University of Manchester

CS3282 : Digital Communications

Section 4: Introduction to digital transmission

- Transmission of digital signals by suitably shaped pulses over wire-lines or radio channels.
- Such pulses often visualised as being rectangular in shape.
- Not unrealistic for base-band transmission over short distances.
- However, a rectangular pulse shape requires infinite bandwidth
- Undesirable for transmission over a wire-line or channel where economy of bandwidth utilisation is important.
- Usually important with long distance high speed transmission.
- More typical pulse shape is rounded with ringing before & after main part of the pulse to reduce its bandwidth.

Data (bit-) rate and signalling rate:

- Data-rate or bit-rate is number of bits per second.
- Signalling-rate is number of 'symbols' per second [baud-rate]
- Symbol is voltage pulse chosen from 2 or more possibilities.
- With binary signalling, there are 2 possible symbols,
- Say rectangular pulses of amplitude +V or -V.
- Then signalling (baud) rate can be equal to data rate .
- Ternary signalling is used with +V, 0 & –V pulses.
- Now signalling(baud) rate can be less than bit-rate.
- Could send 3-bits using 2 ternary symbols.
- Hence bit-rate could be 1.5 times the signalling (baud) rate.
- Quaternary signalling has 4 possible symbols.
- Symbol period = T seconds, & signalling-rate = 1/T baud.

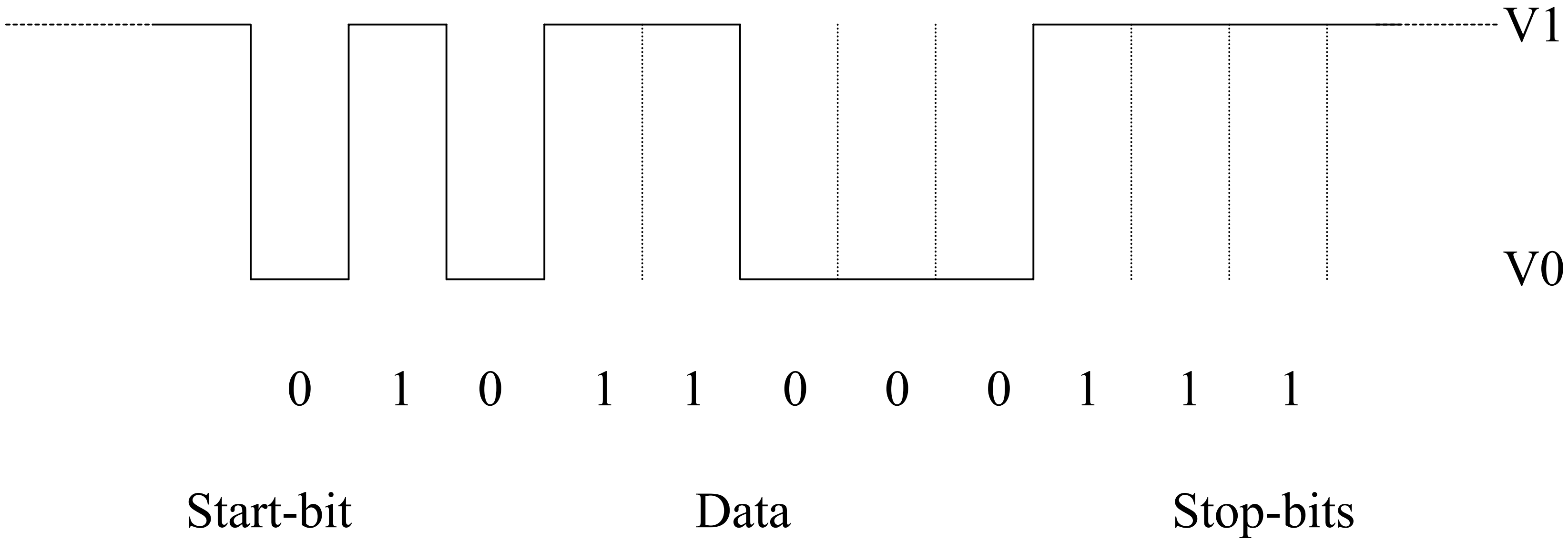
Asynchronous transmission (low data rates):

- Transmitted symbols synchronised to a timing waveform or clock which is normally not transmitted.
- Receiver must extract a clock signal from received signal.
- For lengthy transmissions, receiver clock must be accurately matched to transmitter clock allowing for channel delay.
- Any small discrepancy will accumulate to large timing errors.
- For short transmissions, e.g. of 8-bit numbers between computers & peripherals over short distances, clocks need only be approximately matched.
- They may resynchronise at the beginning of each short block.
- This is 'asynchronous' transmission & basis of RS232.

