

# Generating Tableau Provers Using METTEL<sup>2</sup>

## *Tutorial*

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THE 22ND INTERNATIONAL CONFERENCE  
ON AUTOMATED REASONING WITH ANALYTIC TABLEAUX AND RELATED METHODS

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# Outline

- 1 Introduction
- 2 Overview and motivation
- 3 Distribution kit and installation
- 4 Demo
- 5 Specification
  - Syntax specification
  - Tableau specification
  - Options
- 6 Optimisations and controlling derivations
  - Search strategies
  - Rule priorities
- 7 Rewriting
- 8 Blocking
  - Unrestricted blocking rule
  - Specifying the blocking mechanism
- 9 Further tricks
  - Labelled tableaux
  - Signed tableaux
  - Embedding sorts
  - Eliminating expressions
  - Reducing branching factor
  - Tabular logics
- 10 Testing and benchmarking utilities
- 11 Conclusion

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The screenshot shows a web browser window titled "MetTeL2 Online Demo | MetTeL tableau prover generation platform - Mozilla Firefox". The address bar shows the URL "www.mettel-prover.org/online-demo/mettel2-demo/". The page content includes the MetTeL logo, a navigation menu with "Home", "Online Demo", "Download", "Documents", "About Us", and "Contact", and a main heading "MetTeL 2 online demo". Below the heading, there is a paragraph about reading the "system description for MetTeL2" and a link for "Show quick help". A code editor displays the "1.Logic's syntax" section, showing predefined syntax rules for logic specifications. On the right side, there are two download buttons: "DOWNLOAD THE NIGHTLY VERSION" (with a version number 2.0-614) and "DOWNLOAD THE LATEST VERSION" (with a version number 1.0-197).

MetTeL  
A Generic Tableau Prover

MetTeL<sup>2</sup>: Towards a Prover Generation Platform

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## MetTeL 2 online demo

Reading the [system description for MetTeL2](#) may help you in using the following tool.

[Show quick help](#)

### 1.Logic's syntax

You may select one of the predefined syntax for following list:

User Defined syntax ...

```

1 /* every syntax starts with 'specification'
2  * keyword followed by the name of logic */
3 specification myLogic;
4
5 /* immediately after there should be
6  a block for introducing the syntax */
7 syntax myLogic{
8
9 //syntax definition should be initiated with definition of sort
10
11 }

```

DOWNLOAD THE NIGHTLY VERSION  
experimental  
2.0-614

DOWNLOAD THE LATEST VERSION  
1.0-197

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# Prover Development Problem

- Different applications require different (logical) formalisms
- Logics need reasoning tools
- Implementation of provers is expensive
- Altering existing provers is hard
- Translational approach requires additional knowledge and skills for the user

## Solution

Generation of a prover code from a specification of a logic.

# Goals and Objectives of Implementation of METTEL<sup>2</sup>

- An easy to use prover generator
- Modularity of generated code
- Hierarchy of public JAVA classes and interfaces that can be easily extended and integrated with other systems

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# Requirements and Installation

Requires JAVA. Tested on JRE 1.6 and 1.7.

## Installation

- Download and install JRE 1.7 from <http://www.java.com>.
- Download **mettel-2.0-XXX.zip** from <http://www.mettel-prover.org>.
- Place all the files from **mettel-2.0-XXX.zip** into some directory, e.g. **~/mettel2**.

# Distribution Kit and Library Dependencies

## mettel-2.0-XXX.zip

- **mettel2.jar** — main **jar**-file for the generator, includes all the packages, depends on the ANTLR libraries:
  - **antlr.jar**,
  - **antlr3.jar**,
  - **antlr3-runtime.jar**,
  - **stringtemplate4.jar**,and the CSV formatting library:
  - **opencsv.jar** — required for running benchmark suites.
- **mettel2-core.jar** — required for running the generated provers, depends on:
  - **antlr3-runtime.jar** — required for running parsers generated by ANTLR.
- **mettel2-util.jar** — required for running expression generators and benchmark suites.

