Elimination of TBox and ABox

**ABox elimination**

- **In extensions of ALCQ:** ABox consistency problem is reduced to (and hence has the same complexity as) the concept satisfiability problem, as follows: an ABox $A$ is consistent w.r.t. a TBox $T$ iff the following concept is satisfiable w.r.t. the same TBox $T$:

$$
\bigwedge_{a \text{ occurs in } A} \exists U. \left( \{a\} \sqcap \bigwedge_{a \in C \in A} C \sqcap \bigwedge_{aRb \in A} \exists R. \{b\} \right)
$$

where $U$ is a fresh role name (i.e., not occurring in $A, T$).

**TBox elimination**

Given a TBox $T$, denote $C_T := \bigwedge_{(D \sqsubseteq E) \in T} (\neg D \sqcup E)$. So, $T$ is equivalent to the TBox $\{\top \sqsubseteq C_T\}$.

In the following cases, a general TBox can be “internalized”, so that reasoning w.r.t. TBox can be reduced to (and hence has the same complexity as) reasoning without TBox.

- **In extensions of ALCQ:** a concept $C$ is satisfiable w.r.t. a TBox $T$ iff the following concept is satisfiable (w.r.t. empty TBox):

$$
C \sqcap \{a\} \sqcap \exists U.\{a\} \sqcap \forall U.C_T \sqcap \bigwedge_{R \in \text{Roles}} \forall U.\forall R.\exists U^{-}.\{a\},
$$

where the role name $U$ and the nominal $\{a\}$ are fresh (i.e., not occurring in $C, T$) and Roles is the set of role names occurring in $C$ and $T$ and inverses thereof.

- **In extensions of SHI:** a concept $C$ is satisfiable w.r.t. a TBox $T$ and RBox $R$ iff the concept $C \sqcap C_T \sqcap \forall U.C_T$ is satisfiable w.r.t. empty TBox and the following RBox:

$$
R_U := R \cup \{ \text{Trans}(U) \} \cup \{ R \sqsubseteq U \mid R \in \text{Roles} \},
$$

where $U$ is a role name not occurring in $C, T, R$, and Roles is the set of all role names occurring in $C, T, R$ (and their inverses, if the language under consideration has the inverse role constructor).

- **In extensions of ALC($\sqcup, \ast$):** a concept $C$ is satisfiable w.r.t. a TBox $T$ iff the following concept is satisfiable (w.r.t. empty TBox):

$$
C \sqcap \forall (R_1 \sqcup \ldots \sqcup R_n)\ast.C_T,
$$

where $\{R_1, \ldots, R_n\}$ is the set of role names occurring in $C, T$ (and their inverses, if the language under consideration has the role inverse constructor).