Asynchronous transmission (cont)

- Data sent in short words with synchronising start & stop bits.
- Receiver clock resynchronises itself at each start-bit.
- Consider transmission of 8-bit ASCII characters by RS232.
- When idle, the line remains high at voltage V_1.
- A start-bit of ‘0’ is sent to signify start of a transmission.
- The eight bits of data are then transmitted using “non-return to zero” (NRZ) pulses.
- Finally a number of “1” stop-bits are transmitted to ensure that next character is not sent immediately.

Asynchronous transmission (cont)



- **A**dvantage is simplicity of transmitter & especially receiver.
- Disadvantage is that it is inefficient in its utilisation of the channel capacity.
- Receiver waits for transition from 'idle state' "1" to "0” indicating a 'start bit'.
- Delays for half a symbol period according to its own clock with approx same frequency as transmitter clock
- Then samples channel eleven times at intervals of T seconds.
- Samples will lie close to centre of each symbol,
- Timing will drift over the 11 samples
- Acceptable because of the frequent resynchronisation.

Synchronous transmission:

- For efficient transmission of continuous data for long periods of time, often at data rates close to maximum possible over a channel of specified bandwidth.
- Synchronising code (10101010) sent at start of transmission.
- Thereafter, receiver clock kept synchronised in frequency & symbol-timing from the transmission itself.
- To achieve required bandwidth efficiency, pulses shaped.
- Use a filter whose impulse-response is the required shape.
- To detect presence or absence of a pulse, receiver samples received waveform at the correct symbol timing points.

Base-band synchronous transmission over wire-lines:

Two factors must be borne in mind:-

(i) Keep average voltage level as close as possible to zero since any voltage offset carries no data & just wastes power.
In many cases, DC component of a signal lost over wire lines

(ii) Ensure that signal always has a frequency component at the signalling rate (or an exact multiple or sub-multiple of it) to allow a timing waveform to be extracted at the receiver for synchronising the detection process.

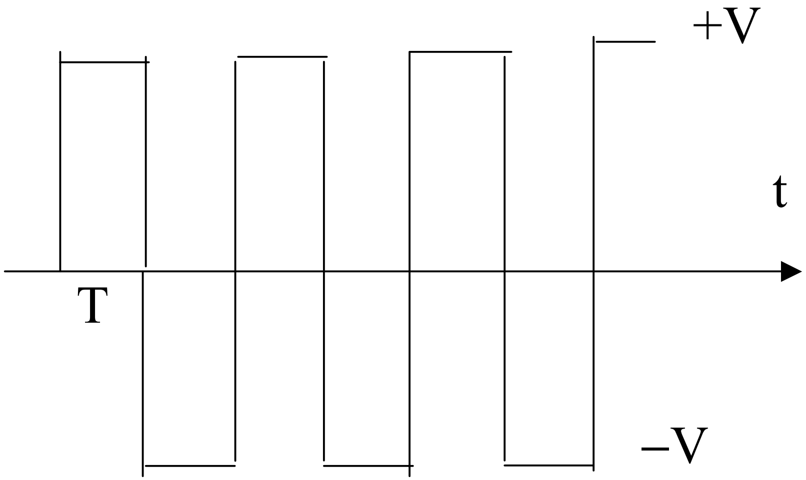
Two approaches which do not work well

- Try to achieve zero average voltage by using +V & -V, hoping that, on average, same number of ones & zeros will occur.
  Long sequence of '0 0 0 0 ' or '1 1 1 1 1 ... 1' would fail.