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# Generating and Running Prover

- Prepare a specification file **S4.s**.
- Generate the prover:

```
> java -jar ~/mettel2/mettel2.jar -i S4.s
```

- Place the generated **S4.jar** into **~/mettel2**.
- Run the prover:

```
> java -jar ~/mettel2/S4.jar
```

# Package Structure of Generated Provers

## S4.java

- Syntax related classes: `S4.language.S4`
- Tableau related classes: `S4.tableau.S4`
- Executable classes:
  - Main class for the prover: `S4.tableau.S4.S4TableauProver`
  - Main class for running benchmark suites: `S4.tableau.S4.S4Benchmark`
  - Generator of random expressions for specified syntax:  
`S4.language.S4.util.S4RandomExpressionGenerator`
  - Statistical analyser of expressions:  
`S4.language.S4.S4ProblemAnalyzer`
- Additional resources and examples: *etc*

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# Specification

## Short syntax

```

specification <name>;
options{
...
}
syntax <name>{
...
}
tableau <name>{
...
}

```

## Full syntax

```

specification <path>;
options{
...
}
syntax <name>
options{
...
}
tableau <name> in syntax <name>
options{
...
}
}
<other syntax and tableau blocks>

```

# Inside Syntax Specification

- Declaration of sorts of a (multi-sorted) propositional language:

**sort** formula, world;

- Connective specifications as simple BNF statements:

formula at = ' @ ' world formula;  
 formula disjunction = formula ' | ' formula;

## Formula examples

$P \sim Q, @I P,$   
 $@I(P \sim Q),$   
 $@I \sim(p|(P \sim Q)),$   
 $@I \sim p|P \sim Q$

$P \vee \neg Q, @_{\ell} P,$   
 $@_{\ell}(P \vee \neg Q),$   
 $@_{\ell} \neg(p \vee (P \vee \neg Q)),$   
 $@_{\ell} \neg p \vee P \vee \neg Q$



# Example

$$\begin{aligned} \phi &\stackrel{\text{def}}{=} p \mid \perp \mid \neg\phi \mid \diamond\phi \mid @_w\phi \mid \phi \vee \phi \mid w \approx w \mid R(w, w) \\ w &\stackrel{\text{def}}{=} i \mid f(w, \phi) \end{aligned}$$

## syntax S4{

**sort** formula, world;

formula false = 'false';

formula negation = '~' formula;

formula diamond = '<>' formula;

formula at = '@' world formula;

formula disjunction = formula '|' formula;

formula equality = '[' world '=' world ']' ; //Equality

formula relation = 'R' '(' world ',' world ')' ; //Relation

world f = 'f' '(' world ',' formula ')' ; //Skolem function

}

# Inside Tableau Specification

- Tableau rule declaration in a premise-conclusion syntax:

$$@l(P \mid Q) / @lP \$ | @lQ \$;$$

$$\frac{@l(P \vee Q)}{@lP \mid @lQ}$$

- Branch separator \$| and rule separator \$;
- Rule priorities:

$$@l(P \mid Q) / @lP \$ | @lQ \textit{priority} 2 \$;$$

- Default priority is 0

# Options for Tableau Specification

- Redefinable separators \$| and \$;

```

options{
  ...
  tableau.rule.delimiter =;
  tableau.rule.branch.delimiter=|
}
...
tableau S4{
  ...
  @| (P | Q) / @| P || @| Q priority 3;
}

```

- The delimiter / between premises and conclusions can be redefined via **tableau**.rule.premise.delimiter property.
- Must be redefined in the global **options** block or in the **options** block for the corresponding syntax.

# Example

```

tableau S4{
  @i ~(~P) / @i P priority 1; //Double–negation removal
  @i(P|Q) / @i P || @i Q priority 3; //Disjunction rule
  @i~(P|Q) / @i~P @i~Q priority 1; //"Conjunction" rule
  @i<>P / R(i,f(i,P)) @f(i,P)P priority 7; //Diamond rule
  @i~(<>P) R(i,j) / @j~P priority 2; //"Box" propagation rule
  @i P / R(i,i) priority 1; //Reflexivity
  R(i,j) R(j,k) / R(i,k) priority 2; //Transitivity
  @i P @i~P / priority 0; //Closure rule
  R(i,j) / [i=j] || ~([i=j]) priority 6; //Ancestor blocking rule
  ~([i=i]) / priority 0; //Closure rule for inequality
}

```

Notice separators in tableau rules!

