University of Manchester

CS3282: Digital Communications '06

Section 9: Multi-level digital modulation & demodulation

9.1. Introduction:

• So far, mainly binary signalling using ASK, FSK & PSK.

• Bandwidth efficiency up to 2 b/s/ Hz at base-band.

• Multiplying by carrier doubles bandwidth,

• Max bandwidth efficiency of binary ASK & PSK is 1 b/s/Hz.
i.e. about 3.1 kb/s over 300-3400 kHz telephone link.

• Less than one tenth of what we know to be achievable.

• With binary MSK & QPSK maximum is 2 b/s/Hz.

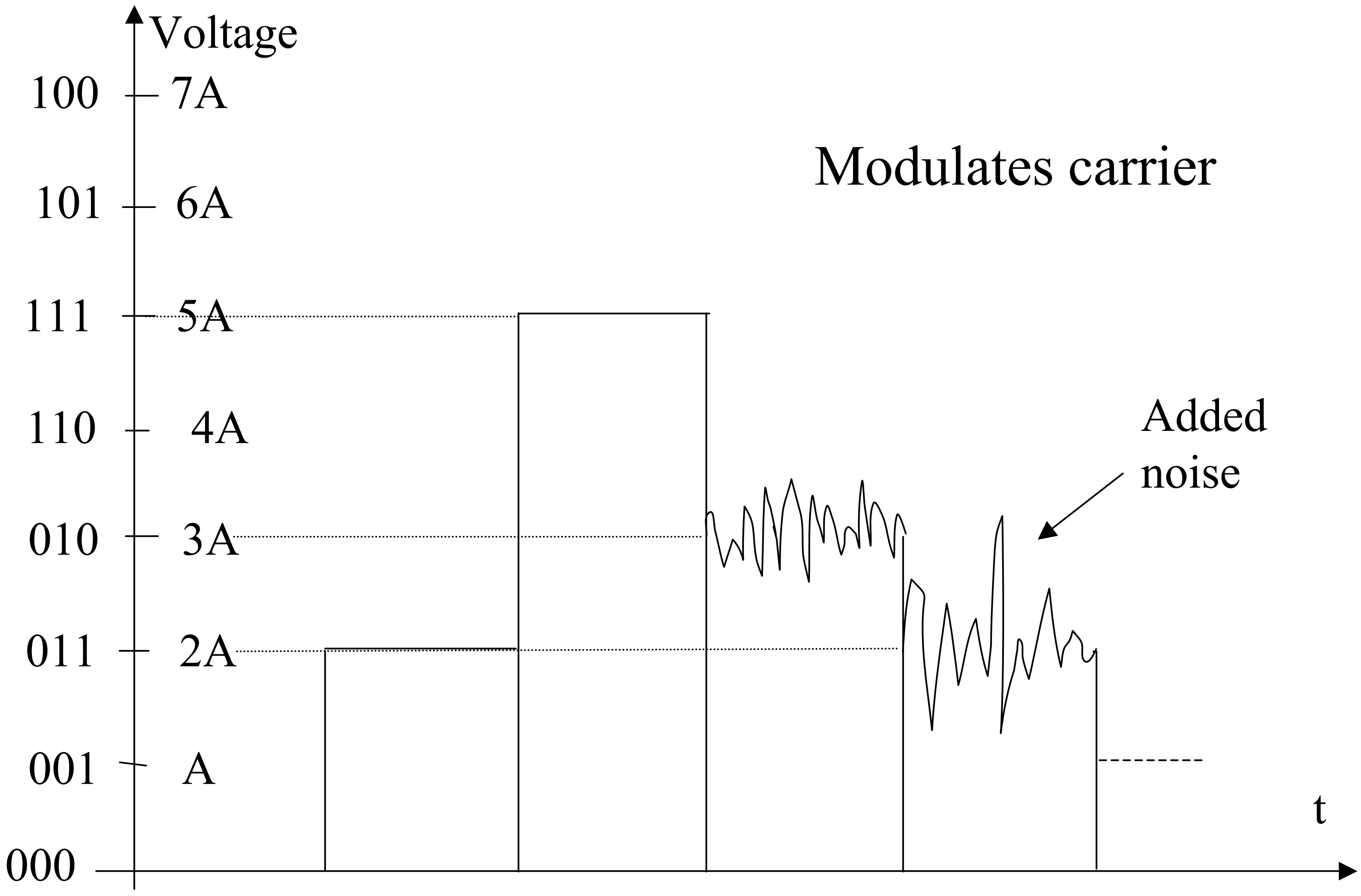
• To increase band-width efficiency further, use multi-level modulation where each symbol represents more than one bit.

9.2. Multi-level ASK & Gray coding:

- Most obvious multi-level technique, rarely used, is 'M-ary ASK'.
- Modulates amplitude of carrier with M different amplitudes.
- To encode N bits/symbol, choose $M=2^N$ rather than M=2.
- To encode 3 bits/symbol, could have 8 rect symbols of heights

  0, A, 2A, 3A, 4A, 5A, 6A & 7A for 000, 001, 010, 011, etc.
- Consider 'bit-error prob.', P_B , with this new signalling strategy.
- Compare with binary ASK with same voltage separation (A).
- Noise level giving 1 bit-error with binary signalling could change

  000 to 001 (1 bit-error) or 011 to 100 (3 bit-errors).
- Better to use a 3-bit "Gray-code" as follows.


Voltage
100
7A
101
6A
111
5A
110
4A
010
3A
011
2A
001
A
000
Modulates carrier
Added
noise
t

- Noise changing kA to (k-1)A or (k+1)A results in only 1 bit-error.
- Neglect possibility of noise changing 3A to 5A or A
- Then ‘ no. of bit-errors / s ‘ ≈ ‘no. of symbol errors /s’ .
- ∴ 'bit-error prob.' ≈ 'symbol error prob.' ÷ ‘no. of bits / symbol’.
- BER is number of bit-errors in a quantity of bits (e.g. 1 in 1000)
- Not ‘per second’.
- .Keeping same value of A for 8-ary ASK as for binary ASK increases power.
- To keep average power same, must reduce A.
- Increases P_B as voltage between levels reduces.
- M-ary reduces bandwidth for given bit-rate but increases P_B
- Note that noise power reduces as bandwidth reduces.

Detection of multi-level ASK

Assume that after demodulation, the input Z to the threshold detector has possible 8 levels: 0, A, 2A, 3A, ...