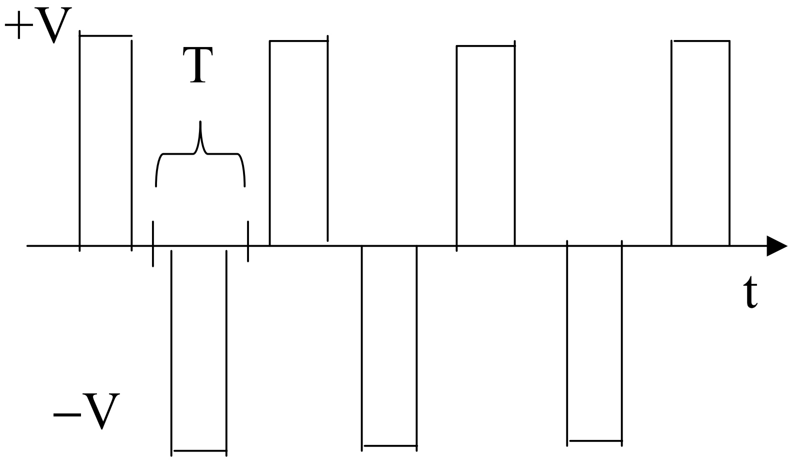
- Use ternary coding with alternate mark inversion (AMI)
  i.e. send 0 volts for "0" & ±V volts alternately, for logic "1".

To transmit: ' 1, 0, 1, 1, 0 , 1, 1 ' ,:
we send V, 0, -V, V, 0, -V, V

Average voltage ("dc level") now guaranteed to be zero.
Timing waveform extraction easy for '1 1 1 1 1 1 1 ...'
However for '0 0 0 0 0 0 .... 0' , receiver can lose synchronisation as received signal will be zero for a while.



'..1111111..' by NRZ AMI



..1111111' by RZ AMI

HDB3 coding: (high density bipolar, order 3):

- Uses ternary coding to send binary coded data, as for AMI, but places incorrectly signed pulse in place of any 4th consec. zero.

For ' 1 1 1 0 0 0 0 0 0 0 0 0 1 0 1 … '

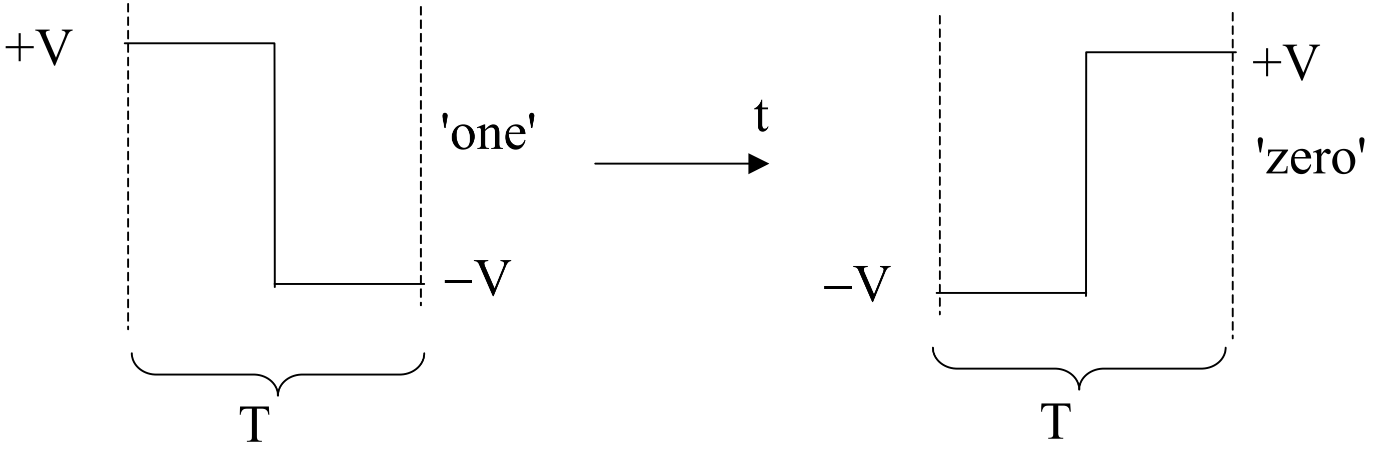
' +V -V +V 0 0 0 **+V** 0 0 0 **+V** 0 -V 0 +V … '

- Incorrect "**+V**" pulse included only for clock synchronisation.
- It is taken to be a “0” at the receiver.
- Average voltage is no longer zero in the short block above, but over a longer time-span the average will still remain zero since incorrect +V pulses and -V pulses will occur equally often.