# Options

```
options{  
  branch.bound=  
  branch.selection.strategy=  
  
  equality.keywords={equality}  
  
  tableau.rule.delimiter=$;  
  tableau.rule.branch.delimiter=$|  
  tableau.rule.premise.delimiter=/  
}
```

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# Features of Generated Provers

- Dynamic backtracking
- Conflict directed backjumping
- DFLR or BF search strategy
- Rule applications are controlled via rule priorities
- Equality reasoning via backward and forward rewriting

# DFLR and BF Search Strategies

- Option for depth-first left-to-right search strategy:

```
branch.selection.strategy = \  
    mettel.core.tableau.MettelSimpleLIFOBranchSelectionStrategy
```

- Some logics, e.g.  $ALBO^{id}$ , require fair branch selection strategy for termination.
- Option for breadth-first search strategy:

```
branch.selection.strategy = \  
    mettel.core.tableau.MettelSimpleFIFOBranchSelectionStrategy
```

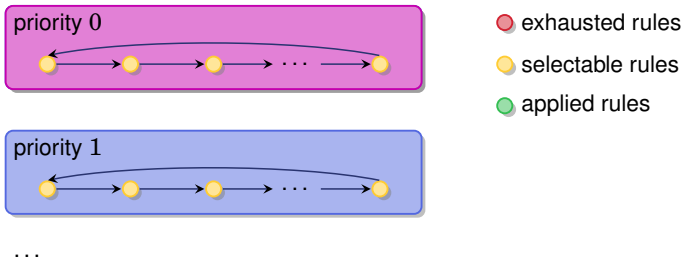
- Other implementations of the interface `MettelBranchSelectionStrategy` are allowed.



# Generic Protocol for Rule Priorities

- A rule can be selected only if the queue of expressions associated with the rule is not empty (rule is *selectable*).
- Selection must be fair within each priority group: if a rule is selectable then it will be selected eventually.
- A rule can be selected only if all rules with smaller priority values (higher priority) are not selectable.
- Note that selectable rule is not necessarily applicable.

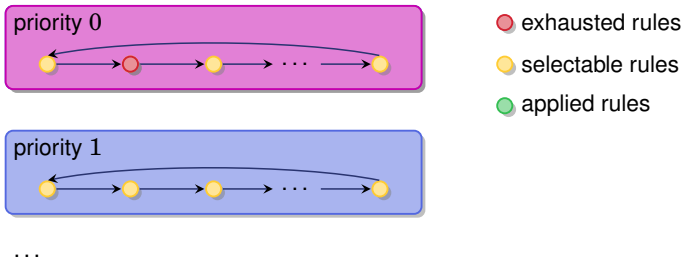
# Implemented Rule Selection Algorithm



## Example

In order to achieve that the closure rule is applied immediately after any new information is added to a branch assign to the closure rule the priority value 0 and to all other rule values higher than 0, e.g., 1.

