‘Good’ and ‘Bad’ Diversity in Classifier Ensembles

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Overview

- The perspective...
- Good Diversity, Bad Diversity
- Toy data & Results
- Relation to Patterns of Success/Failure
- Questions

Perspective: Diversity should be natural

Diversity should be a **natural consequence** of:

1. The loss function
2. The combiner function

and nothing more.

Perspective: Diversity should be natural

Loss function: **quadratic**
Combiner: **linear**

$h \in \mathcal{R}$

$\bar{h} = \frac{1}{T} \sum_t h_t$

$$(\bar{h} - y)^2 = \frac{1}{T} \sum_t (h_t - y)^2 - \underbrace{\frac{1}{T} \sum_t (h_t - \bar{h})^2}_{diversity} .$$

Perspective: Diversity should be natural

Loss function: **KL-divergence**
Combiner: **product rule**

$h \in \mathcal{R}$

$\bar{h} = Z^{-1} \prod_t h_t^{\frac{1}{T}}$

$$D_{KL}(y||\bar{h}) = \frac{1}{T}\sum_t D_{KL}(y||h_t) - \underbrace{\frac{1}{T}\sum_t D_{KL}(\bar{h}||h_t)}_{diversity}$$

Perspective: Diversity should be natural

Loss function: **0/1 loss**
Combiner: **majority vote**

$h_t \in \{-1, +1\}$

$\bar{h} = sign\left(\frac{1}{T}\sum_t h_t\right)$

$$\delta(\bar{h}, t) = \quad ..... \quad + \quad .....$$

Decomposing Majority Vote Error

$y, h_t \in \{-1, +1\}$

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— Boring proof goes here —

Cut to the Chase

$0/1$ loss can be re-written:

$$\delta(H, y) = \frac{1}{T} \sum_t \delta(h_t, y) - yH \frac{1}{T} \sum_t \delta(h_t, H)$$

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Integrate over $p(\mathbf{x})$

$$\int_{\mathbf{x}} \delta(H, y) = \int_{\mathbf{x}} \frac{1}{T} \sum_t \delta(h_t, y) \quad - \quad \int_{\mathbf{x}(+)} \frac{1}{T} \sum_t \delta(h_t, H)$$
$$+ \quad \int_{\mathbf{x}(-)} \frac{1}{T} \sum_t \delta(h_t, H)$$

$\mathbf{x}(+)$ ... data subspace where ensemble is correct.
$\mathbf{x}(-)$ ... data subspace where ensemble is wrong.

Good Diversity, Bad Diversity

$$\ldots\ldots \underbrace{- \int_{\mathbf{x}(+)} \frac{1}{T} \sum_t \delta(h_t, H)}_{\text{'good' diversity}} \quad \underbrace{+ \int_{\mathbf{x}(-)} \frac{1}{T} \sum_t \delta(h_t, H)}_{\text{'bad' diversity}}$$

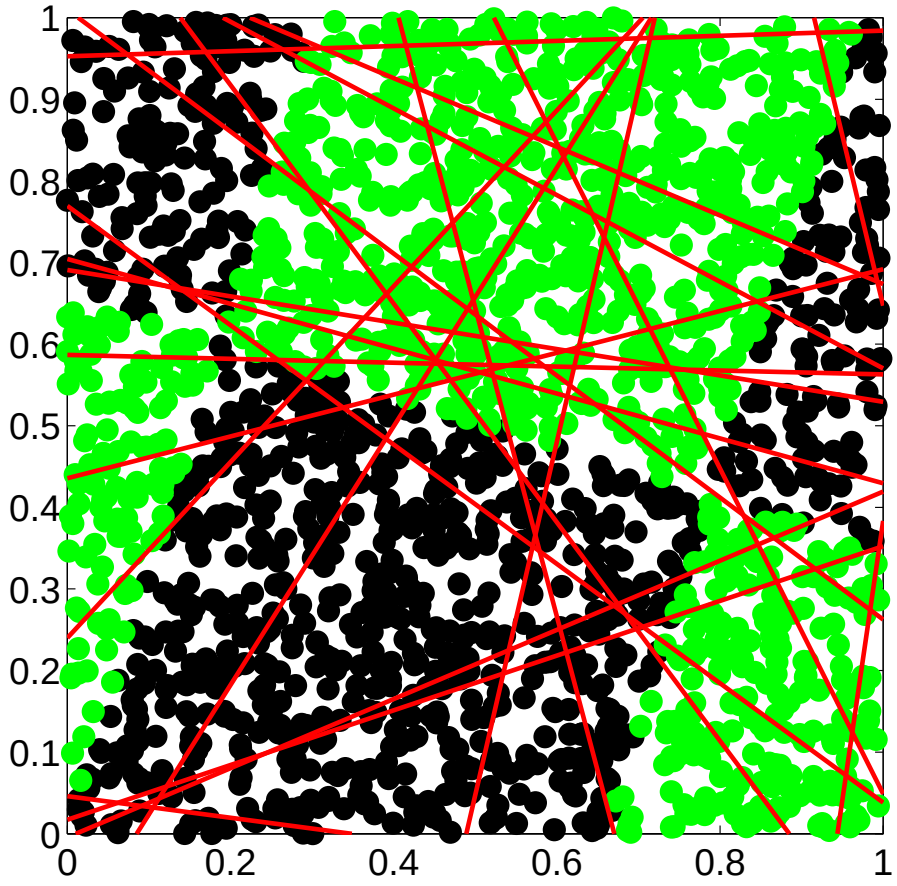
Good diversity subtracts error...

