COMPUTER VISION, GRAPHICS, AND IMAGE PROCESSING 24, 255–269 (1983)

NOTE

Digital Image Smoothing and the Sigma Filter

JONG-SEN LEE

Digital Image Processing Laboratory, Naval Research Laboratory, Washington, D.C. 20375

Received January 7, 1982; revised September 24, 1982

A conceptually simple but effective noise smoothing algorithm is described. This filter is motivated by the sigma probability of the Gaussian distribution, and it smooths the image noise by averaging only those neighborhood pixels which have the intensities within a fixed sigma range of the center pixel. Consequently, image edges are preserved, and subtle details and thin lines such as roads are retained. The characteristics of this smoothing algorithm are analyzed and compared with several other known filtering algorithms by their ability to retain subtle details, preserving edge shapes, sharpening ramp edges, etc. The comparison also indicates that the sigma filter is the most computationally efficient filter among those evaluated. The filter can be easily extended into several forms which can be used in contrast enhancement, image segmentation, and smoothing signal-dependent noisy images. Several test images 128 × 128 and 256 × 256 pixels in size are used to substantiate its characteristics. The algorithm can be easily extended to 3-D image smoothing.

1. INTRODUCTION

Generally, digital image smoothing techniques fall into two categories. In the first category, the noisy image is processed globally in the sense that the whole or a large section of a noisy image is correlated to obtain a smoothed image. Techniques in the transform domain using Wiener or least squares filtering [1, 2] and techniques applying one-dimensional or two-dimensional Kalman filter are in this category. Statistical models for the signal (noise free image) and the noise are required for the implementation of these techniques. Unfortunately, the statistical model for most images is either unknown or impossible to describe adequately with a simple random process. The smoothed images display blurred edges and conceal subtle details. In addition, the techniques are computationally costly. In the second category local operators are applied to noisy images. Only those pixels in a small neighborhood of the concerned pixel are involved in the computation. The immediate advantage of these techniques is their efficiency. They have great potential for real-time or near real-time implementation, because several pixels can be processed in parallel without waiting for their neighboring pixels to be processed. Recent research in image smoothing and segmentation favors the local techniques.

There are many local smoothing methods. The well-known median filter [4] in one or two dimensions has attracted much attention. The edge preserving smoothing of Nagao and Matsuyama [5], the gradient inverse weighting scheme of Wang *et al.* [6], the box filtering algorithm [7], and the local statistics method of Lee [8, 9] are just a few other algorithms in this category. Obviously, it is nearly impossible to rank them, because an algorithm may be effective for a class of images, but ineffective for others. In this paper a new class of local smoothing schemes is introduced. It is motivated by the sigma probability of the Gaussian distribution. The basic idea is to replace the pixel to be processed by the average of only those neighboring pixels having their intensity within a fixed sigma range of the center pixel. Replacing the

0734-189X/83 $3.00

center pixel by the average of selected neighboring pixels has been explored by many algorithms. Nagao's filter [5] replaces the center pixel by the average of a subregion which has minimum variance. Lee [9] in his refined local statistics method selected the region by using gradient information. Graham [10] and Prewitt [11] replace a pixel by the average of the surrounding area if the absolute value of their difference is smaller than some threshold. Rosenfeld [1] in his region growing and tracking algorithm excludes high contrast edges, lines, and points from the average by judging the gray level difference between the average of the region and the new pixel. The extended box-filtering algorithm [7] restricts the average to only neighboring pixels within a fixed intensity range. The main difference between the box filter and the sigma filter of this paper is that the former has the intensity range fixed throughout the whole image, while the latter lets the intensity range float with the intensity of the center pixel. The advantages are numerous: (1) noise near edge areas will be smoothed without blurring the edge because only pixels on one side of the edge are included in the average; (2) subtle details of several pixel clusters and linear features of one to three pixels in width will be preserved since only those pixels and not the background are included in the average; (3) it will not create artifacts and will retain shapes, because no directional masks are used, unlike the algorithms of Nagao [5] and Lee [9]; (4) it is computationally efficient, since only simple compare and fixed point add instructions are involved.

A comparison of the sigma filter, the gradient inverse filter, Nagao's filter, and the median filter are conducted in this paper. Comparisons are based on the following criteria: (1) effectiveness in smoothing noise; (2) preservation of subtle details and linear features; (3) immunity from shape distortion; (4) retention of step edges and sharpening of ramp edges; (5) removal of high-contrast spot noise; (6) computational efficiency. For the smoothing algorithms to be effective they are applied iteratively three times to test images of dimension 128×128. In many respects the sigma filter performs better than other algorithms except as regards the ability to remove sharp spot noise. Some methods of reducing this deficiency are presented.

The sigma filter and its extended forms are discussed in the next section. A comparison study of the algorithms is given in Section 3. Section 4 is devoted to presenting the characteristics of the sigma filter and its extensions to images with multiplicative noise.

2. THE SIGMA FILTER

The noise in an image is generally considered as spatially uncorrelated and with continuous intensity spectrum. White Gaussian noise is an example. We shall regard as noise any random clutter of the size of three or fewer pixels. It is well known that the "straight" averaging filter will smooth noise at the expense of blurring edges and smearing subtle details. An indiscriminate average of pixels in a window is the cause of the problem. As mentioned in Section 1 many schemes have been developed to overcome this problem. The merits of these algorithms will be explored in more detail in the next section. In this section, a conceptually simple algorithm is developed which easily excludes significantly different pixels from the average.