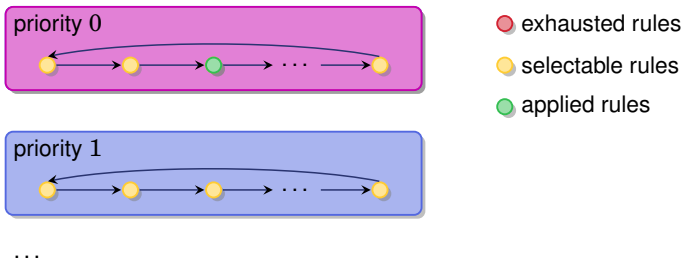
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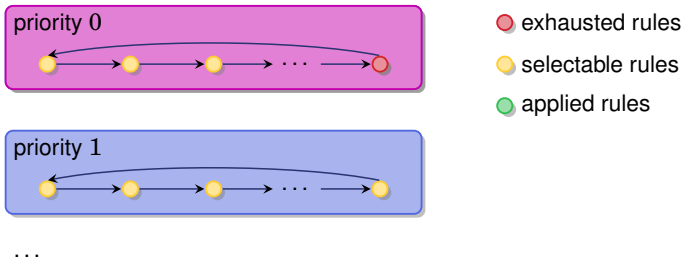
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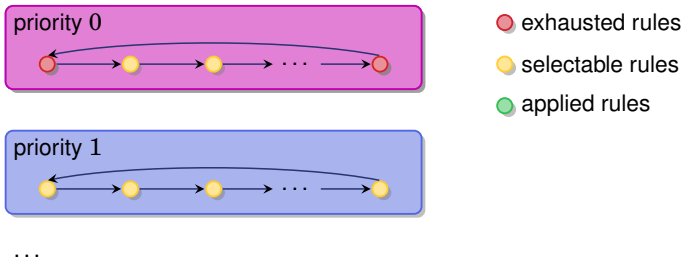
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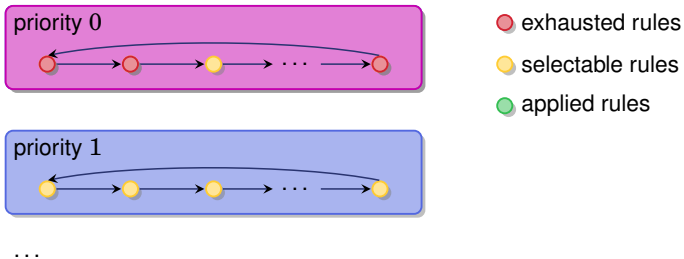
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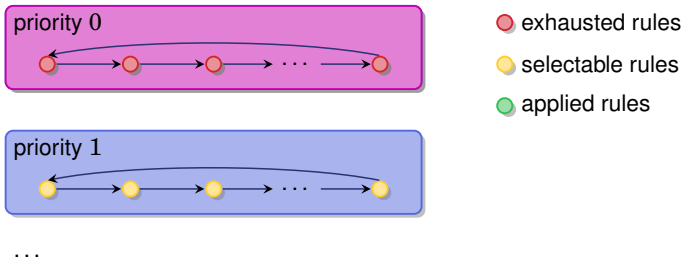
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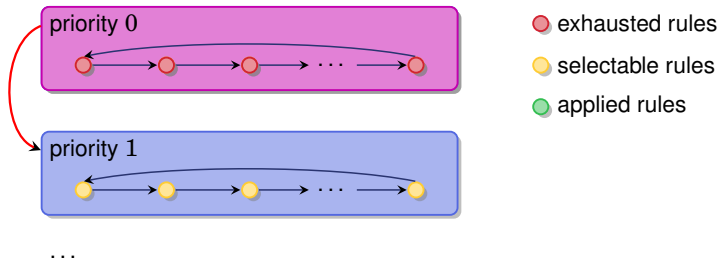


