
\]
Blocking

- When creating new node, check ancestors for equal (superset) label
- If such a node is found, new node is **blocked**

\[
\begin{align*}
\mathcal{L}(w) &= \{\text{human}, (\neg\text{human} \sqcup \exists\text{has-mother} \cdot \text{human}), \exists\text{has-mother} \cdot \text{human}\} \\
\text{has-mother} \\
\mathcal{L}(x) &= \{\text{human}, (\neg\text{human} \sqcup \exists\text{has-mother} \cdot \text{human})\}
\end{align*}
\]
Blocking

- When creating a new node, check ancestors for equal (superset) label
- If such a node is found, the new node is **blocked**

\[ L(w) = \{ \text{human}, (\neg \text{human} \sqcup \exists \text{has-mother}.\text{human}), \exists \text{has-mother}.\text{human} \} \]

block represents **cyclical** model
Simple subset blocking may not work with more complex logics.
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics

- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
Simple subset blocking may not work with more complex logics

E.g., reasoning with inverse roles
- Expanding node label can affect predecessor
- Label of blocking node can affect predecessor
- E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

\[ T = \{ \top \sqsubseteq \forall R^-. (\forall S^- . \neg C), \top \sqsubseteq \exists R.C \} \]
Simple subset blocking may not work with more complex logics

E.g., reasoning with inverse roles
- Expanding node label can affect predecessor
- Label of blocking node can affect predecessor
- E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

\[ T = \{ \top \sqsubseteq \forall R^-.(\forall S^-.\neg C), \top \sqsubseteq \exists R.C \} \]

\[ \mathcal{L}(w) = \{ C, \exists S.C \} \]
Simple subset blocking may not work with more complex logics

E.g., reasoning with inverse roles
- Expanding node label can affect predecessor
- Label of blocking node can affect predecessor
- E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \subseteq \forall R^\neg.(\forall S^\neg \neg C), \top \subseteq \exists R.C \}$$

$$\mathcal{L}(w) = \{ C, \exists S.C, \forall R^\neg.(\forall S^\neg \neg C), \exists R.C \}$$
Simple subset blocking may not work with more complex logics

E.g., reasoning with inverse roles
- Expanding node label can affect predecessor
- Label of blocking node can affect predecessor
- E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \subseteq \forall R^-.(\forall S^- \rightarrow C), \top \subseteq \exists R.C \}$$

$$L(w) = \{C, \exists S.C, \forall R^-.(\forall S^- \rightarrow C), \exists R.C \}$$

$$L(x) = \{C, \forall R^-.(\forall S^- \rightarrow C), \exists R.C \}$$
Simple subset blocking may not work with more complex logics

E.g., reasoning with inverse roles
- Expanding node label can affect predecessor
- Label of blocking node can affect predecessor
- E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \sqsubseteq \forall R^-. (\forall S^- \to C), \top \sqsubseteq \exists R.C \}$$

$$\mathcal{L}(w) = \{ C, \exists S.C, \forall R^- . (\forall S^- \to C), \exists R.C \}$$

$$\mathcal{L}(x) = \{ C, \forall R^- . (\forall S^- \to C), \exists R.C \}$$

Blocked
Simple subset blocking may not work with more complex logics

E.g., reasoning with inverse roles
- Expanding node label can affect predecessor
- Label of blocking node can affect predecessor
- E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

$T = \{ \top \subseteq \forall R^-. (\forall S^- \rightarrow \neg C), \top \subseteq \exists R.C \}$

$L(w) = \{C, \exists S.C, \forall R^-.(\forall S^- \rightarrow \neg C), \exists R.C\}$

$L(y) = \{C, \forall R^-.(\forall S^- \rightarrow \neg C), \exists R.C\}$

$L(x) = \{C, \forall R^-.(\forall S^- \rightarrow \neg C), \exists R.C\}$

Blocked
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

\[ T = \{ \top \sqsubseteq \forall R^-.(\forall S^-.\neg C'), \top \sqsubseteq \exists R.C \} \]