The detector must have 8 thresholds:

$Z<A/2$	: 000
$A/2 < Z < 3A/2$	: 001
$3A/2 < Z < 5A/2$	: 011
…	
$Z > 15A/2$	: 100

Matched filter detection for 8-ary ASK:

- Identical pulse shapes (e.g. rect) after demodulation.
- Amplitudes 0, A, 2A, …, 7A, for $s_0(t)$, $s_1(t)$, …, $s_7(t)$,
- Single matched filter $H((f))$ matched to $s_1(t)-s_0(t)$.
- Equal to $s_2(t)-s_1(t)$, $s_3(t)-s_2(t)$, etc.
- Responses of $H((f))$ to $s_0(t)$, $s_1(t)$, $s_3(t)$ ... are $a_0(t)$, $a_1(t)$…
- Output z(t) of matched filter sampled at t=T & decision made
- Define 7 thresholds $\gamma_{0.5} = (a_1(T)+a_0(T))/2$

  $\gamma_{1.5} = (a_2(T)+a_1(T))/2$, ...,

  $\gamma_{6.5}=(a_7(T)+a_6(T))/2$.

- With rect symbols of duration T, $\gamma_{0.5}= A^2T/2$, $\gamma_{1.5}= 3A^2T/2$, $\gamma_{2.5}= 5A^2T/2$, etc.

Received symbol is decided to be:

$s_0(t)$ if $z(T) < \gamma_{0.5}$

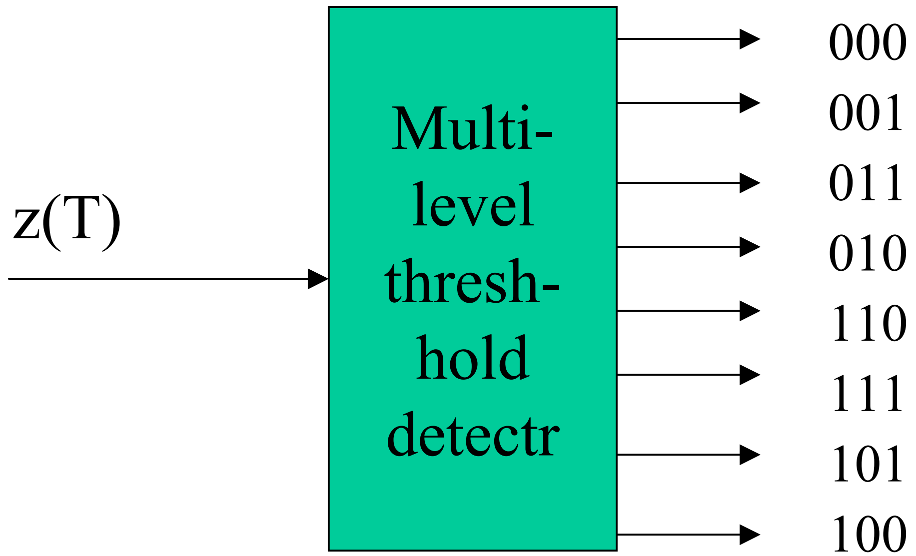
$s_1(t)$ if $\gamma_{0.5} \le z(T) < \gamma_{1..5}$

$s_2(t)$ if $\gamma_{1.5} \le z(T) < \gamma_{2..5}$

…

$s_6(t)$ if $\gamma_{5.5} \le z(T) < \gamma_{6..5}$

$s_7(t)$ if $\gamma_{6.5} \le z(T)$



Matched filter detection of QPSK

Apply 2 matched filters:

- to ‘in-phase’ signal after mult by $\cos(2\pi f_C t)$
- to ‘quad’ signal after x by sin

Exercise: A binary ASK transmitter & receiver communicate at 1000 b/s with $P_B = 10^{-9}$. To increase bit-rate to 3000 b/s over same channel, binary ASK is replaced by 8-ary ASK with same average transmitter power. Estimate new P_B. Assume there is no MF.

Solution:

With 0 and A rectangular pulses without a MF,

if $P_B = 10^{-9} = Q(A/(2\sigma))$, $A/(2\sigma) \approx 6$

Av power $= E_B \times (1/T) = (A^2/2)T/T = A^2/2$

With 8-ary ASK (0, B, 2B, ... 7B), av energy per symbol is:

$\mathrm{Av}\{0, B^2T, (2B)^2T, ..., (7B)^2T\} = (140/8)B^2 = 17.5B^2T$

So av power $= 17.5B^2$

Same av power, so $17.5B^2 = A^2/2$ & $B = A / 5.9$

Same noise power, so Prob of symbol error $\approx 2Q(B/(2\sigma))$

$= 2Q((A/35) / (2\sigma)) = 2Q(6/5.9) = 2Q(1.02) \approx 0.24$

Solution continued

Note that Prob of symbol error $\approx 2Q(B/(2\sigma))$ because a noise spike > B/2 or < -B/2 will cause a symbol error for 6 out of 8 symbols. It would be more accurate to replace 2 by (14/8) since the minimum voltage gives a symbol error only when a noise spike >B/2 and the max voltage only for -ve noise spike <-B/2.

Assuming Grey coding used, $P_B \approx$ (Symbol error prob) /3

$= 0.08$

Another exercise:

A binary ASK transmitter & receiver communicate at 1000 b/s with $P_B = 10^{-9}$. To reduce channel bandwidth required for same 1000 b/s, binary ASK is replaced by an 8-ary ASK with same transmitter power. Estimate new P_B.

- Advantage of multi-level signalling over binary is increase in bandwidth efficiency (b/s per Hz).
- Disadvantage is increase in P_B for given transmission power.
- Multi-level ASK is effective where noise is relatively low so that we can safely reduce voltage between detection thresholds without incurring large increases in bit-errors.
- Works with modem transmissions over telephone lines.