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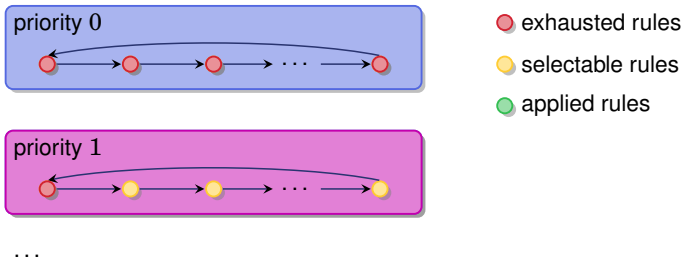
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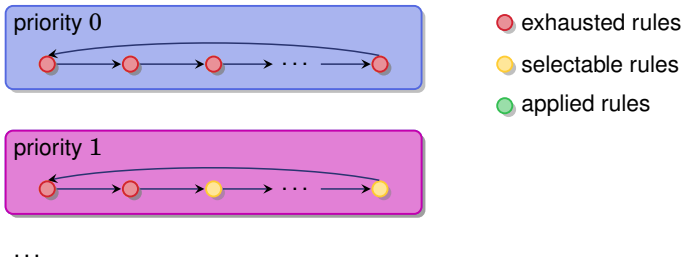
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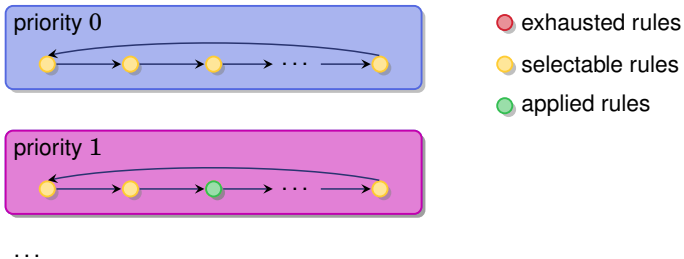
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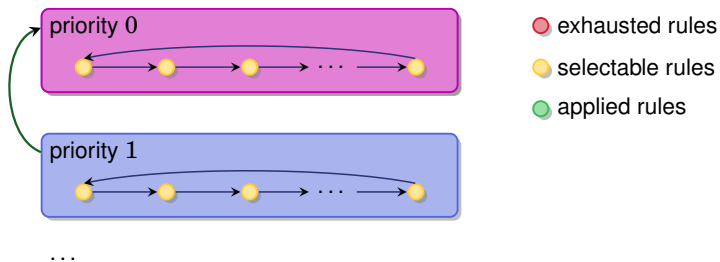
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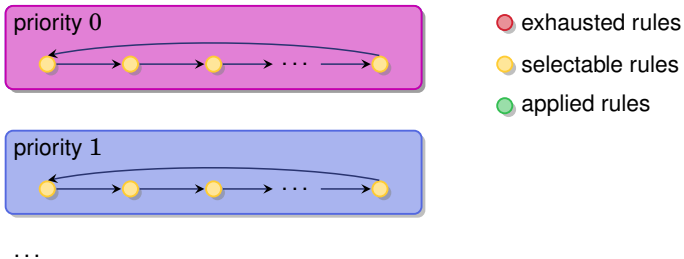
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# A Guideline for Choosing Rule Priorities

Given two rules  $\rho$  and  $\rho'$ , set priority value of  $\rho$  less than the priority value of  $\rho'$  if

- $\rho$  has lower branching factor than  $\rho'$ , or
- branching factors of  $\rho$  and  $\rho'$  are equal but  $\rho$  has less premises.

Additional optimisation is possible, if each set of expressions has a complexity measure.

## Example

```

tableau S4{
  @i ~(~P) / @i P priority 1; //Double–negation removal
  @i(P|Q) / @i P || @i Q priority 3; //Disjunction rule
  @i~(P|Q) / @i~P @i~Q priority 1; //"Conjunction" rule
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# Why Rewriting?

- Considerably simplifies derivations:

- Exhaustive rewriting:

$$\text{if } f(i, P) \xrightarrow{\mathcal{R}} i \text{ then } f(f(f(i, P), P), P) \xrightarrow{\mathcal{R}} i.$$

- On-the-fly simplification — forward rewriting.
- Rewrite system can be efficiently maintained.
- Equality reasoning is required for the generic blocking mechanism.

# Equality Reasoning via Forward and Backward Rewriting

- A lexicographic path ordering  $\prec$  on all expressions of current tableau branch.
- Rewrite relation  $\mathcal{R}$ .
- Forward rewriting: every new expression in a branch is rewritten wrt  $\mathcal{R}$ .
- Backward rewriting is triggered by equality expressions:
  - every equality expression  $E(\alpha, \beta)$  in current branch is oriented wrt  $\prec$ , and
  - either  $\alpha \xrightarrow{\mathcal{R}} \beta$  or  $\beta \xrightarrow{\mathcal{R}} \alpha$  is added to  $\mathcal{R}$ , and
  - all the expressions in the branch are rewritten wrt  $\mathcal{R}$ .

# Specifying Equality Expressions

- Option `equality.keywords`:

Default: `equality.keywords = {equality}`