$L(w) = \{ C, \exists S.C, \forall R^-.(\forall S^-\neg C'), \exists R.C \}$

$L(y) = \{ C, \forall R^-.(\forall S^-\neg C'), \exists R.C \}$

$L(x) = \{ C, \forall R^-.(\forall S^-\neg C'), \exists R.C \}$

Blocked
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \sqcap \exists S. C$ w.r.t. Tbox

\[ T = \{ \top \sqsubseteq \forall R^- . (\forall S^- . \neg C'), \top \sqsubseteq \exists R. C \} \]

\[ L(w) = \{ C, \exists S. C, \forall R^- . (\forall S^- . \neg C'), \exists R. C \} \]

**cyclical** model?
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

$$\mathcal{T} = \{ \top \sqsubseteq \forall R^-.(\forall S^-.\neg C'), \top \sqsubseteq \exists R.C \}$$

\[ \mathcal{L}(w) = \{ C, \exists S.C, \forall R^-.(\forall S^-.\neg C'), \exists R.C \} \]

cyclical model?
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \sqsubseteq \forall R^-(\forall S^- . \neg C), \top \sqsubseteq \exists R.C \}$$

$\mathcal{L}(w) = \{ C, \exists S.C, \forall R^-(\forall S^- . \neg C'), \exists R.C, \forall S^- . \neg C \}$

**cyclical** model?
Blocking with More Expressive DLs

- Simple subset blocking may not work with more complex logics
- E.g., reasoning with inverse roles
  - Expanding node label can affect predecessor
  - Label of blocking node can affect predecessor
  - E.g., testing $C \sqcap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \sqsubseteq \forall R^-. (\forall S^- \neg C), \top \sqsubseteq \exists R.C \}$$

$\mathcal{L}(w) = \{ C, \exists S.C, \forall R^- . (\forall S^- \neg C'), \exists R.C, \forall S^- . \neg C \}$

cyclical model?
Simple subset blocking may not work with more complex logics. E.g., reasoning with inverse roles:
- Expanding node label can affect predecessor
- Label of blocking node can affect predecessor
- E.g., testing $C \cap \exists S.C$ w.r.t. Tbox

$$T = \{ \top \sqsubseteq \forall R^-. (\forall S^- . \neg C), \top \sqsubseteq \exists R.C \}$$

$$\mathcal{L}(w) = \{ C, \exists S.C, \forall R^- . (\forall S^- . \neg C), \exists R.C, \forall S^- . \neg C, \neg C \}$$

Cyclical model?

Clash
Solution (for inverse roles) is **dynamic blocking**
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**
- Blocks can be established broken and re-established
Dynamic Blocking

- Solution (for inverse roles) is **dynamic blocking**
  - Blocks can be established broken and re-established
  - Continue to expand $\forall R.C$ terms in blocked nodes
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts
Dynamic Blocking

Solution (for inverse roles) is \textit{dynamic blocking}

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

$\mathcal{L}(w) = \{C, \exists S.C\}$
Dynamic Blocking

- Solution (for inverse roles) is **dynamic blocking**
  - Blocks can be established broken and re-established
  - Continue to expand $\forall R.C$ terms in blocked nodes
  - Check that cycles satisfy $\forall R.C$ concepts

$$L(w) = \{C, \exists S.C, \forall R^-(\forall S^{-}.\neg C'), \exists R.C\}$$
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**
- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[
\mathcal{L}(w) = \{C, \exists S.C, \forall R^-(\forall S^- \neg C), \exists R.C\}
\]

\[
\mathcal{L}(x) = \{C, \forall R^-(\forall S^- \neg C), \exists R.C\}
\]
Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established, broken, and re-established.
- Continue to expand $\forall R.C$ terms in blocked nodes.
- Check that cycles satisfy $\forall R.C$ concepts.

\[
\mathcal{L}(w) = \{ C, \exists S.C, \forall R^{-} \cdot (\forall S^{-} \cdot S.C), \exists R.C \}
\]

\[
\mathcal{L}(x) = \{ C, \forall R^{-} \cdot (\forall S^{-} \cdot \neg C), \exists R.C \}
\]
Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand \( \forall R.C \) terms in blocked nodes
- Check that cycles satisfy \( \forall R.C \) concepts

\[
\mathcal{L}(w) = \{C, \exists S.C, \forall R^-.(\forall S^- .\neg C), \exists R.C\}
\]

\[
\mathcal{L}(y) = \{C, \forall R^-.(\forall S^- .\neg C), \exists R.C\}
\]

\[
\mathcal{L}(x) = \{C, \forall R^-.(\forall S^- .\neg C), \exists R.C\}
\]
Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand \( \forall R.C \) terms in blocked nodes
- Check that cycles satisfy \( \forall R.C \) concepts