9.3. Orthogonality:

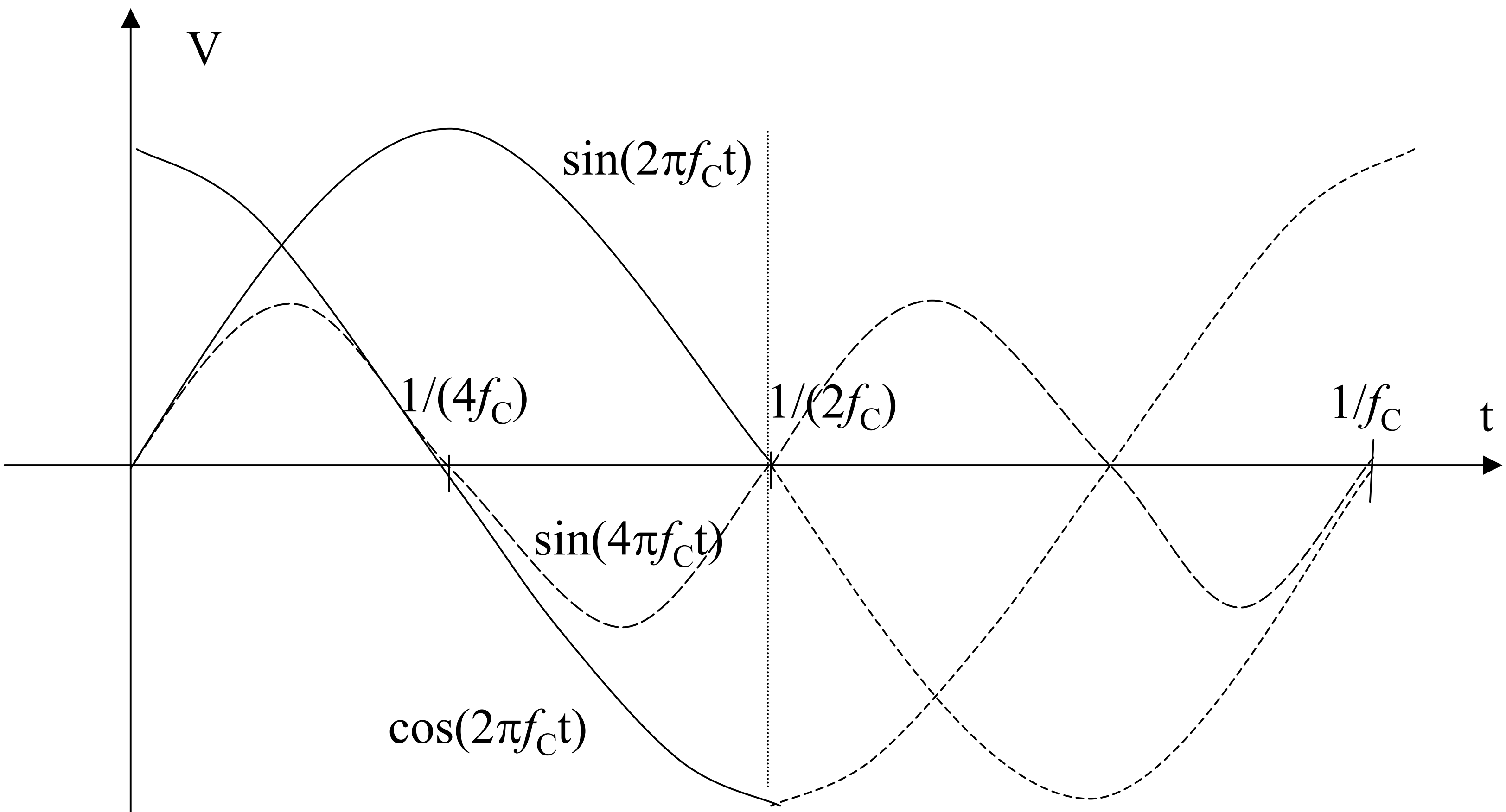
- Increase in P_B incurred by changing from binary to multi-level signalling avoided if additional bits are conveyed by signalling which is 'orthogonal' to the original binary signalling.
- Achieved with change from PSK to '4-phase' PSK (QPSK) .
- Can also be achieved with multi-level FSK.
- Orthogonality is definitely not present with multi-level ASK.

- Signals a(t) and b(t) are orthogonal over a period R seconds if:

$$\int_0^R a(t)b(t)dt = 0$$

- $\cos(2\pi f_C t)$ & $\sin(2\pi f_C t)$ orthogonal if R integer multiple of $1/(2f_C)$. When $R=1/(2f_C)$, R is duration of one half-cycle of the carrier.

Remember that $\cos(2\pi f_C t)\sin(2\pi f_C t) = 0.5\sin(4\pi f_C t)$
& examine following graph noting the equal area of $0.5\sin(4\pi f_C t)$ above & below t axis in range $0 < t < 1/(2f_C)$.
This also applies for the range $0 < t < 1/f_C$.



•QPSK can transmit 2 bits/ symbol without increasing P_B over what is obtained, for given E_B/N_0 , with binary PSK at 1 bit / symbol.

•Made possible by orthogonality of in-phase & quadrature components of sinusoidal carrier.

•Orthogonality requires suitable choice of T as integer multiple of $1/(2f_C)$ & detector that integrates over one or more symbol intervals.

• Use vector-demodulator with low-pass filters removing spectral power above $2f_C - B/2$ where B is bandwidth of QPSK signal

•This will recover complex base-band of QPSK signal
with real & imag parts invisible to each other.

• QPSK still works even if exact orthogonality not achieved,

• But lowest possible bit-error probability requires orthogonality.

•Binary FSK can be extended to multi-level FSK without increasing P_B if frequencies & symbol interval chosen such that all FSK symbols are orthogonal.

• It may be shown graphically that:

(i) $\cos(2\pi f_0 t + \phi)$ & $\cos(2\pi f_1 t)$ orthogonal over integer multiple of $1/|f_1 - f_0|$ for any ϕ .

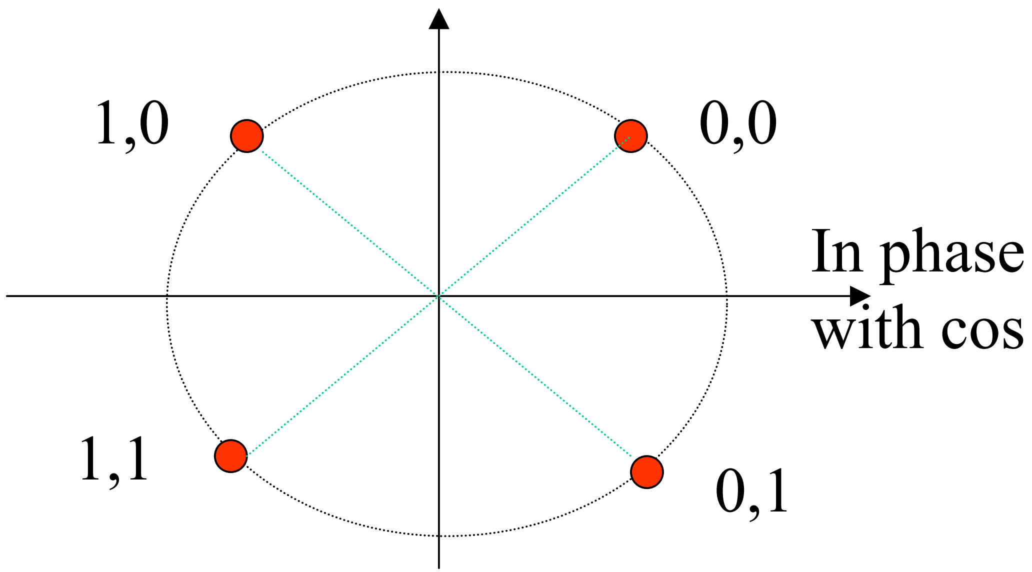
(ii) $\cos(2\pi f_0 t)$ & $\cos(2\pi f_1 t)$ orthogonal over integer multiple of $1/(2|f_1 - f_0|)$.