Other base-band signalling waveforms (line codes):

NRZ-AMI, RZ-AMI, NRZ-HDB3 and RZ-HDB3 commonly used.
NRZ-L (level) is straightforward with +V for 1 & -V for 0.
NRZ-M (mark) has 1 represented by change & 0 by no change.
NRZ-S (space) represents 0 by a change, and 1 by no change.
Uni-polar RZ has binary 'return to zero' pulses (0 and +V).
Bi-polar RZ has +V & -V 'return to zero' pulses.
Bi-phase-L, bi-phase-M & bi-phase-S used in magnetic recording systems, optical communications, satellite links & Ethernet.

Bi-phase-L is “Manchester coding” as shown below:



Manchester coding

Question: What are the advantages and disadvantages of Manchester coding as compared with NRZ-HDB3?

4B3T coding: (4-bits re-coded as 3 ternary digits):

- In 4 bits, have 16 possible numbers.
- In 3 ternary digits have 27 possible numbers.
- Represent each 4-bit number by a 3 ternary digit number,
  & have some 3-ternary digit numbers left over.
- Allocate alternative codes to some binary numbers & use these
  - (i) to keep average signal level zero
  - (ii) to ensure significant carrier content.
- The ternary codes are:

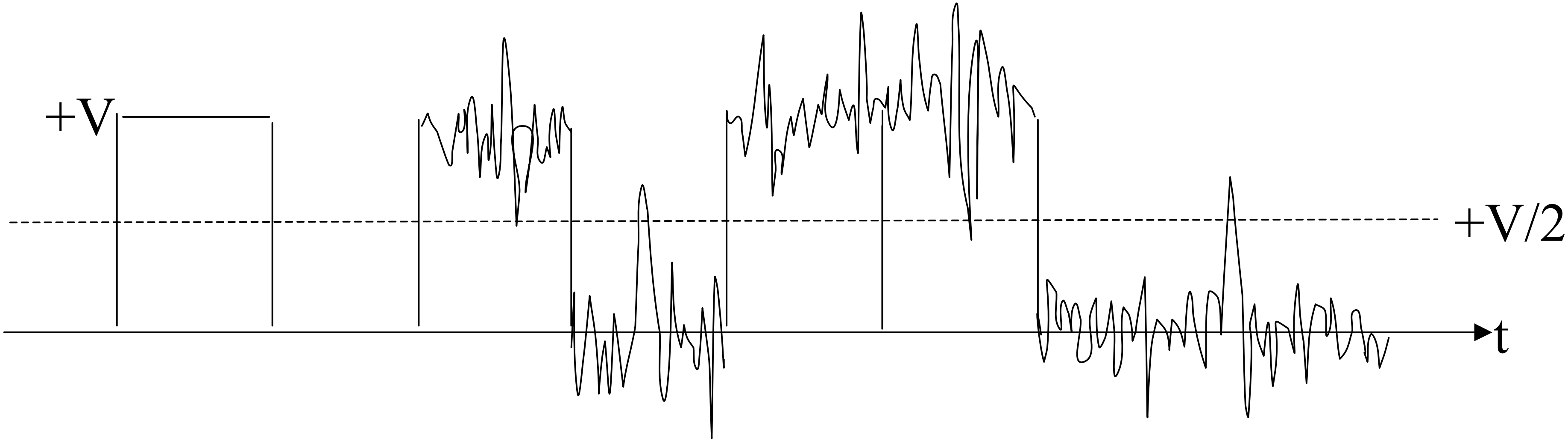
BINARY	(a) TERNARY	(b)
0000	– – –	+ + +
0001	– – 0	+ + 0
0010	– 0 –	+ 0 +
0011	0 – –	0 + +
0100	– – +	+ + –
0101	– + –	+ – +
0110	+ – –	– + +
0111	– 0 0	+ 0 0
1000	0 – 0	0 + 0
1001	0 0 -–	0 0 +
1010	0 + –	
1011	0 – +	
1100	+ 0 –	
1101	– 0 +	
1110	+ – 0	
1111	– + 0	

- Represent +V by "+", –V by "–", & “0 volts” by “0”.
- Choose either column (a) or (b) where there is a choice.
- When decoded they give same sequence of 4-bits.
- If "accumulated disparity" is "+"choose column (a) to redress balance. Otherwise choose column (b).
- Given same pulse shaping, 4B3T requires less bandwidth than AMI as it makes better use of the 3 levels –V, 0 and +V
- If pulses of width T shaped so that bandwidth is 2/(3T) Hz, AMI has 'bandwidth efficiency' of 1/T b/s in 2/(3T) Hz i.e. 1.5 b/s per Hz.
- 4B3T would have 4/(3T) b/s in 2/(3T) Hz, i.e. 2 b/s per Hz.