- any disagreement on these $\mathbf{x}$ *increases gain of the ensemble relative to the average individual error.*

Bad diversity adds error...

- any disagreement on these $\mathbf{x}$ *reduces gain of the ensemble relative to the average individual error.*

A Toy Dataset



Experimental Protocol

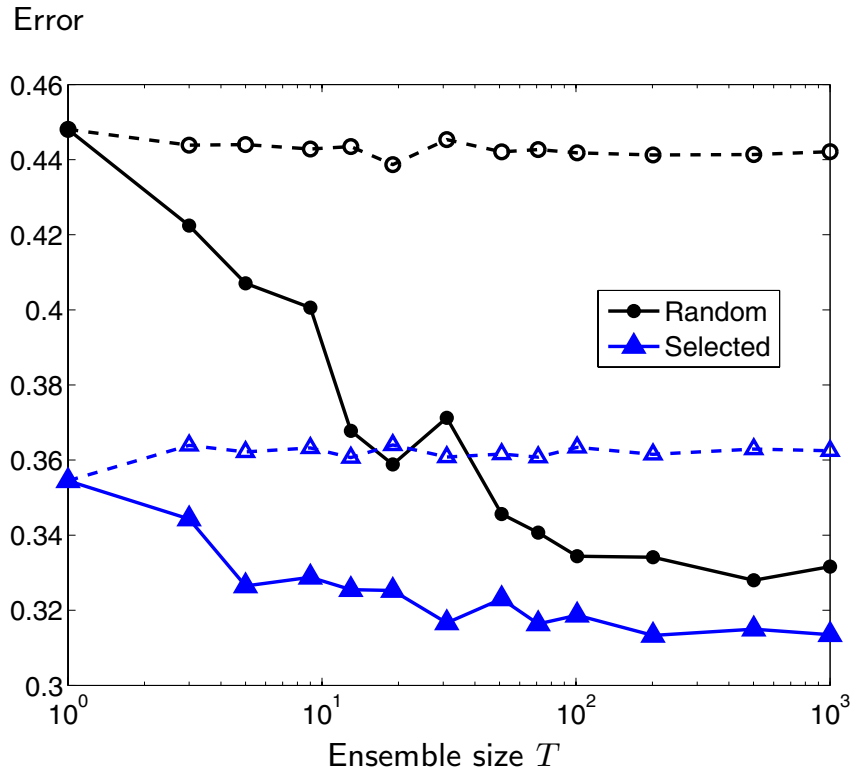
1. Generate pool Φ : 2000 random linear classifiers.
2. The ensemble size T was varied as

$$T \in \{1, 3, 5, 9, 13, 19, 31, 51, 71, 101, 201, 501, 1001\}.$$

3. Generate/test 50 ensembles for each value of T :
   - 3.1 Sample T classifiers without replacement from Φ.
   - 3.2 Generate test set of 1000 2-d points.
   - 3.3 Statistics on ensemble and individual performance stored.
4. The stored values were averaged across the 50 runs.