Most image noise is Gaussian in distribution. The two-sigma probability is defined as the probability of a random variable being within two standard deviations of its mean. The two-sigma probability for a one-dimensional Gaussian distribution is 0.955. This can be interpreted as meaning that 95.5% of random samples lie within the range of two standard deviations. In image smoothing, any pixel outside the two-sigma range most likely comes from a different population and, therefore, should be excluded from the average. If we assume that the a priori mean is the gray level of the pixel to be smoothed, we can establish a two-sigma range from the gray level and include in the average only those pixels within the two-sigma intensity. Let $x_{i,j}$ be the intensity or gray level of pixel (i, j), and $\hat{x}_{i,j}$ be the smoothed pixel (i, j). Also we assume that the noise is additive with zero mean and standard deviation σ. The sigma filter procedure is then described as follows:

(1) Establish an intensity range $(x_{i,j} + \Delta, x_{ij} - \Delta)$, where $\Delta = 2\sigma$.

(2) Sum all pixels which lie within the intensity range in a $(2n + 1, 2m + 1)$ window.

(3) Compute the average by dividing the sum by the number of pixels in the sum.

(4) Then $\hat{x}_{i,j}$ = the average. (To reduce sharp spot noise, step (4) will be modified later in this section.)

Or, mathematically, let

$$\begin{aligned} \delta_{k,l} &= 1, \qquad \text{if } (x_{i,j} - \Delta) \leq x_{k,l} \leq (x_{i,j} + \Delta) \\ &= 0, \qquad \text{otherwise.} \end{aligned}$$

Then

$$\hat{x}_{i,j} = \sum_{k=i-n}^{n+i} \sum_{l=j-m}^{m+j} \delta_{k,l} x_{k,l} \Big/ \sum_{k=i-n}^{n+i} \sum_{l=j-m}^{m+j} \delta_{k,l}. \tag{2}$$

The two-sigma range is generally large enough to include 95.5% of the pixels from the same distribution in the window, yet in most cases it is small enough to exclude pixels representing high-contrast edges and subtle details. Linear features such as roads one or two pixels wide are retained, because only those pixels with intensity near that of the feature are included in the average. The main drawback is that sharp spot noise represented by clusters of one or two pixels will not be smoothed. This could be very annoying especially for a fairly noisy image. To remedy this, we shall replace the two-sigma average with the center pixel's immediate neighbor average, if M, the number of pixels within the intensity range, is less than a prespecified value K. In other words, step (4) is replaced by

$$\begin{aligned} \hat{x}_{i,j} &= \text{two-sigma average}, \qquad \text{if } M > K \\ &= \text{immediate neighbor average}, \qquad \text{if } M \leq K. \end{aligned} \tag{4}$$

The value of K should be carefully chosen to remove isolated spot noise without destroying thin features and subtle details. For a 7×7 window, K should be less than 4, and it should be less than 3 for a 5×5 window. It should be noted that subtle textures within the two-sigma range will be wiped out after a few iterations. If conservation of texture information is required, a small Δ range and one or two iterations should be used.

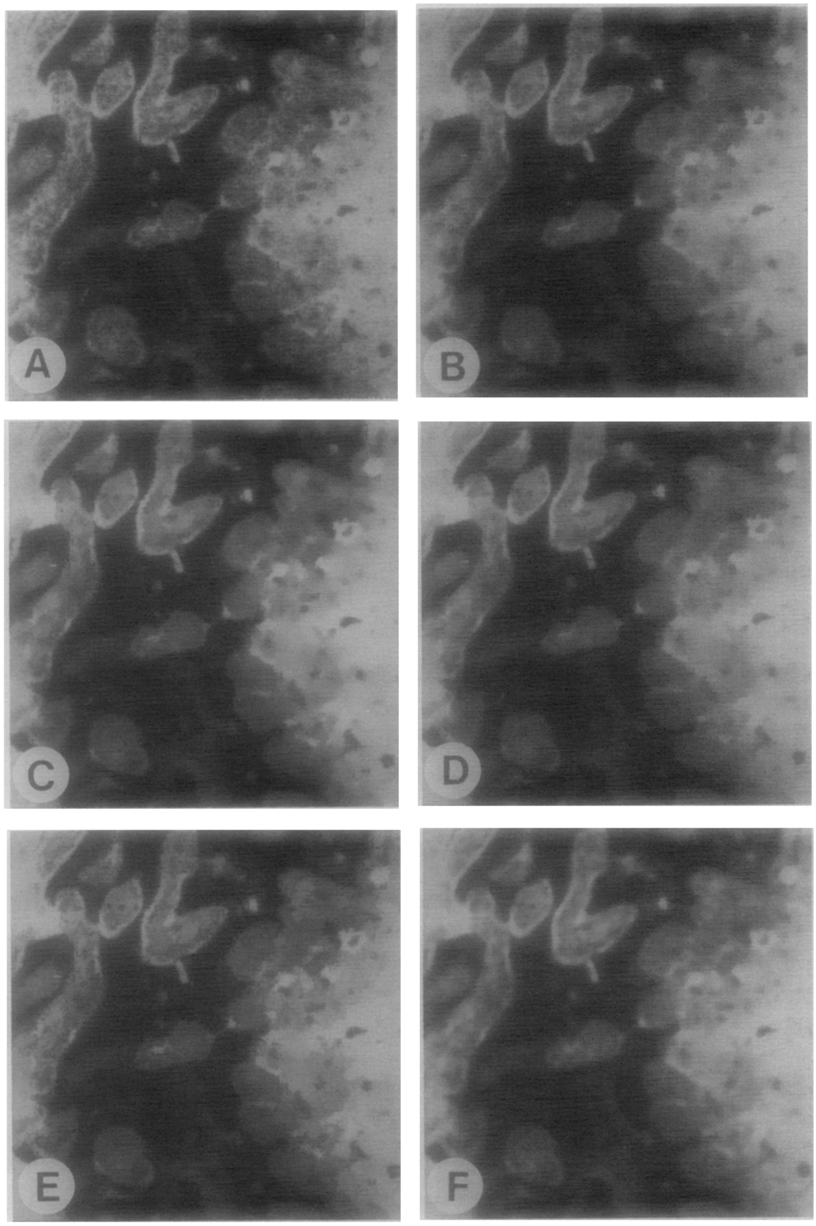

FIG. 1. This figure shows the results of the 7 × 7 sigma filter when applied once, twice, and three times to a medical image. The biased sigma filter when applied to (D) is shown in (E), and the result of the 3 × 3 median filter is shown in (F) for comparison.