```
options{  
  equality.keywords = {equality, equivalence}  
}
```

- Corresponding name in a binary connective specification:

```
syntax SomeLogic{  
  ...  
  formula equality = ' [ ' nominal '=' nominal ' ] ' ;  
  formula equivalence = formula '<->' formula ;  
}
```

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# Blocking Mechanisms

- There are periodicities in tableau derivations.
- They are usually detected with some form of loop checking mechanisms:
  - subset or equality blocking,
  - ancestor or anywhere blocking,
  - static or dynamic blocking,
  - pattern-based blocking, etc.
- The standard loop-checking mechanisms are tied to particular logics.
- In METTEL<sup>2</sup>, a generic blocking mechanism, called **unrestricted blocking mechanism**, can be specified.

# Unrestricted Blocking Rule

$$(ub): \frac{}{x \approx y \mid x \not\approx y}$$

## Termination Condition

In every open branch there is some node from which point onwards, all possible applications of the (ub) rule have been performed before any application of any term-generating rule.

- Early blocking:

Standard loop checking:



- Relies on tableau backtracking
- Provides termination for logics with FMP

# Unrestricted Blocking Rule

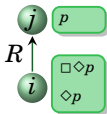
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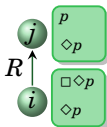
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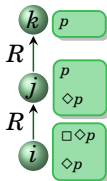
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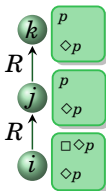
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# Unrestricted Blocking Rule

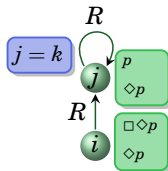
$$(ub): \frac{}{x \approx y \mid x \not\approx y}$$

## Termination Condition

In every open branch there is some node from which point onwards, all possible applications of the (ub) rule have been performed before any application of any term-generating rule.

- Early blocking:

Standard loop checking:



- Relies on tableau backtracking
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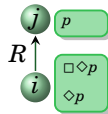
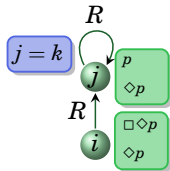
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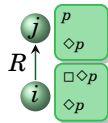
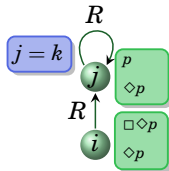
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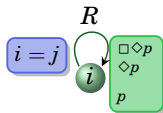
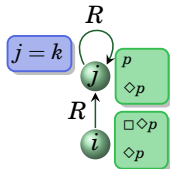
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## Specifying the Unrestricted Blocking Mechanism

- Ensure that equality reasoning is turned on.
- Append equality expressions to the syntax specification.
- Specify the unrestricted blocking rule:

```

tableau S4{
  ...
  @i P @j Q / [i=j] || ~[i=j];
}

```

- Satisfy the termination condition by using rule priorities:

```

tableau S4{
  ...
  @i<>P / R(i,f(i,P)) @f(i,P)P priority 7; //Diamond rule
  ...
  @i P @j Q / [i=j] || ~([i=j]) priority 6; //Blocking rule
}

```



# Example

```

syntax S4{
  sort formula, world;
  formula false = 'false';
  formula negation = '~' formula;
  formula diamond = '<>' formula;
  formula at = '@' world formula;
  formula disjunction = formula '|' formula;
  formula equality = '[' world '=' world ']' ; //Equality
  formula relation = 'R' '(' world ',' world ') ' ; //Relation
  world f = 'f' '(' world ',' formula ') ' ; //Skolem function
}

tableau S4{
  @i ~(~P) / @i P priority 1; //Double-negation removal
  @i(P|Q) / @i P || @i Q priority 3; //Disjunction rule
  @i~(P|Q) / @i~P @i~Q priority 1; //"Conjunction" rule
  @i<>P / R(i,f(i,P)) @f(i,P)P priority 7; //Diamond rule
  @i~(<>P) R(i,j) / @j~P priority 2; //"Box" propagation rule
  @i P / R(i,i) priority 1; //Reflexivity
  R(i,j) R(j,k) / R(i,k) priority 2; //Transitivity
  @i P @i~P / priority 0; //Closure rule
  R(i,j) / [i=j] || ~([i=j]) priority 6; //Ancestor blocking rule
  ~([i=j]) / priority 0; //Closure rule for inequality
}

```

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# Labelled Tableaux