- Assuming "accumulatd disparity" to be 0 at start, encode in 4B3T:
  0010, 0000, 1111, 0001, 0001, 0001, ...
- Answer: + 0 +, - - -, - + 0, + + 0, - - 0, + + 0, ...

Estimation of 'bit-error probability' using Q(z)

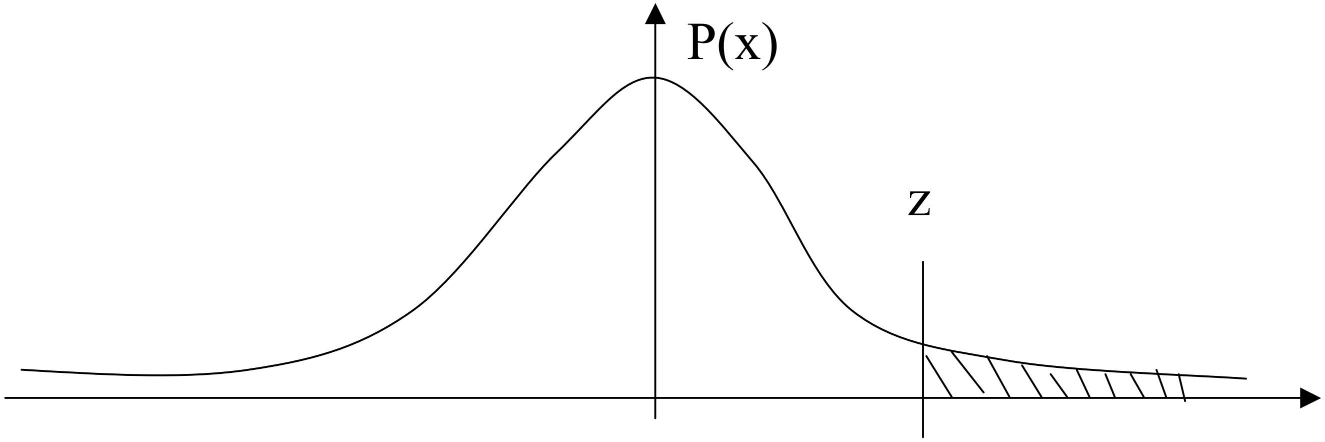
- Channel & receiver may be assumed to add Gaussian noise of zero mean & fixed variance σ^2 to transmitted signal.
- Receiver of +V & 0 volt rect pulses may set threshold at +V/2.
- Decide whether this is exceeded at sampling points in centre of each rectangular pulse.
- If amplitude of noise exceeds +V/2 error may occur.