For images with unknown noise characteristics, the intensity range Δ can be determined either from a rough estimation of the noise standard deviation in a flat area, or from the desirability of retaining the gray level difference between the desirable features and its background. The sigma filter can be applied repeatedly with reduced σ after each iteration. Two or three iterations are generally sufficient to reduce the noise level significantly.

As an illustration, Fig. 1(A) shows a medical image of cell structure. The results of applying the 7 × 7 sigma filter once, twice and three times are shown in Fig. 1(B), (C), and (D), respectively. The result of applying the median filter twice is shown in (F). It should be noted that (E) is the result of applying a derivative version of the sigma filter, to be discussed in Section 4.

3. A COMPARISON OF LOCALLY SMOOTHING ALGORITHMS

Numerous local image smoothing algorithms have been developed recently. It is impractical to compare all of them in detail. The straight local average method is known to blur edges and details. Lev *et al.* [12] applied a template matching technique to detect edges and lines and then replaced the pixel by a weighted average corresponding to the particular pattern detected. Twelve 3 × 3 masks are created and relatively complicated weighting schemes are proposed. This algorithm is not computationally efficient, nor is it very effective in smoothing noise, since the window size is small. Lee [9, 13], using a local statistics method, produced good results for images corrupted by both additive and multiplicative noise. However, artifacts are observed in some cases, and the computation of the local variance makes this algorithm somewhat inefficient. These two filtering algorithms are excluded in the present comparison. The recently published gradient inverse method [6], the edge preserving smoothing scheme of Nagao and Matsuyama [5], and the well-known median filter are chosen instead.

For completeness, brief descriptions of these three algorithms are given in this section. The gradient inverse weighting scheme employs a 3 × 3 window and computes for each pixel its inverse gradient weighted average with its neighboring pixels. The idea is to weight less those pixels having greater absolute differences with their center pixel. The procedure for processing $x_{i,j}$ in a 3 × 3 window is given as follows:

(1) Compute the inverse gradients of the eight neighboring pixels:

$$\begin{aligned} g_{kl} &= 1/|x_{i+k,j+l} - x_{i,j}| \qquad \text{if } x_{i+k,j+l} \neq x_{i,j} \\ &= 1/2, \qquad \text{if } x_{i+k,j+l} = x_{i,j} \end{aligned}$$

where $k, l = \{-1, 0, +1\}$.

(2) Compute weights for the eight neighbors:

$$w_{k,l} = \frac{1}{2} \cdot \frac{g_{k,l}}{\Sigma g} \quad \text{and} \quad w_{i,j} = \frac{1}{2}$$

(3) $\hat{x}_{i,j} = \sum_k \sum_l w_{k,1} x_{i+k,j+l}$.

Nagao and Matsuyama [5] proposed an algorithm which selects the most homogeneous neighborhood and replaces the pixel by its neighborhood average. They

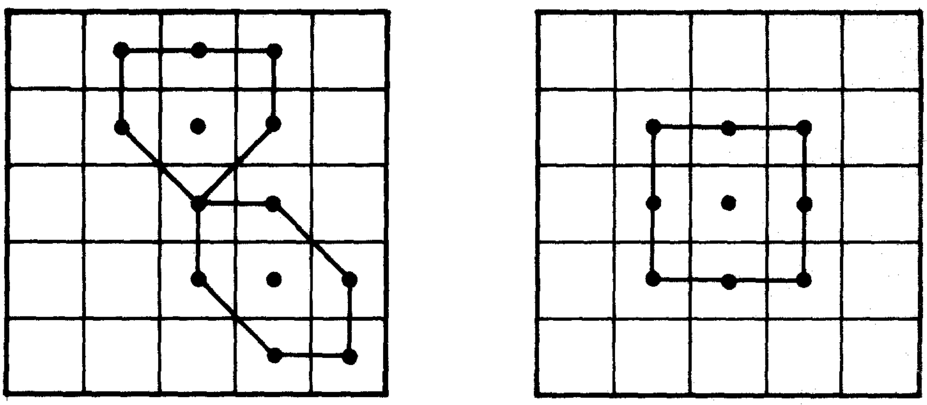

FIG. 2. Nagao's filter with its nine subregions in a 5 × 5 window. Two of the eight directional subregions are shown on the left, and the ninth mask is on the right. The center pixel is replaced by the mean of the subregion having the minimum variance.

created nine overlapped subregions in a 5 × 5 window as shown in Fig. 2. The means and variances of the nine subregions are computed, and the center pixel is replaced by the mean of the subregion having the minimum variance.

The median filter is more flexible. It can be applied columnwise, rowwise, and areawise. In our study, a 3 × 3 window is used, and the median of the nine pixels in the window represents the smoothed pixel. The reason for not using a large window is that a large window will smear details and edges, not to mention the higher computational load.

Two test images shown in Figs. 3 and 4, of dimension 128 × 128 pixels, are used in our comparison. In Fig. 3, a computer generated pattern of bars with increasing width (one pixel, three pixels,..., 15 pixels) is created, and corrupted with noise to test the ability to preserve linear features, the ability to smooth noise along edges, and the effectiveness of noise reduction in general. The average intensity of the bar is 150 and of the background is 50. Figure 4 is a natural aerial scene artificially corrupted with noise. The intensity levels in all images in this paper are between 0 and 255. Each algorithm is applied to the noisy image repeatedly 3 times. The sigma filter is applied in a 7 × 7 window with the intensity intervals 2σ, σ, and $\sigma/2$, and $K = 2$.