\[
\mathcal{L}(w) = \{ C, \exists S.C, \forall R^-.(\forall S^-.\neg C), \exists R.C, \forall S^- . \neg C \}
\]

\[
\mathcal{L}(x) = \{ C, \forall R^-.(\forall S^- . \neg C), \exists R.C \}
\]
Solution (for inverse roles) is **dynamic blocking**
- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[ \mathcal{L}(w) = \{ C, \exists S.C, \forall R^-. (\forall S^- . \neg C), \exists R.C, \forall S^- . \neg C \} \]

\[ \mathcal{L}(y) = \{ C, \forall R^-. (\forall S^- . \neg C), \exists R.C \} \]

\[ \mathcal{L}(x) = \{ C, \forall R^-. (\forall S^- . \neg C), \exists R.C \} \]
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**
- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[
\mathcal{L}(w) = \{ C, \exists S.C, \forall R^{-}.(\forall S^{-}.\neg C), \\
\quad \exists R.C, \forall S^{-}.\neg C \} \\
\mathcal{L}(y) = \{ C, \forall R^{-}.(\forall S^{-}.\neg C), \\
\quad \exists R.C \} \\
\mathcal{L}(x) = \{ C, \forall R^{-}.(\forall S^{-}.\neg C), \\
\quad \exists R.C \} \\
\mathcal{L}(z) = \{ C, \forall R^{-}.(\forall S^{-}.\neg C), \\
\quad \exists R.C \} 
\]
Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[
L(w) = \{ C, \exists S.C, \forall R^-.(\forall S^- . \neg C), \\
\exists R.C, \forall S^- . \neg C \}
\]

\[
L(y) = \{ C, \forall R^-.(\forall S^- . \neg C), \\
\exists R.C \}
\]

\[
L(x) = \{ C, \forall R^-.(\forall S^- . \neg C), \\
\exists R.C, \forall S^- . \neg C \}
\]

\[
L(z) = \{ C, \forall R^-.(\forall S^- . \neg C), \\
\exists R.C \}
\]
Dynamic Blocking

Solution (for inverse roles) is **dynamic blocking**

- Blocks can be established broken and re-established
- Continue to expand $\forall R.C$ terms in blocked nodes
- Check that cycles satisfy $\forall R.C$ concepts

\[
\mathcal{L}(y) = \{C, \forall R^-.(\forall S^-.\neg C), \exists R.C\}
\]

\[
\mathcal{L}(x) = \{C, \forall R^-.(\forall S^-.\neg C), \exists R.C, \forall S^- . \neg C\}
\]

\[
\mathcal{L}(z) = \{C, \forall R^-.(\forall S^-.\neg C), \exists R.C\}
\]

\[
\mathcal{L}(w) = \{C, \exists S.C, \forall R^-.(\forall S^- . \neg C), \exists R.C, \forall S^- . \neg C, \neg C\}
\]
Non-finite Models

With number restrictions some satisfiable concepts have only non-finite models
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing \( \neg C \) w.r.t. \( T = \{ \top \sqsubseteq \exists R.C, \top \sqsubseteq \leq 1R^- \} \)
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{ \top \sqsubseteq \exists R.C, \top \sqsubseteq \leq 1R^- \}$

\[ \omega \mathcal{L}(w) = \{ \neg C \} \]
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing \( \neg C \) w.r.t. \( T = \{ \top \subseteq \exists R.C, \top \subseteq \leq 1R^- \} \)

\[
\bigwedge_w L(w) = \{ \neg C, \exists R.C, \leq 1R^- \}
\]
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models.
- E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{\top \subseteq \exists R.C, \top \subseteq \leq 1R\}$

\[
\begin{align*}
\mathcal{L}(w) &= \{\neg C, \exists R.C, \leq 1R\} \\
R \\
\mathcal{L}(x) &= \{C\}
\end{align*}
\]
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models