```

sort labelledFormula, formula, label;
...
labelledFormula at = '@' label formula;
...

```

## Example

```

syntax S4{
  sort tableauFormula, formula, world;
  formula false = 'false';
  formula negation = '~' formula;
  formula diamond = '<>' formula;
  tableauFormula at = '@' world formula;
  formula disjunction = formula '|' formula;
  tableauFormula equality = '[' world '=' world ']' ; //Equality
  tableauFormula inequality = '~' '[' world '=' world ']' ; //Equality
  tableauFormula relation = 'R' '(' world ',' world ') ' ; //Relation
  world f = 'f' '(' world ',' formula ') ' ; //Skolem function
}

```

## Signed Tableaux

```

sort signedFormula, formula;
...
signedFormula trueValue = 'T' formula;
signedFormula falseValue = 'F' formula;
...

```

## Example

```

syntax Int{
  sort formula, individual;
  formula false = 'false';
  formula trueValue = 'T' formula;
  formula falseValue = 'F' formula;
  formula at = '@' individual formula;
  formula negation = '~' formula;
  formula conjunction = formula '&' formula;
  formula disjunction = formula '|' formula;
  formula implication = formula '->' formula;
  individual successorImp = 'F' '(' individual ',' formula ',' formula ')';
  formula relation = 'R' '(' individual ',' individual ')';
  formula equality = '[' individual '=' individual ']';
}

```

# Embedding Sorts

**sort** formula, proposition;

...

formula proposition = '# ' proposition;

...

**syntax** IEL{

**sort** formula, individual, prop, agent;

formula true = 'true';

formula false = 'false';

formula singleton = '{ ' individual ' }';

formula atom = '# ' prop;

formula negation = '~ ' formula;

formula diamondQ = '<q>' agent formula;

formula diamondK = '<k>' agent formula;

formula diamondX = '<x>' agent formula;

formula at = '@' individual formula;

formula query = '[ ? ' formula ' ]' agent formula;

formula resol = '[ ! ]' agent formula;

formula disjunction = formula ' | ' formula;

formula equality = '[ ' individual '=' individual ' ]';

individual fq = 'fq' (' individual ', ' agent ', ' formula ');

individual fk = 'fk' (' individual ', ' agent ', ' formula ');

individual fx = 'fx' (' individual ', ' agent ', ' formula ');

}

# Eliminating Expressions

## Example