•Binary MSK orthogonal at 1/T b/s with $f_1 = f_0 \pm 1/(2T)$.

•Multi-level MSK at 1/T baud: use $f_0, f_1, f_2, f_3 \ldots$ at 1/(2T) spacing.

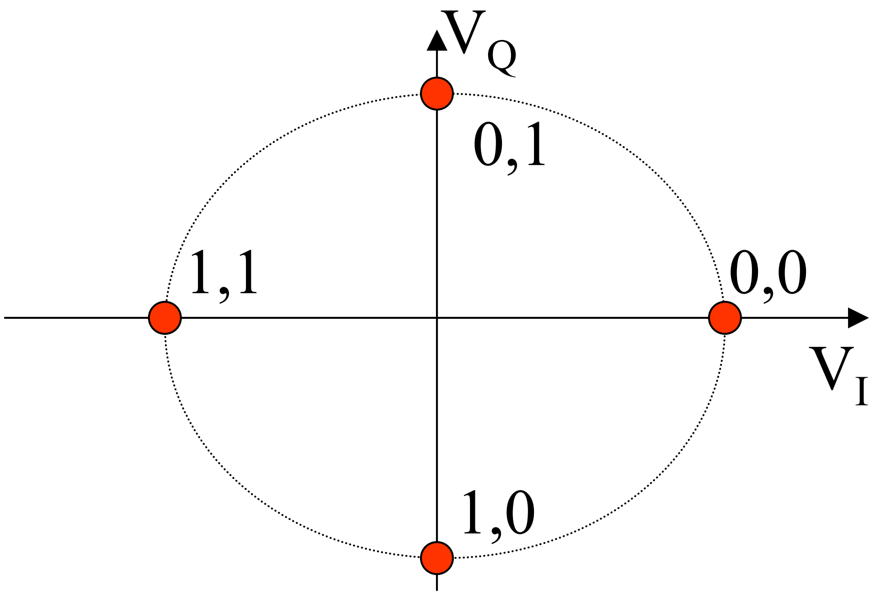
9.4. QPSK and orthogonality

- Constellation diagram below describes QPSK with 2-bits/ symbol.
- Each symbol is T second segment of sine-wave, of amplitude A.
- Same frequency f_C Hz = $2\pi\Omega_C$ as carrier
- Phase lead of $\pm 45^O$, $\pm$ 135 with respect to carrier.
- Modulation achieved by applying pairs of bits to symbol allocation unit producing pair of voltages as described by table.
- Apply the two voltages obtained to vector-modulator.



Bit1	Bit2	VI	VQ
0	0	A	A
0	1	A	-A
1	0	-A	A
1	1	-A	-A

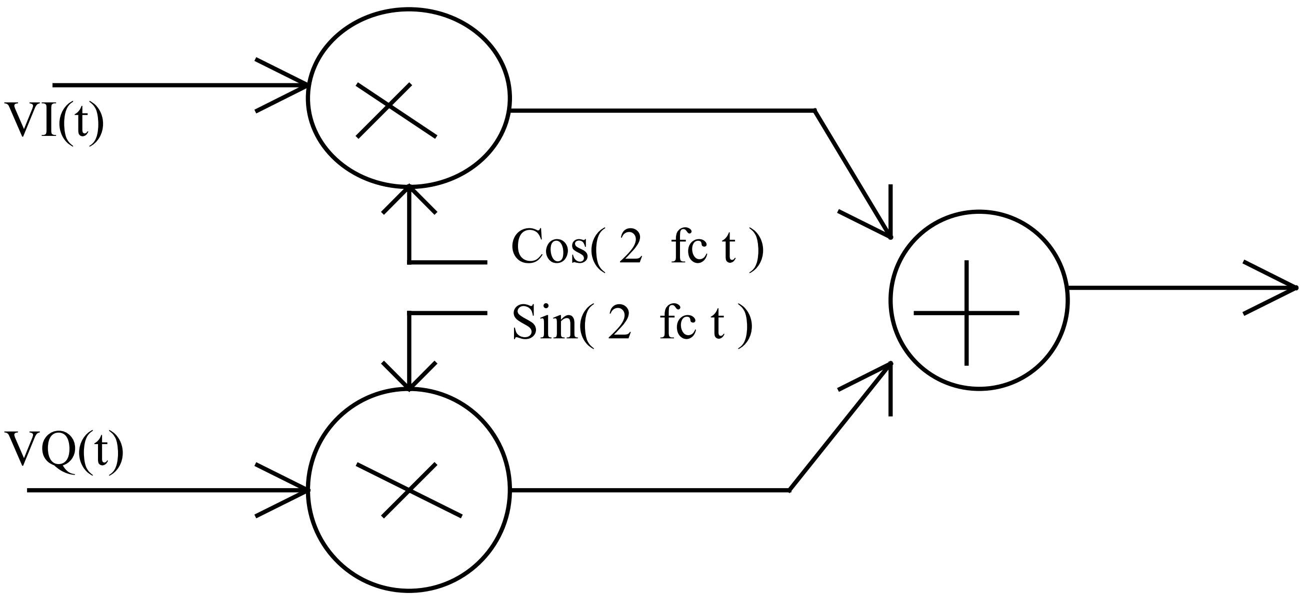
Constellation below is alternative form for QPSK with phase leads of 0, 90, 180 & 270^{O}.
Call it "St George's cross" QPSK constellation
Pulse-shaping, to minimise ISI applied to voltages $v_I(t)$ & $v_Q(t)$ before they are multiplied by the carrier.



