Elimination of TBox and ABox

ABox elimination

- **<u>In extensions of $\mathcal{ALCO}$:</u>** ABox consistency problem is reduced to (and hence has the same complexity as) the concept satisfiability problem, as follows: *an ABox $\mathcal{A}$ is consistent w.r.t. a TBox $\mathcal{T}$ iff the following concept is satisfiable w.r.t. the same TBox $\mathcal{T}$:*

$$\bigsqcap_{a \text{ occurs in } \mathcal{A}} \exists U.\Big(\{a\} \sqcap \bigsqcap_{a:C \in \mathcal{A}} C \sqcap \bigsqcap_{aRb \in \mathcal{A}} \exists R.\{b\} \Big)$$

where U is a *fresh* role name (i.e., not occurring in $\mathcal{A}, \mathcal{T}$).

TBox elimination

Given a TBox $\mathcal{T}$, denote $C_{\mathcal{T}} := \bigsqcap_{(D \sqsubseteq E) \in \mathcal{T}} (\neg D \sqcup E)$. So, $\mathcal{T}$ is equivalent to the TBox $\{\top \sqsubseteq C_{\mathcal{T}}\}$.

In the following cases, a general TBox can be "internalized", so that reasoning w.r.t. TBox can be reduced to (and hence has the same complexity as) reasoning without TBox.

- **<u>In extensions of $\mathcal{ALCIO}$:</u>** *a concept C is satisfiable w.r.t. a TBox $\mathcal{T}$ iff the following concept is satisfiable (w.r.t. the empty TBox):*

$$C \sqcap \{a\} \sqcap \exists U.\{a\} \sqcap \forall U.C_{\mathcal{T}} \sqcap \bigsqcap_{R \in \mathsf{Roles}} \forall U.\forall R.\exists U^{-}.\{a\},$$

where the role name U and the nominal $\{a\}$ are *fresh* (i.e., not occurring in $C, \mathcal{T}$) and Roles is the set of role names occurring in C and $\mathcal{T}$ and inverses thereof.

- **<u>In extensions of $\mathcal{SH}$:</u>** *a concept C is satisfiable w.r.t. a TBox $\mathcal{T}$ and RBox $\mathcal{R}$ iff the concept $C \sqcap C_{\mathcal{T}} \sqcap \forall U.C_{\mathcal{T}}$ is satisfiable w.r.t. empty TBox and the following RBox:*

$$\mathcal{R}_U := \mathcal{R} \cup \{ \mathsf{Trans}(U) \} \cup \{ R \sqsubseteq U \mid R \in \mathsf{Roles} \},$$

where U is a role name not occurring in $C, \mathcal{T}, \mathcal{R}$, and Roles is the set of all role names occurring in $C, \mathcal{T}, \mathcal{R}$ (and their inverses, if the language under consideration has the inverse role constructor).

- **<u>In extensions of $\mathcal{ALC}(\sqcup, *)$:</u>** *a concept C is satisfiable w.r.t. a TBox $\mathcal{T}$ iff the following concept is satisfiable (w.r.t. empty TBox):*

$$C \sqcap \forall (R_1 \sqcup \ldots \sqcup R_n)^*.C_{\mathcal{T}},$$

where $\{R_1, \ldots, R_n\}$ is the set of role names occurring in $C, \mathcal{T}$ (and their inverses, if the language under consideration has the role inverse constructor).