(1) Effectiveness in Noise Smoothing

The efficiency of smoothing noise can be measured by the reduction in noise standard deviation or variance. For the images of Fig. 3 the standard deviations of each smoothed image are computed from a flat area in the lower left corner. The results are listed in Table 1.

The gradient inverse filter is apparently the least efficient smoothing algorithm due to its small mask and the nature of its weighting scheme. The sigma filter is significantly superior in smoothing noise with a reduction of standard deviation by approximately a factor of ten. The Nagao and median filters are comparable in their ability to reduce noise.

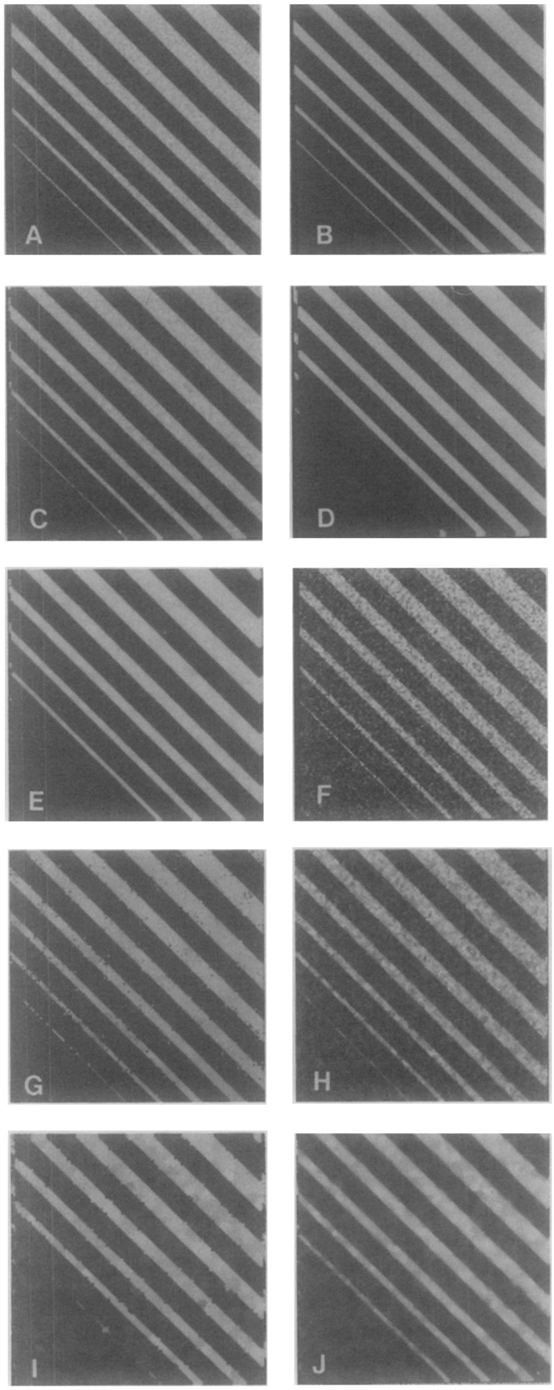

FIG. 3. The noise corrupt images ((A) and (F)) of bars with increasing width (one pixel, three pixels,..., 15 pixels). Several noise smoothing algorithms are applied, and the results are shown in (B)–(E) and (G)–(J), respectively. (B) and (G) are the results of applying the sigma filter, while (C) and (H), (D) and (I), and (E) and (J) are the results of the gradient inverse filter, Nagao's filter and the 3×3 median filter, respectively.