```

options{
  ...
  equality.keywords={equality,equivalence}
}
syntax LTL{
  ...
  formula equivalence = formula '<->' formula;
  ...
  formula eventuality = 'E' ' (' world ' , ' formula ' ) ' ;
}
tableau LTL{
  ...
  @i (<>P) / @i (E(i,P)) priority 1;
  @i (E(j,P)) / @i P ((E(j,P)) <-> (<>P)) || @f(i) (E(j,P)) priority 7;
  ...
  @i E(j,P) / priority 8;//"Bad" loop check
}

```

# Reducing Rule Branching Factor

Move negated conclusion to premises if it cannot be instantiated to become “non-atomic”.

$$\frac{}{\neg R(i, j) \mid \neg R(j, k) \mid R(i, k)}$$

$$\frac{@_i \neg \diamond p}{\neg R(i, j) \mid @_j \neg p}$$

# Reducing Rule Branching Factor

Move negated conclusion to premises if it cannot be instantiated to become “non-atomic”.

$$\frac{R(i,j), R(j,k)}{R(i,k)}$$

$$\frac{@_i \neg \diamond p}{\neg R(i,j) \mid @_j \neg p}$$



# Reducing Rule Branching Factor

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# Reducing Rule Branching Factor

Move negated conclusion to premises if it cannot be instantiated to become “non-atomic”.

$$\frac{R(i,j), R(j,k)}{R(i,k)}$$

$$\frac{@_i \neg \diamond p, R(i,j)}{@_j \neg p}$$

# Tabular Logics

**specification** Lukasiewicz3;

**syntax** Lukasiewicz{

**sort** valuation, formula;

valuation true = 'T' formula | unknown = 'U' formula | false = 'F' formula;

formula true = 'true' | false = 'false';

formula negation = '~' formula;

formula conjunction = formula '&' formula;

formula disjunction = formula '|' formula;

formula implication = formula '->' formula;

}

**tableau** Lukasiewicz{

T P F P / **priority 0** \$; T P U P / **priority 0** \$;

U P F P / **priority 0** \$; U P F P / **priority 0** \$;

T ~P / F P **priority 1** \$; U ~P / U P **priority 1** \$; F ~P / T P **priority 1** \$;

T (P & Q) / T P T Q **priority 2** \$; F (P & Q) / F P \$ | F Q **priority 1** \$;

U (P & Q) / T P U Q \$ | U P T Q \$ | U P U Q **priority 3** \$;

T (P | Q) / T P \$ | T Q **priority 2** \$; F (P | Q) / F P F Q **priority 1** \$;

U (P | Q) / F P U Q \$ | U P F Q \$ | U P U Q **priority 3** \$;

F (P -> Q) / T P F Q **priority 1** \$; U (P -> Q) / U P F Q \$ | T P U Q **priority 2** \$;

T (P -> Q) / T Q \$ | F P \$ | U P U Q **priority 3** \$;

T false / **priority 0** \$; U false / **priority 0** \$;

U true / **priority 0** \$; F true / **priority 0** \$;

}

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# Options for Random Problem Generators

Copy `S4RandomExpressionGenerator.properties` from `output/S4/etc` and edit it.

```
world.f.frequency = 1

world.variable.frequency = 1
world.depth = 1
world.variables = i, j, k
world.variables.number = 3

world.generate = 0

world.top.connectives =

formula.false.frequency = 1
formula.negation.frequency = 1
formula.diamond.frequency = 1
formula.at.frequency = 1
formula.disjunction.frequency = 1
formula.worldEquality.frequency = 1
formula.relation.frequency = 1

formula.variable.frequency = 1
formula.depth = 10
formula.variables = p, q, r
formula.variables.number = 3

formula.generate = 1000

formula.top.connectives = at
```

# Generating Random Problems and Benchmarking

- Generate problems

```
> java -cp S4.jar:mettel2-util.jar \  
S4.language.S4.util.S4RandomExpressionGenerator \  
-p S4RandomExpressionGenerator.properties
```

- Run a benchmark

```
> java -cp S4.jar:mettel2-util.jar:opencsv.jar \  
S4.language.S4.util.S4Benchmark \  
-d random_problems
```

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# Summary

- METTEL<sup>2</sup> is easy-to-use and flexible tool
- METTEL<sup>2</sup> allows specification of various types of tableau calculi
- Case studies:
  - Boolean logic
  - S4
  - IPC
  - *ALCO*
  - *ALBO*<sup>id</sup>
  - *SHOI*
  - LTL and temporal logic with cardinality constraints
  - Tree-valued Łukasiewicz logic
  - K with global counting operators
  - Interrogative epistemic logics
  - etc



# Outlook

- Local aims:
  - Further optimisations of generated provers.
  - More options to allow the user to control the generation process.
  - Extending specification languages.
  - Increase level of abstraction of the tableau core.
- An *ultimate goal* is to enable automatic generation of provers from other definitions of logics. In particular:
  - Implement the tableau synthesis framework for synthesis of tableau calculi from semantics of logics.
  - Investigate a possibility to generate tableau provers from Hilbert axiomatisations.



*Thank You!*

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