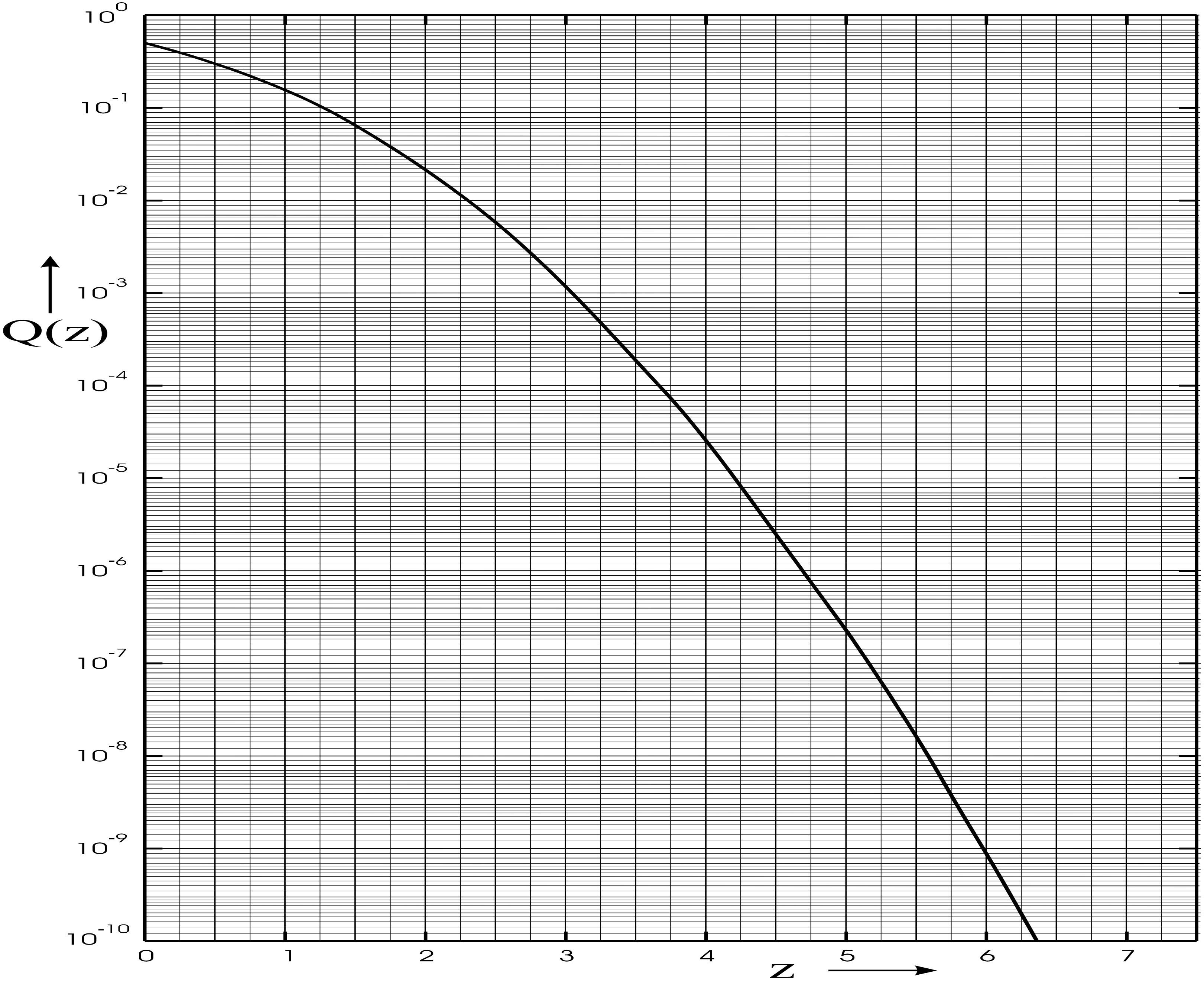
- Probability of Gaussian noise sample with zero mean & variance $\sigma^2=1$ being greater than some voltage z is

$$Q(z) = \int_z^\infty p(x)dx$$

- $p(x) = (1/(\sqrt{(2\pi)}))\exp(-x^2/2)$ is pdf of unit variance ($\sigma^2 = 1$) Gaussian noise as plotted below.



Q(z) is plotted as a graph against z in figure attached.


Q(z)
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
0
1
2
3
4
5
6
7
z

- For variance σ^2, prob of noise sample exceeding z is $Q(z/\sigma)$.
- Therefore probability of a sample exceeding A/2 is $Q(A/(2\sigma))$

.

- Q(z) may be obtained from erfc,

$$Q(z) = 0.5 \text{ erfc}(z / \sqrt{2})$$

- or the following approximation:

$$Q(z) \approx (0.4 / z) \exp(-z^2 / 2) \quad \text{when } z > 3$$

Example 4.1:

A receiver receives +1 & 0 volt pulses with Gaussian noise of variance $\sigma^2 = 0.01$. Estimate bit-error rate assuming a 0.5 volt threshold & an equal number of 1s and 0s.

Solution:

Bit error probability is:

prob(noise > 0.5 v)×prob ("0") + prob(noise< -0.5) × prob ("1")

= prob(noise > 0.5 v)×0.5 + prob(noise< -0.5 v)×0.5

= prob(noise > 0.5 v)

= $Q(0.5/0.1) = 3 \times 10^{-7}$.

[This is bit-error probability].

Bit-error rate = 1 in 0.33×10^{7}.