- E.g., testing \( \neg C \) w.r.t. \( T = \{ \top \subseteq \exists R.C, \top \subseteq \leq 1R^- \} \)

\[
\begin{align*}
    w & : \mathcal{L}(w) = \{ \neg C, \exists R.C, \leq 1R^- \} \\
    x & : \mathcal{L}(x) = \{ C, \exists R.C, \leq 1R^- \}
\end{align*}
\]
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R. C, \top \subseteq \leq 1R^- \}$

\[
\begin{align*}
\mathcal{L}(w) &= \{ \neg C, \exists R. C, \leq 1R^- \} \\
\mathcal{L}(x) &= \{ C, \exists R. C, \leq 1R^- \} \\
\mathcal{L}(y) &= \{ C, \exists R. C, \leq 1R^- \}
\end{align*}
\]
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R.C, \top \subseteq \leq 1R^- \}$

\[ w \\quad \mathcal{L}(w) = \{ \neg C, \exists R.C, \leq 1R^- \} \]

\[ x \quad \mathcal{L}(x) = \{ C, \exists R.C, \leq 1R^- \} \]

\[ R \]

\[ y \quad \mathcal{L}(y) = \{ C, \exists R.C, \leq 1R^- \} \]

\[ \text{Blocked} \]
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{ \top \sqsubseteq \exists R.C, \top \sqsubseteq \leq 1R^- \}$

$$\mathcal{L}(w) = \{ \neg C, \exists R.C, \leq 1R^- \}$$

$$\mathcal{L}(x) = \{ C, \exists R.C, \leq 1R^- \}$$

$$\mathcal{L}(y) = \{ C, \exists R.C, \leq 1R^- \}$$

Cyclical model?
With number restrictions some satisfiable concepts have only non-finite models.

E.g., testing $\neg C$ w.r.t. $T = \{ \top \sqsubseteq \exists R.C, \top \sqsubseteq \leq 1R^- \}$

\[
\ell(w) = \{ \neg C, \exists R.C, \leq 1R^- \}
\]

\[
\ell(x) = \{ C, \exists R.C, \leq 1R^- \}
\]

Cyclical model?
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $T = \{\top \subseteq \exists R.C, \top \subseteq R \leq 1R^-\}$

\[
\begin{align*}
L(w) &= \{\neg C, \exists R.C, \leq 1R^-\} \\
L(x) &= \{C, \exists R.C, \leq 1R^-\}
\end{align*}
\]

\[ R^- \]

\[ \Rightarrow w = x \]

Cyclical model?
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $T = \{ \top \sqsubseteq \exists R.C, \top \sqsubseteq \leq 1R^- \}$

\[ \mathcal{L}(w) = \{ \neg C, \exists R.C, \leq 1R^- , C \} \]

Cyclical model?
Non-finite Models

- With number restrictions some satisfiable concepts have only non-finite models
- E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{\top \sqsubseteq \exists R.C, \top \sqsubseteq \leq 1R^-\}$

```
$\mathcal{L}(w) = \{-C, \exists R.C, \leq 1R^-\}$
```

```
$\mathcal{L}(x) = \{C, \exists R.C, \leq 1R^-\}$
```

```
$\mathcal{L}(y) = \{C, \exists R.C, \leq 1R^-\}$
```

model must be non-finite
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{ \top \sqsubseteq \exists R.(C \cap \exists R^-. \neg C), \top \sqsubseteq \leq 1R^- \}$
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing $\neg C$ w.r.t. $T = \{\top \subseteq \exists R. (C \cap \exists R^+. \neg C), \top \subseteq \leq 1 R^+\}$

$\forall w \quad L(w) = \{\neg C\}$
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R.(C \sqcap \exists R^{-}.\neg C'), \top \subseteq \leq 1R^{-} \}$

\[ w \quad \mathcal{L}(w) = \{ \neg C, \exists R.(C \sqcap \exists R^{-}.\neg C'), \leq 1R^{-} \} \]
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing $\neg C$ w.r.t. $T = \{ \top \subseteq \exists R.(C \sqcap \exists R^- \neg C), \top \subseteq \leq 1R^- \}$

\[
\begin{align*}
L(w) &= \{ \neg C, \exists R.(C \sqcap \exists R^- \neg C), \leq 1R^- \} \\
L(x) &= \{ (C \sqcap \exists R^- \neg C) \}
\end{align*}
\]
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{ \top \sqsubseteq \exists R. (C \sqcap \exists R^- . \neg C), \top \sqsubseteq \leq 1 R^- \} $