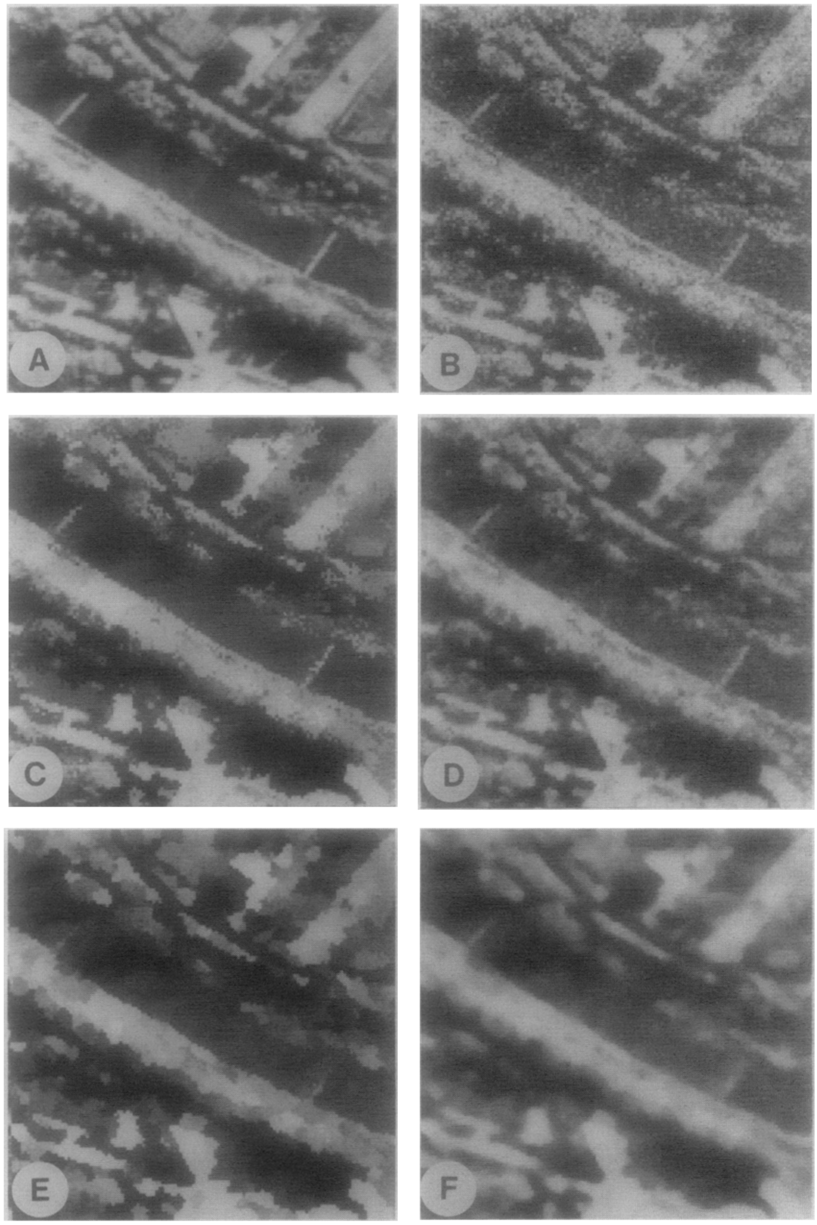

FIG. 4. (A) An aerial image (courtesy of Image Processing Institute, USC); (B) Gaussian noise of standard deviation 20 is artificially added to (A); (C), (D), (E), and (F) are the results applied by the sigma filter, gradient inverse filter, Nagao's filter, and the 3 × 3 median filter, respectively.

TABLE 1
Comparison of Reduction in Noise Standard Deviation

	Noise Standard Deviation	
Smoothing Algorithms	Bar pattern with $\sigma = 10$	Bar pattern with $\sigma = 30$
Sigma filter	0.81	3.54
Gradient inverse	5.74	17.84
Nagao's filter	2.48	10.87
Median filter	2.55	8.11

(2) Preservation of Subtle Details and of Linear Features

In some images it is important to retain highly distinguishable subtle details and line features, such as piers and roads. In other applications, such as image segmentation, it may be desirable to remove subtle details. The sigma filter is effective in preserving subtle details and line features as long as the intensity difference between them and their background is greater than the two-sigma intensity range. The background pixels will be excluded from the average when processing a pixel which represents the road or the subtle detail. In fact, it would preserve even a single outlying pixel, if we were not using the threshold K for spot noise reduction. The gradient inverse method theoretically will smear any feature of any size if applied a sufficient number of times, since it includes all pixels in the average and only weights them less if the difference is large. Similarly the Nagao filter will blur and eventually devour any feature with dimensions of three pixels or less in any direction. This can be easily seen in a noise free one-dimensional case in Fig. 5, in which the Nagao filter is equivalent to replacing the center pixel with the average of itself and its two neighbors on either side, whichever has the minimum variance. The center pixel of the three-pixel-wide pulse will drop in value after one application. The deterioration will continue slowly in the one-dimensional case, but much faster in the two dimensional case. As seen in Figs. 3(D) and (I), the bars of width one and three pixels are almost completely wiped out. The 3×3 median filter will wipe out single pixel lines in one application, since in a 3×3 mask, among the nine pixels, six of them will be background pixels. Thus the median will approach the background pixel value. A bar two pixels wide is a critical case. It has five to six pixels depending on the orientation of the bar. The median filter will swallow slightly curving or broken two-pixel-wide bars. For a 5×5 median filter, a three-pixel-wide bar will be wiped out in one application. The images in Fig. 4 further substantiate the characteristics of these algorithms. Figures 4(C), (D), (E), and (F) are the results of applying the smoothing algorithms three times. The gradient inverse scheme shown in Fig. 4(D) did not do much about the noise and slightly reduced the contrast of the image. As shown in Fig. 4(E), Nagao's filter smeared bridges and subtle detail and created artifacts. The 3×3 median filter smeared the bridges and generally blurred the image. The sigma filter performed fairly well except for the sharp spot noise problem.

(3) Immunity from Shape Distortion

The gradient inverse method is not effective in smoothing noise, but it is relatively free from artifacts and shape distortion. Nagao's filter, on the other hand, as shown

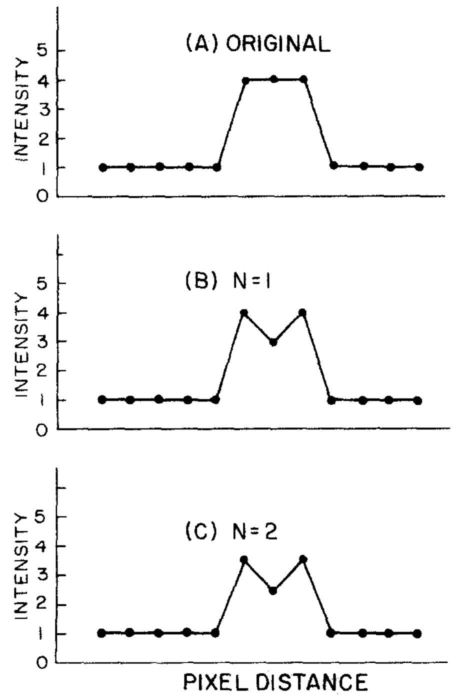

FIG. 5. The effect of Nagao's filter as applied in the one-dimensional case, to illustrate that it will blur and eventually devour any feature with dimension of three pixels or less. (A) shows the original one-dimensional three-pixel-wide pulse; the center pixel drops in value after one application as shown in (B); (C) shows that the value of the pulse decreases even more after another application.

in Fig. 4(E), does create significant distortion because of the directional subregion average. It will round off corners of less than 90°. Median filter is known to create artifacts. The 3 × 3 median filter will round off corners and produce patterns of patches, the same as Nagao's filter. As shown in Fig. 4(C), the sigma filter is practically free of shape distortion.