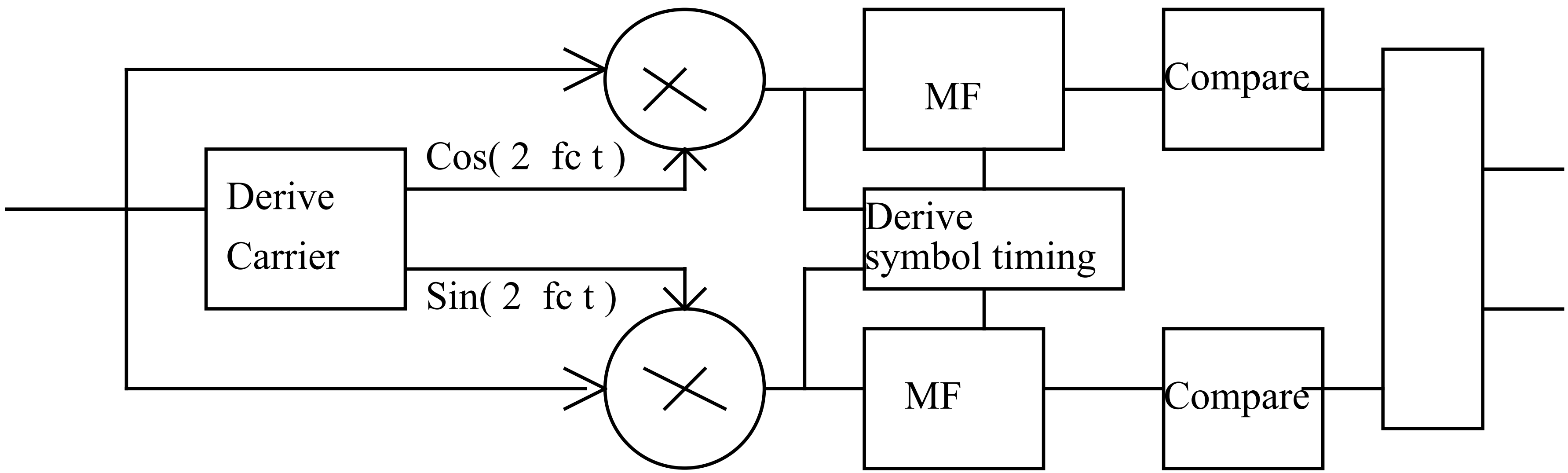
Bit1	Bit2	V_I	V_Q
0	0	A	0
0	1	0	A
1	0	0	-A
1	1	-A	0

(Different from notes .. Gray coded)

QPSK generation



Coherent QPSK detector for St. Andrew's constellation is 2 PSK detectors:


Derive
Carrier
Cos(2 fc t)
Sin(2 fc t)
MF
MF
Derive
symbol timing
Compare
Compare

- Coherent QPSK detector for right constellation a little different.
- Minimises 'Euclidean' distance between received symbol & each of 4 possibilities.
- Must examine I & Q components together,
- No longer have 2 independent PSK detectors.
- Can divide constellation into 4 regions, each region being represented by one symbol.
- Easy with 'St.George' QPSK constellation,
- Adaptable to more complicated constellations.

- With binary PSK, the 2 symbols are not orthogonal.
- They are 'anti-podal' or 'bipolar'.
- QPSK, adds quadrature component to each binary PSK symbol.
- To achieve exact orthogonality, we must use a symbol interval, T, equal to an integer multiple of half the carrier period and use an integrator as a detector.
- The coherent detectors must use matched filters or correlation detectors for the shaped "I" and "Q" symbols.
- This makes "Q-channel" and the "I-channel" truly independent.

- Other forms of QPSK: $\pi/4$-QPSK , DQPSK, etc. See text-book.

•**Carrier derivation**:

•Reasonable method for binary psk is to square incoming signal.

•With $A\cos(2\pi f_C t)$ for '1' & $-A\cos(2\pi f_C t)$ for '0',

this gives $A^2\cos^2(2\pi f_C t) = 0.5A^2(1 + \cos(2\pi 2 f_C t))$.

•Removing constant part gives cosine of twice frequency needed.

•A normal PLL can be locked to this frequency.

•Frequency halving may then be carried out using '÷2' counter".

•Alternatively, use a modified PLL which squares its VCO output before it is multiplied by the incoming signal.

•Now twice the VCO frequency matches the incoming frequency.

•VCO will have half the incoming frequency as required. Clever!!

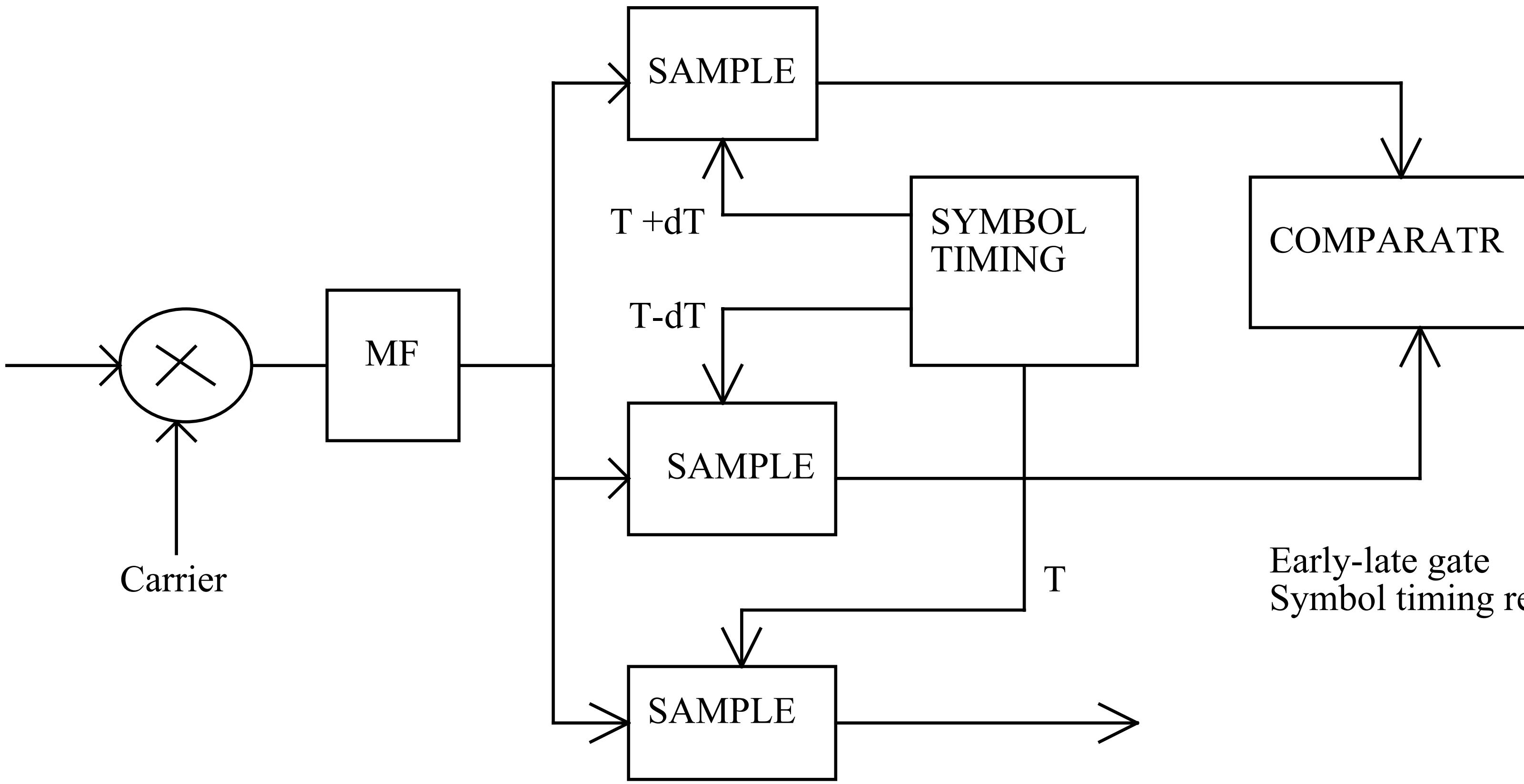
- The halving in frequency, will leave us with a sign ambiguity.
- Known training signal sent from time to time to allow this ambiguity to be resolved.
- For QPSK, carrier derivation as above except that fourth power of incoming signal calculated to eliminate the modulation, & frequency ÷4 used.