\[
\begin{align*}
\mathcal{L}(w) &= \{ \neg C, \exists R. (C \sqcap \exists R^- . \neg C), \leq 1 R^- \} \\
\forall R \\
\mathcal{L}(x) &= \{ (C \sqcap \exists R^- . \neg C), \exists R. (C \sqcap \exists R^- . \neg C), \leq 1 R^-, C, \exists R^- . \neg C \}
\end{align*}
\]
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing $\neg C$ w.r.t. $\mathcal{T} = \{\top \subseteq \exists R. (C \cap \exists R^-. \neg C'), \top \subseteq \leq 1R^-\}$

\[
\begin{align*}
\mathcal{L}(w) &= \{-C, \exists R. (C \cap \exists R^-. \neg C'), \leq 1R^-\} \\
\mathcal{L}(x) &= \{(C \cap \exists R^-. \neg C'), \exists R. (C \cap \exists R^-. \neg C'), \leq 1R^-, C, \exists R^- . \neg C\} \\
\mathcal{L}(y) &= \{(C \cap \exists R^-. \neg C'), \exists R. (C \cap \exists R^-. \neg C'), \leq 1R^-, C, \exists R^- . \neg C\}
\end{align*}
\]
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing $\neg C$ w.r.t. $T = \{\top \subseteq \exists R.(C \cap \exists R^-.\neg C), \top \subseteq \leq 1R^\bot\}$

\[
\begin{align*}
\mathcal{L}(w) &= \{\neg C, \exists R.(C \cap \exists R^-.\neg C), \leq 1R^\bot\} \\
\mathcal{L}(x) &= \{(C \cap \exists R^-.\neg C), \exists R.(C \cap \exists R^-.\neg C), \leq 1R^\bot, C, \exists R^-.\neg C\} \\
\mathcal{L}(y) &= \{(C \cap \exists R^-.\neg C), \exists R.(C \cap \exists R^-.\neg C), \leq 1R^\bot, C, \exists R^-.\neg C\}
\end{align*}
\]
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing \( \neg C \) w.r.t. \( T = \{ \top \subseteq \exists R. (C \sqcap \exists R^-. \neg C), \top \subseteq \leq 1R^- \} \)

\[
\begin{align*}
\mathcal{L}(w) &= \{ \neg C, \exists R. (C \sqcap \exists R^-. \neg C), \leq 1R^- \} \\
\mathcal{L}(x) &= \{ (C \sqcap \exists R^-. \neg C), \exists R. (C \sqcap \exists R^-. \neg C), \leq 1R^-, C, \exists R^- . \neg C \} \\
\mathcal{L}(y) &= \{ (C \sqcap \exists R^-. \neg C), \exists R. (C \sqcap \exists R^-. \neg C), \leq 1R^-, C, \exists R^- . \neg C \} \\
\text{But } &\exists R^- . \neg C \in \mathcal{L}(y) \text{ not satisfied}
\end{align*}
\]
Inadequacy of Dynamic Blocking

- With non-finite models, even dynamic blocking not enough
- E.g., testing \( \neg C \) w.r.t. \( T = \{ \top \subseteq \exists R.(C \land \exists R^- \neg C), \top \subseteq \leq 1R^- \} \)

This diagram illustrates the situation with the following statements:

- \( w \) : \( \mathcal{L}(w) = \{ \neg C, \exists R.(C \land \exists R^- \neg C), \leq 1R^- \} \)
- \( x \) : \( \mathcal{L}(x) = \{(C \land \exists R^- \neg C), \exists R.(C \land \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C\} \)
- \( y \) : \( \mathcal{L}(y) = \{(C \land \exists R^- \neg C), \exists R.(C \land \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C\} \)

**But** \( \exists R^- \neg C \in \mathcal{L}(y) \) **not satisfied**

**Inconsistency due to** \( \leq 1R^- \in \mathcal{L}(y) \) **and** \( C \in \mathcal{L}(x) \)
Double Blocking I

Problem due to $\exists R^- . \neg C$ term only satisfied in predecessor of blocking node

\[ \mathcal{L}(w) = \{ \neg C, \exists R. (C \cap \exists R^- \neg C), \leq 1R^- \} \]

\[ \mathcal{L}(x) = \{(C \cap \exists R^- \neg C), \exists R. (C \cap \exists R^- \neg C), \leq 1R^-, C, \exists R^- \neg C\} \]
Double Blocking I