(4) Retention of Step Edges and Sharpening Ramp Edges

The intensity variations in the direction perpendicular to a sharp edge in the image plane form a step edge. Retaining the sharpness of a step edge is highly desirable in both image smoothing and segmentation. The gradient inverse filter will blur the step edge, as it computes the average on all pixels. The median filter will maintain a noise free step edge, but it will smear a noise step edge. Figure 6 shows a 3 × 3 mask moving through an edge. Assuming the edge is contaminated by noise, the 3 × 3 median filter replaces the center pixel with the fifth least bright pixel of the six pixels on the left side of the edge, while as the window moves right by one pixel, the center pixel is replaced by the fifth brightest among the six pixels on the dark side of the edge. Consequently the sharpness of the edge is degraded. The sigma filter, however, retains its sharpness by replacing the center pixel by the average of the six pixels.

Sharpening a ramp edge is generally of interest in studies of image segmentation by gray level difference. In this application Nagao's filter is excellent due to its

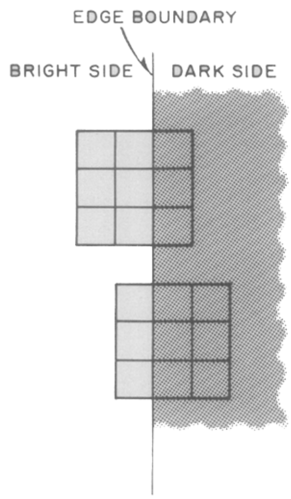

FIG. 6. The 3 × 3 median filter will maintain a noise free step edge, but it will smear a noisy step edge. Assuming the edge is contaminated by noise, the median is the fifth least bright pixel of the six pixels on the left side of the edge as shown in the upper portion of the figure. As the window moves to the right by one pixel (refer to the lower portion of the figure), the median is the fifth brightest pixel of the six in the shaded area.

directional subregion average. The other three algorithms will not sharpen a ramp edge but all will maintain a ramp edge fairly well. A derivative of the sigma filter which will sharpen a ramp edge will be discussed in the next section.

(5) Removing Spot Noise

The median filter is well known for its effectiveness in removing sparsely positioned sharp spot noise, since the spot noise has intensity at either end of the intensity scale. Nagao's filter is also effective, but requires a few iterations. The gradient inverse filter weights the spot noise much higher than its surrounding pixels. Consequently, it is not effective. The sigma filter with large window size is highly suseptible to spot noise, since no other pixel but the spot noise itself is within the two sigma range. The modified version with threshold K (as shown in Fig. 3(G)) discussed in the last section will remove most isolated spot noise. However, spot noise near edges remains because the 7 × 7 mask contains several edge pixels which will fall into the two-sigma range. Increasing the value of K will further reduce the spot noise, but at the expense of blurring edges and subtle details. The spot noise can be further reduced by applying a 3 × 3 sigma filter with $K = 1$, or 2. Figure 7 shows the effect of spot noise reduction by applying it to Figs. 3(G) and 4(C) for $K = 1$ and $K = 2$. Figures 3(G) and 4(C) are repeated in Fig. 7 for comparison. As shown in Fig. 7(C), the spot noise is almost completely removed; however, the one-pixel-wide bar is badly broken up. Figures 6(E) and (F) show the effect on the aerial image.

(6) Computational Efficiency

In our comparison, the algorithms were coded in FORTRAN and no special efforts were devoted to accelerate their executions. The computations were carried