Symbol timing recovery:

- Even when carrier has been derived, there remains the problem of determining when one symbol ends and the next one begins.
- The value of T, will be known.
- In some cases it may be possible to assume that symbols are locked to carrier, so that a symbol is say 1.5 carrier cycles.
- This is not always possible since very slight inaccuracy in any previous demodulation process, say from 900 MHz to some intermediate frequency, or Doppler shifts due to receiving signals from moving vehicles, and/or other effects can remove this synchronism.

“Early-late gate” method.

Aims to adjust its sampling time to maximise samples obtained.
There are 2 detectors, one fed with slightly early timing reference T + dT & the other with a slightly late timing reference T - dT.
Outputs of the 2 detectors periodically compared to see which is producing the larger output.
Timing then advanced or retarded slightly according to whether the output is larger at T+dT or T-dT.
This should increase the larger output, and as this process is repeated it will be maximised. Further adjustments will cause the two samplers to settle around a situation where both produce approximately the same output, the true sampling point being equidistant between the early one T+dT and the late one T-dT.
This true sampling point is used for a third accurate sampler.



Early-late gate
Symbol timing recovery

Non-coherently detected M-ary FSK:

Have seen how binary FSK may be non-coherently detected & how it is advantageous to have $|f_1 - f_0| = 1/T$ Hz.

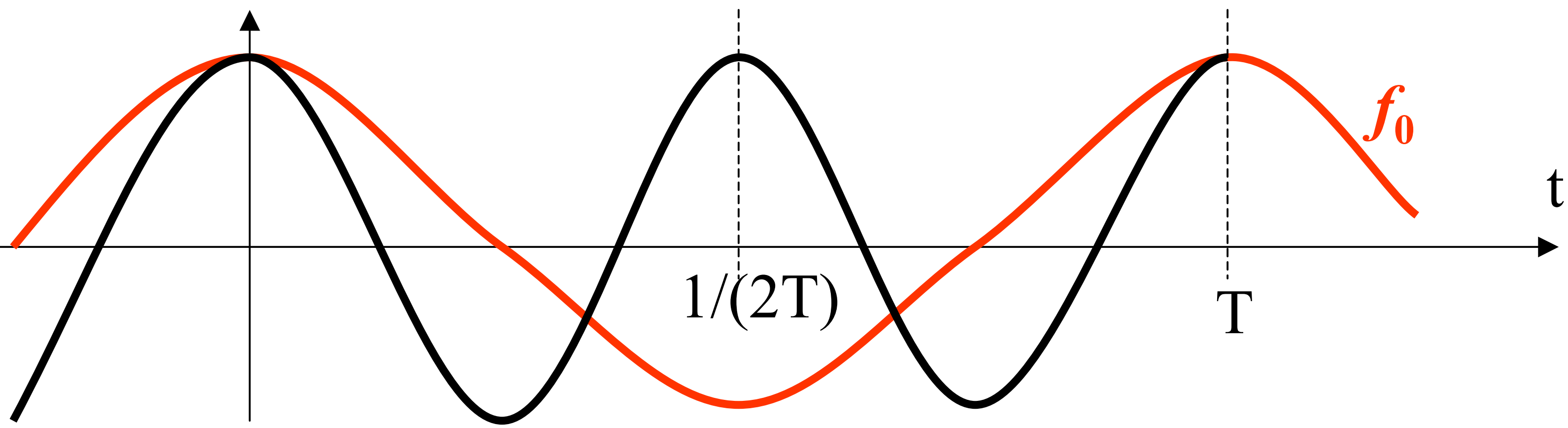
Mag-spectrum for transmissions at f_0 has null at f_1 & vice-vsa.

Non-phase synchronised binary FSK symbols:

$$A\cos(2\pi f_0 t + \phi) \;\;\&\;\; A\cos(2\pi f_1 t)$$

orthogonal over T seconds when $|f_1 - f_0| = 1/T$ regardless of ϕ .