Problem due to $\exists R^- . \neg C$ term only satisfied in predecessor of blocking node

\[ \mathcal{L}(w) = \{ \neg C, \exists R . (C \cap \exists R^- . \neg C), \leq 1R^- \} \]

\[ \mathcal{L}(x) = \{(C \cap \exists R^- . \neg C), \exists R . (C \cap \exists R^- . \neg C), \leq 1R^- , C, \exists R^- . \neg C \} \]

Solution is **Double Blocking** (pairwise blocking)
Problem due to $\exists R^- . \neg C$ term only satisfied in predecessor of blocking node

Solution is Double Blocking (pairwise blocking)
- Predecessors of blocked and blocking nodes also considered
Double Blocking I

Problem due to $\exists R^-. \neg C$ term only satisfied in predecessor of blocking node

Solution is **Double Blocking** (pairwise blocking)

- Predecessors of blocked and blocking nodes also considered
- In particular, $\exists R.C$ terms satisfied in predecessor of blocking node must also be satisfied in predecessor of blocked node $\neg C \in \mathcal{L}(w)$
Due to pairwise condition, block no longer holds

\[
\begin{align*}
\mathcal{L}(w) & = \{ \neg C, \exists R. (C \sqcap \exists R^- \neg C), \leq 1 R^- \} \\
\mathcal{L}(x) & = \{(C \sqcap \exists R^- \neg C), \exists R. (C \sqcap \exists R^- \neg C), \leq 1 R^-, C, \exists R^- \neg C\} \\
\mathcal{L}(y) & = \{(C \sqcap \exists R^- \neg C), \exists R. (C \sqcap \exists R^- \neg C), \leq 1 R^-, C, \exists R^- \neg C\}
\end{align*}
\]
Due to pairwise condition, block no longer holds
Expansion continues and contradiction discovered

\[ \mathcal{L}(w) = \{ \neg C, \exists R. (C \sqcap \exists R^- . \neg C), \leq 1R^- \} \]

\[ \mathcal{L}(x) = \{ (C \sqcap \exists R^- . \neg C), \exists R. (C \sqcap \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C \} \]

\[ \mathcal{L}(y) = \{ (C \sqcap \exists R^- . \neg C), \exists R. (C \sqcap \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C \} \]
Double Blocking II

- Due to pairwise condition, block no longer holds
- Expansion continues and contradiction discovered

\[ \mathcal{L}(w) = \{ \neg C, \exists R. (C \sqcap \exists R^- . \neg C), \leq 1R^- \} \]

\[ \mathcal{L}(x) = \{ (C \sqcap \exists R^- . \neg C), \exists R. (C \sqcap \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C \} \]

\[ \mathcal{L}(y) = \{ (C \sqcap \exists R^- . \neg C), \exists R. (C \sqcap \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C \} \]

\[ \mathcal{L}(z) = \{ \neg C \} \]
Double Blocking II

- Due to pairwise condition, block no longer holds
- Expansion continues and contradiction discovered

\[ \mathcal{L}(w) = \{ \neg C, \exists R. (C \sqcap \exists R^{-}. \neg C), \leq 1R^{-} \} \]

\[ R \]

\[ \mathcal{L}(x) = \{(C \sqcap \exists R^{-}. \neg C), \exists R. (C \sqcap \exists R^{-}. \neg C), \leq 1R^{-}, C, \exists R^{-}. \neg C \} \]

\[ R^{-} \]

\[ \mathcal{L}(y) = \{(C \sqcap \exists R^{-}. \neg C), \exists R. (C \sqcap \exists R^{-}. \neg C), \leq 1R^{-}, C, \exists R^{-}. \neg C \} \]

\[ R^{-} \]

\[ \mathcal{L}(z) = \{ \neg C \} \]

\[ \Rightarrow z = x \]
Double Blocking II

- Due to pairwise condition, block no longer holds
- Expansion continues and contradiction discovered

\[
\begin{align*}
\mathcal{L}(w) &= \{ \neg C, \exists R. (C \sqcap \exists R^-. \neg C), \leq 1R^- \} \\
\mathcal{L}(x) &= \{(C \sqcap \exists R^- . \neg C), \exists R. (C \sqcap \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C, \neg C\} \\
\mathcal{L}(y) &= \{(C \sqcap \exists R^- . \neg C), \exists R. (C \sqcap \exists R^- . \neg C), \leq 1R^-, C, \exists R^- . \neg C\}
\end{align*}
\]

Clash