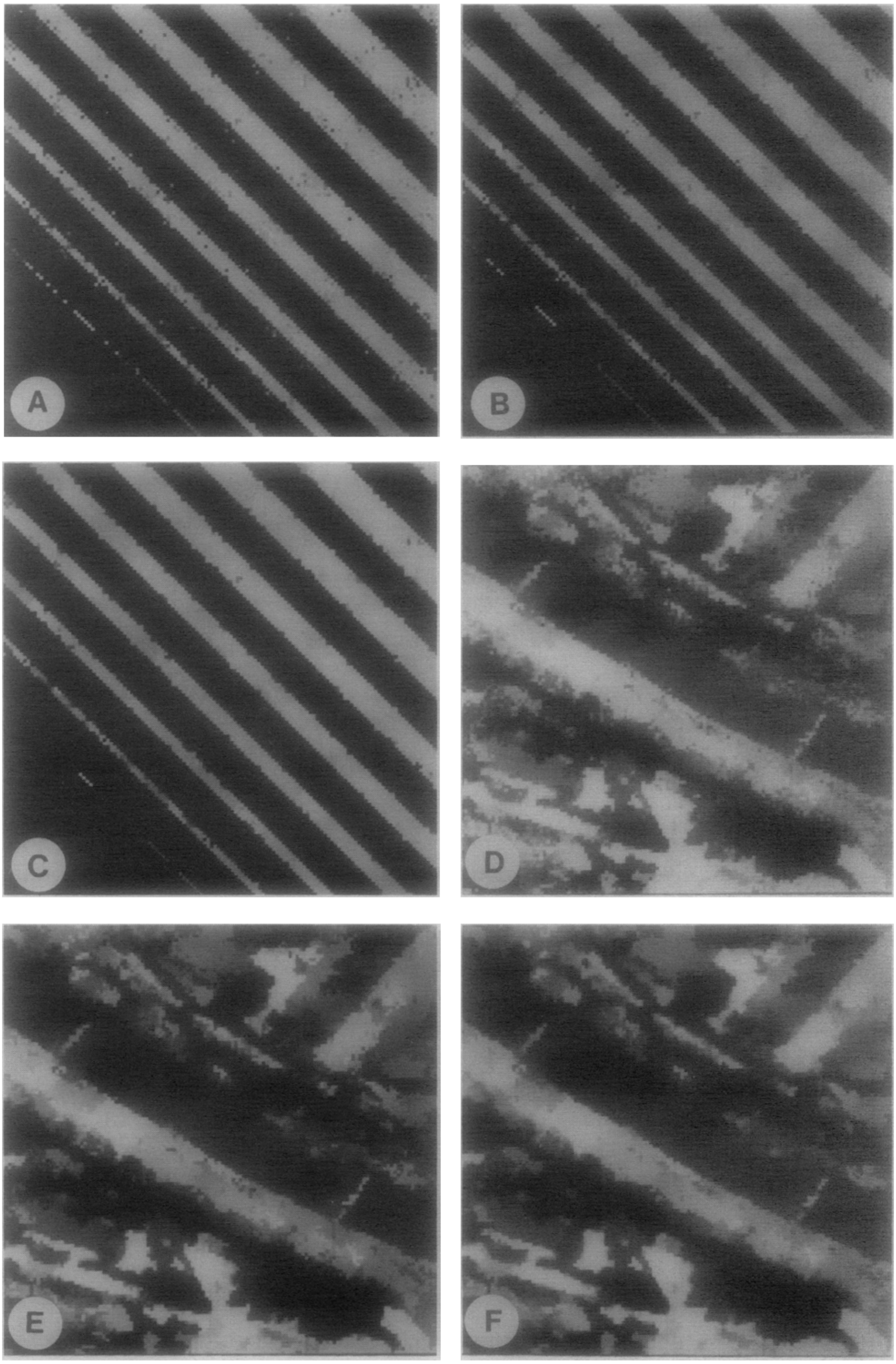

FIG. 7. The sigma filter is susceptible to spot noise as indicated in (A) and (D). The problem is overcome by applying the 3 × 3 sigma filter with $K = 1$ and 2 as shown in (B) and (C) for the bar pattern, and (E) and (F) for the aerial scene. (A) and (D) repeat Figs. 2(G) and 3(C), respectively.

out on a Data General NOVA 800 with a Comtal 8000 image display. The ratio of computational time required for images of size 128×128 for each filter (per iteration) is listed in increasing order as follows:

(1) The sigma filter (7×7), 1 unit of time;

(2) The median filter (3×3), 1.5 units of time;

(3) The gradient inverse filter, 4.0 units of time;

(4) Nagao's filter, 11.0 units of time.

The sigma filter is the fastest algorithm in this group even with a 7×7 window. In our simulation, it took no more time than computing a straight 7×7 average. Nagao's filter is extremely slow, since it requires the computation of variance for nine subregions.

4. THE EXTENDED SIGMA FILTER

The sigma filter can be easily extended to perform image enhancement, segmentation, smoothing of signal-dependent noise, and even 3-D images. Here, only a few of these possibilities will be mentioned.

(1) The Biased Sigma Filter

This extended sigma filter will sharpen a ramp edge and also enhance the contrast of subtle detail. The bias is introduced by separately averaging pixels in the upper intensity range of $(x_{i,j}, x_{i,j} + \Delta)$ and in the lower intensity range of $(x_{i,j}, x_{i,j} - \Delta)$. The absolute difference between the upper average and $x_{i,j}$, and also the absolute difference between the lower average and x_{ij}, are computed. The center pixel is replaced by the average which has the smaller absolute difference. The function of the biased sigma filter can be easily explained in a one-dimensional case. Figure 8(A) shows the effect of a seven-pixel-wide biased sigma filter with $\Delta = 3$. With one application, the ramp edge becomes much sharper, and it will approach a step edge as the number of applications increase. It should be cautioned that the intensity range Δ should be chosen to be relatively large. As shown in Fig. 8(B) a ramp edge may become a two step edge with $\Delta = 1$. This algorithm is useful in sharpening edges in preprocessing for image segmentation by gray level difference and also in bringing out subtle details in a smoothed image. Figure 1(E) shows the image of Fig. 1(D) processed by the biased sigma filter.

(2) Signal-Dependent Noise

Signal-dependent noise or speckles occur in coherent optical images as well as in synthetic aperture radar images [13]. To deal with this noise, a reasonably effective method based on local statistics was recently proposed by Lee [14]. In our experiment the sigma filter modified for signal-dependent noise performs better in many cases and requires much less computational time. The intensity range will not only float up and down with $x_{i,j}$ but also shrink or grow with $x_{i,j}$, since σ is a function of $x_{i,j}$. A more detailed discussion will be given in a separate study [15].

(3) Extension to 3-D Images

It is straightforward to extend the sigma filter and its derivatives to 3-D image smoothing. The two-dimensional window will be replaced by a three-dimensional

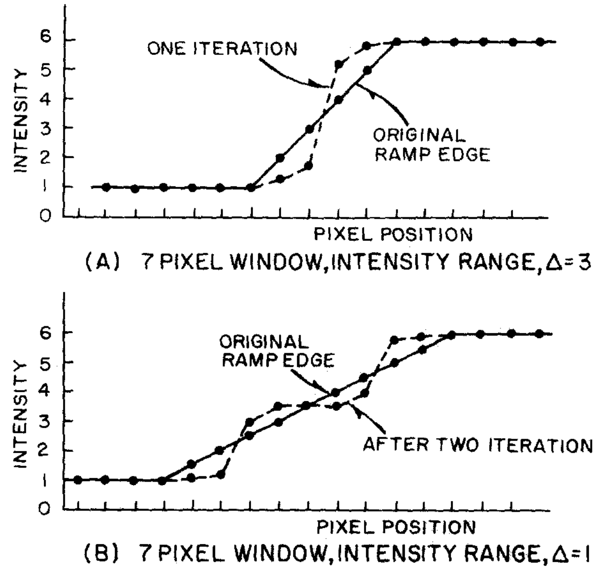

FIG. 8. A ramp edge as shown in (A) is sharpened by applying the biased sigma filter. The intensity range in the biased filter should be chosen to be relatively large. As shown in (B), the ramp edge becomes a two step edge with the intensity range reduced from 3 to 1.

cube. Pixels within the cube are processed by the same procedures established for the two-dimensional case.