Results

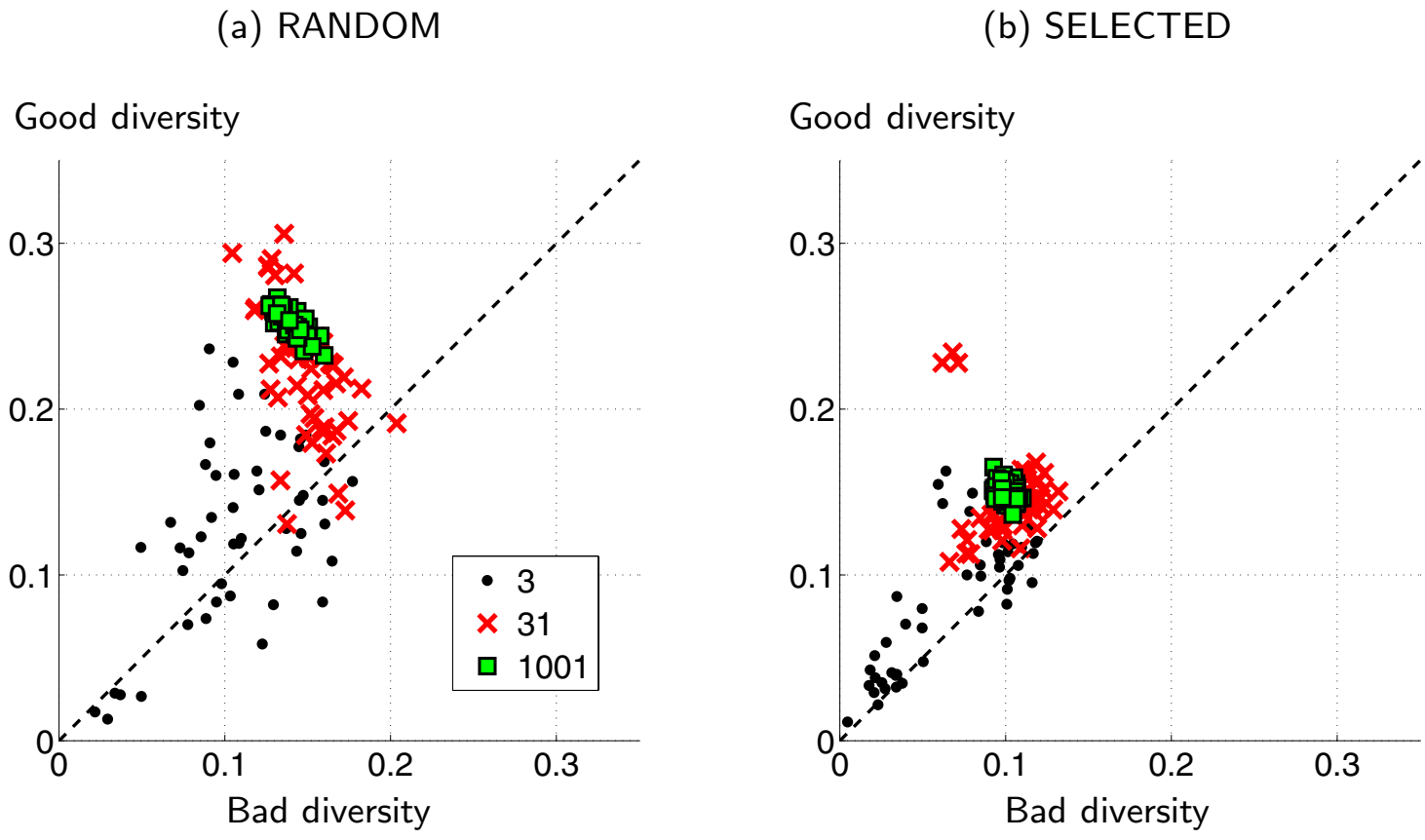
Majority vote error (solid lines). Average individual error (dashed).



Random : 2000 random classifiers.

Selected : 2000 best classifiers from 8000 random ones.

Plotting Good vs Bad



Random classifiers (left)

Classifiers selected for good accuracy (right)

Patterns of Success/Failure

Assume T classifiers each with accuracy p.

L. Kuncheva, C. Whitaker, C. Shipp, R. Duin: *Limits on the majority vote accuracy in classifier fusion*. Pattern Analysis & Applications 6(1), 22-31 (2003)

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(best gain possible relative to p, no wasted votes!)

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$$\max\left\{0,\ \ E_{\text{ind}} - \frac{T-1}{T+1}(1 - E_{\text{ind}})\right\} \leq \mathbf{E_{maj}} \leq p\left(1 + \frac{T-1}{T+1}\right)$$

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Patterns of Success/Failure

$$E_{maj} = E_{ind} - \int_{\mathbf{x}(+)} \frac{1}{T} \sum_t \delta(h_t, H) + \int_{\mathbf{x}(-)} \frac{1}{T} \sum_t \delta(h_t, H)$$

For each point in $\mathbf{x}(+)$, imagine $\frac{T+1}{2}$ classifiers correct (**PoS**)

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$$E_{maj} = p\left(1 + \tfrac{T-1}{T+1}\right).$$

... recovers *exactly* the upper bound :-)

Conclusion

- voting error can be decomposed.
- good diversity helps, bad diversity hinders.
- strong relation to PoS/PoF.
- large variance when T small, stabilize with large T
- not a magic solution - how can we make use of it!?

Questions?