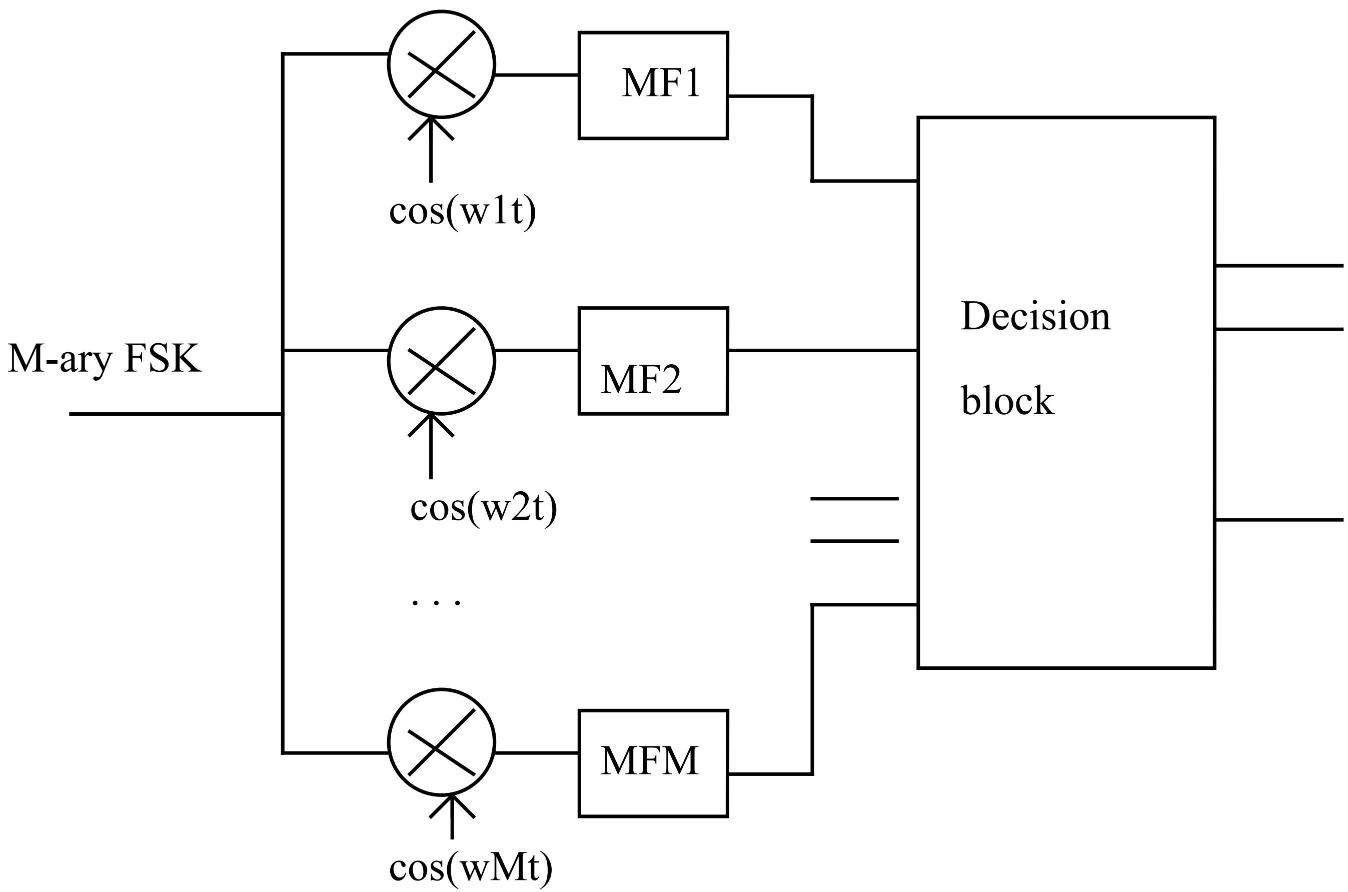
Exercise: Illustrate this graphically when f_0=1/T, f_1=2/T, ϕ=0



- If symbol rate is 1/T, min tone separation for non-coherent orthog FSK is 1/T Hz.
- Non-orthog FSK could be used, but effect of noise & ISI greater.
- Idea extended to M-ary FSK by introducing more tones.
- 8-ary FSK at 1800 b/s ($\therefore$ 600 baud) could have tones at 1000Hz (for 000) 1600Hz (001), 2200 Hz (011), 2800Hz (010), 3400Hz (100), 4000Hz (101), 4600Hz (111), 5400Hz (110).
- BER for this orthog scheme about the same as for binary FSK though bandwidth is wider.
- <u>Exercise</u>: Why are the symbols Gray-coded?

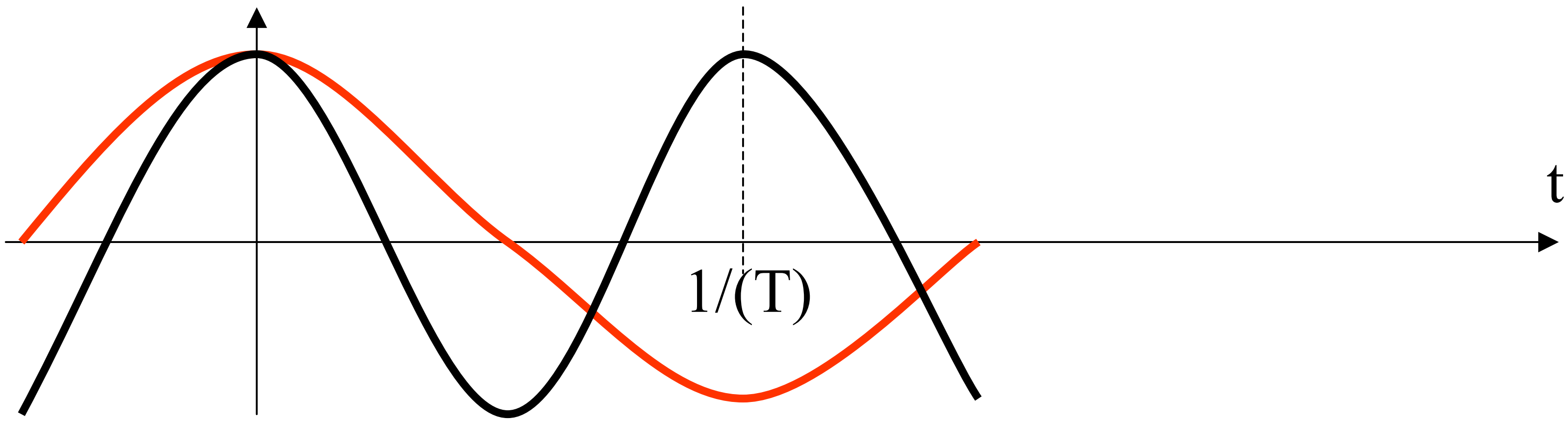
Coherently detected orthogonal M-ary FSK

Where locally generated tones locked to the received tones can be generated at the receiver, greater efficiency can be obtained.



FSK symbols A $\cos(2\pi f_0 t)$ & A $\cos(2\pi f_1 t)$ orthog over symbol period T when $|f_1 - f_0| = 0.5/T$.

Exercise: Illustrate this when $f_0 = 1/(2T)$ & f_1=1/T.



- With coherent detection, min frequency spacing for orthog M-ary FSK is 0.5/T Hz.
- Significant saving in bandwidth with no increase in BER.
- M-ary MSK.
- For given SNR & required bit-rate below H-S limit, M-ary MSK with increasing M may approach any arbitrarily low BER.
- The lower BER, the higher M & the longer T.

- Not necessarily best practical approach but shows it can be done in theory.
- If $E_b/N_o < -1.6$ dB there is no bit-rate for which any given BER can be achieved by this or any other method.
- Binary MSK with Gaussian symbol shaping used in GSM mobile.

Combined amplitude and phase shift keying (QAM and APK)

•To approach bandwidth efficiency of modern computer modems, combination of ASK, FSK & PSK used.