Example 4.2:

A receiver receives +1 volt & 0 volt pulses and has a bit- error probability of 10^{-3}. What is the variance of the received noise?

Solution:

$Q(0.5/\sigma) = 10^{-3}$.

From graph, $0.5/\sigma \approx 3.2$

Therefore, $\sigma = 0.16$, $\sigma^2=0.026$.

There is often little one can do to reduce the noise level, and hence, if we need to decrease the error probability, our only option is to increase the signalling pulse amplitude as it reaches the receiver.

This can be done by increasing the gains of the regenerative repeaters, or, if this is not possible because it would increase the level of interference (cross-talk), the repeater spacing may be reduced.

Example 4.3:

A synchronous transmission system using cable with 30 dB attenuation per km and regenerative repeaters every 5 km is affected by additive Gaussian noise.
A binary bipolar line-code is used with equally spaced detection thresholds at the sampling points.
If the repeater gains cannot be increased because of the interference this would cause to other lines, how can the error probability be reduced from its current value of 10^{-5} to 10^{-7} PER REPEATER.
(Use the graph of Q(z) against z).

Solution:

Each regenerative repeater has receiver & re-transmitter.
At each sampling point, repeater gets +V or –V with AWGN .
Must decide whether "0" or "1" was intended & re-transmit perfectly shaped symbol.
Take threshold to be 0 volts.
Probability of an error is:
(Prob of a "0")x (Prob of noise sample being greater than +V)
+ (Prob of "1") x (Prob of noise sample being less than –V))
We assume an equal probability of "1"s and "0"s.
Therefore probability of an error is:

$$0.5 \times Q(V/\sigma) + 0.5 \times Q(V/\sigma)$$

where σ^2 = variance of the noise.
Prob(noise sample < –V) is same as prob(noise sample > V).

- $Q(z/\sigma)$ = prob (Gaussian noise sample > z) when noise has zero mean & variance σ^2.
- If $Q(V/\sigma) = 10^{-5}$, from graph we find that $V/\sigma = 4.25$
- Therefore $\sigma = V/4.25$.
- This is standard deviation of noise that is causing the errors.
- With same noise level, to increase error probability by 10^{-7} rather than 10^{-5}, each regenerative repeater must receive higher voltage levels for the symbols.
- . Assume received amplitudes raised from ±V to ±U at the sampling points.
- If $Q(U/\sigma) = 10^{-7}$, from the graph, we find that $U/\sigma = 5.2$.
- This means that $U = 5.2\sigma = (5.2/4.25)V = 1.22$ V.
- $20\log_{10}(1.22) = 1.75$dB.
- Therefore we need to arrange that the voltages as received at the input of each regen repeater are raised by 1.75 dB.

- This could be achieved by increasing the output from each repeater by 1.75 dB.

- If we cannot raise the voltages transmitted by the regenerative repeaters because of cross-talk, we can only reduce the distance between the repeaters so that less attenuation occurs between one regenerative repeater and the next.

- Attenuation must be reduced from 5x30 = 150 dB to 148.25 dB. Distance between repeaters must be reduced to 148.25/30 = 4.94 km instead of 5 km.

- Wow!

Problems

4.1. How would bit-error rate be affected by sampling at end or beginning of each rectangular pulse rather than in the centre.

4.2. How would the bit-error rate be affected by sampling each rectangular pulse 3 times & averaging?

4.3: A mobile phone receives a signal over "line of sight" radio (no reflections) from a base station and is affected by AWGN. Received power from base-station decreases with increasing distance according to an "inverse square power law", i.e. P(d) is proportional to $1/d^2$ where P(d) is the received power at distance d. Binary bipolar signalling is used with optimal detection threshold at the sampling points. If the distance from the base-station is currently 800 metres, how much nearer must we move towards it if the error probability is be reduced from its current value of 10^{-5} to 10^{-7}.

4.4. A signalling system has eight symbols which are rectangular pulses of amplitude -4, -3, -2, -1, 0, 1, 2, 3 volts. If the signalling rate is 10 kbaud, what is the maximum achievable bit-rate.

4.5. A system has a bit-rate of 32 kbits/second. How could you achieve this with a signalling rate of 1 kbaud.

4.6. What is the bandwidth efficiency of (i) AMI and (ii) 4B3T coding if binary or ternary pulses of duration T seconds require a bandwidth of 3T/4 Hz.