5. REMARKS

(a) Most local smoothing algorithms do not require prespecified parameters. Clearly, this is a distinct advantage if the algorithm is to be effective for all image categories. The sigma filter does require specification of the intensity range and the size of the window. However, these parameters permit us to fine-tune the filter to a specific image or class of images. Once the characteristics of the sigma filter with respect to the parameters are understood, it is fairly easy to determine the appropriate values. In addition, the computational efficiency of this algorithm permits us to adjust the parameters interactively.

(b) The basic principle of the sigma filter can be incorporated into other algorithms to modify the characteristics of these filters. For example, it could be included in Nagao's filter or Lee's local statistics algorithm [9] to obtain the two-sigma average in the directional subregion after it has been chosen by the procedures of these algorithms.

6. CONCLUSION

A simple, effective, and computationally efficient noise smoothing algorithm has been developed. Detailed comparisons with a few local smoothing algorithms are made to substantiate the basic characteristics of this filter. The procedure and strategy of utilizing this filter has been explored. Applications of this filter to image segmentation and other problems are currently under investigation. It is hoped that the sigma filter will be accepted as a basic digital image processing technique because of its simplicity and effectiveness.

ACKNOWLEDGMENTS

The author thanks Dr. I. Jurkevich and Dr. A. F. Petty for many helpful discussions.

REFERENCES

1. A. Rosenfeld and A. C. Kak, *Digital Picture Processing*, Academic Press, New York, 1976.
2. H. C. Andrews and B. R. Hunt, *Digital Image Restoration*, Prentice-Hall, Englewood Cliffs, N.J., 1977.
3. A. K. Jain, A semicausal model for recursive filtering of two dimensional images, *IEEE Trans. Comput.* **C-26**, 1977.
4. W. K. Pratt, *Digital Image Processing*, Wiley, New York, 1978.
5. M. Nagao and T. Matsuyama, Edge preserving smoothing, *Computer Graphics Image Processing* **9**, 394–407, 1979.
6. D. Wang, A. Vagnucci, and C. Li, Image enhancement by gradient inverse weighted smoothing scheme, *Computer Graphics Image Processing* **15**, 167–181, 1981.
7. M. J. McDonnall, Box-filtering techniques, *Computer Graphics Image Processing* **17**, 65–70, 1981.
8. J. S. Lee, Digital image enhancement and noise filtering by use of local statistics, *IEEE Trans. Pattern Anal. Mach. Intell.* **PAMI-2**, 1980.
9. J. S. Lee, Refined filtering of image noise using local statistics, *Computer Graphics Image Processing* **15**, 380–389, 1981.
10. R. E. Graham, Snow removal—A noise-stripping process for picture signal, *IRE Trans. Inf. Theor.* **8**, 129–144, 1966.
11. J. M. S. Prewitt, Object enhancement and extraction, in *Picture Procession and Psychopictorics* (B. S. Lipkin and A. Rosenfeld, Eds.), Academic Press, New York, 1970.
12. A. Lev, S. Zucker, and A. Rosenfeld, Interactive enhancement of noisy images, *IEEE Trans. Syst. Man Cybern.* **SMC-7**, 435–442, 1977.
13. J. W. Goodman, Some fundamental properties of speckles, *J. Opt. Soc. Amer.* **66**, 1976.
14. J. S. Lee, Speckle analysis and smoothing of synthetic aperture radar image, *Computer Graphics Image Processing* **17**, 24–32, 1981.
15. J. S. Lee, A simple speckle smoothing algorithm for synthetic aperture radar images, *IEEE Trans. Syst. Man Cybern.*, vol. **SMC-13**, No. 1, 85–89, 1983.

Statement of ownership, management and circulation required by the Act of October 23, 1962, Section 4369, Title 39, United States Code: of

COMPUTER VISION, GRAPHICS, AND IMAGE PROCESSING

Published monthly by Academic Press, Inc., 111 Fifth Avenue, New York, N.Y. 10003. Number of issues published annually: 12. Editor: Dr. Azriel Rosenfeld, Computer Science Center, University of Maryland, College Park, MD 20742. Owned by Academic Press, Inc., 111 Fifth Avenue, New York, N.Y. 10003. Known bondholders, mortgagees, and other security holders owning or holding 1 percent or more of total amount of bonds, mortgages, and other securities: None.
Paragraphs 2 and 3 include, in cases where the stockholder or security holder appears upon the books of the company as trustee or in any other fiduciary relation, the name of the person or corporation for whom such trustee is acting, also the statements in the two paragraphs show the affiant's full knowledge and belief as to the circumstances and conditions under which stockholders and security holders who do not appear upon the books of the company as trustees, hold stock and securities in a capacity other than that of a bona fide owner. Names and addresses of individuals who are stockholders of a corporation which itself is a stockholder or holder of bonds, mortgages or other securities of the publishing corporation have been included in paragraphs 2 and 3 when the interests of such individuals are equivalent to 1 percent or more of the total amount of the stock or securities of the publishing corporation.
Total no. copies printed: average no. copies each issue during preceding 12 months: 1659; single issue nearest to filing date: 1718. Paid circulation (a) to term subscribers by mail, carrier delivery or by other means: average no. copies each issue during preceding 12 months: 1213; single issue nearest to filing date: 1218. (b) Sales through agents, news dealers, or otherwise: average no. copies each issue during preceding 12 months: 0; single issue nearest to filing date: 0. Free distribution by mail, carrier delivery, or by other means: average no. copies each issue during preceding 12 months: 58; single issue nearest to filing date: 58. Total no. of copies distributed: average no. copies each issue during preceding 12 months: 1271; single issue nearest to filing date: 1276.

(Signed) Roselle Coviello, Senior Vice President