•Most popular: M-ary "amplitude-phase keying" (APK)

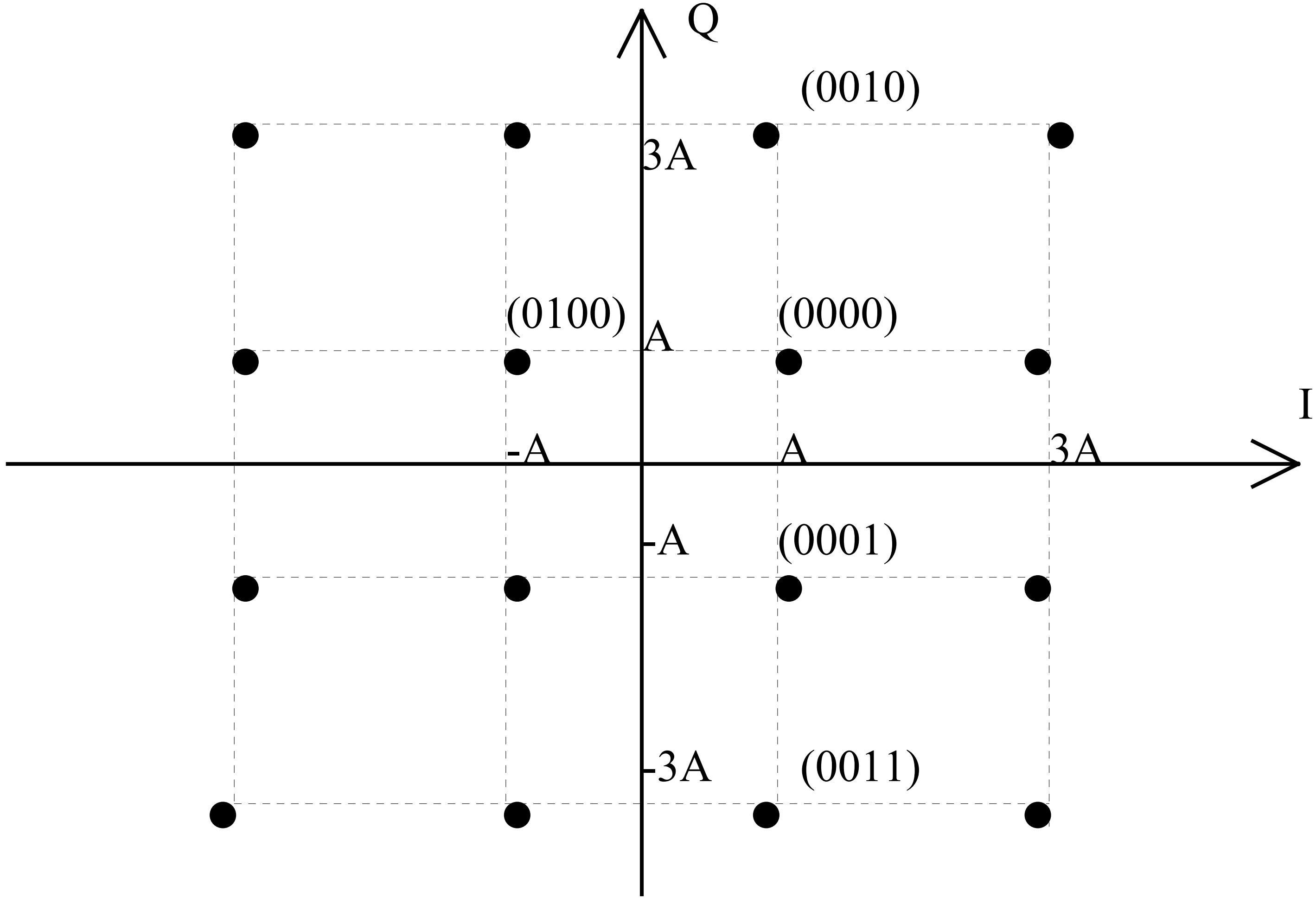
"quadrature amplitude modulation" (QAM).

•Extension of QPSK where different amplitudes (apart from +A, 0 and -A) used to modulate I & Q carriers.

•Consider 16-QAM system below with 4 bits per symbol:

Bit1	bit2	bit3	bit4	VI	VQ	Bit1	bit2	bit3	bit4	VI	VQ
0	0	0	0	A	A	1	0	0	0	3A	A
0	0	0	1	A	-A	1	0	0	1	3A	-A
0	0	1	0	A	3A	1	0	1	0	3A	3A
0	0	1	1	A	-3A	1	0	1	1	3A	-3A
0	1	0	0	-A	A	1	1	0	0	-3A	A
0	1	0	1	-A	-A	1	1	0	1	-3A	-A
0	1	1	0	-A	3A	1	1	1	0	-3A	3A
0	1	1	1	-A	-3A	1	1	1	1	-3A	-3A

Applying VI & VQ (after shaping) to vector-modulator gives a signal whose constellation diagram is shown below:



Exercise: Is this constellation Gray coded? If not, recode it.

Exercise: Specify the 16 symbols in terms of amplitude & phase.

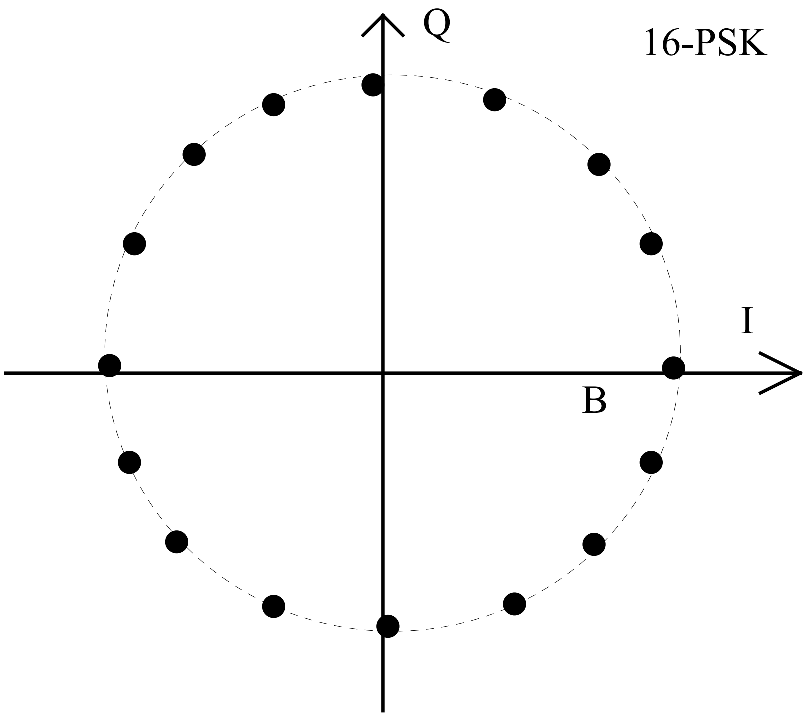
Exercise: If bit-rate is 1200 b/s & carrier is 600 Hz, sketch waveform for 0000 0101 1010 1011.

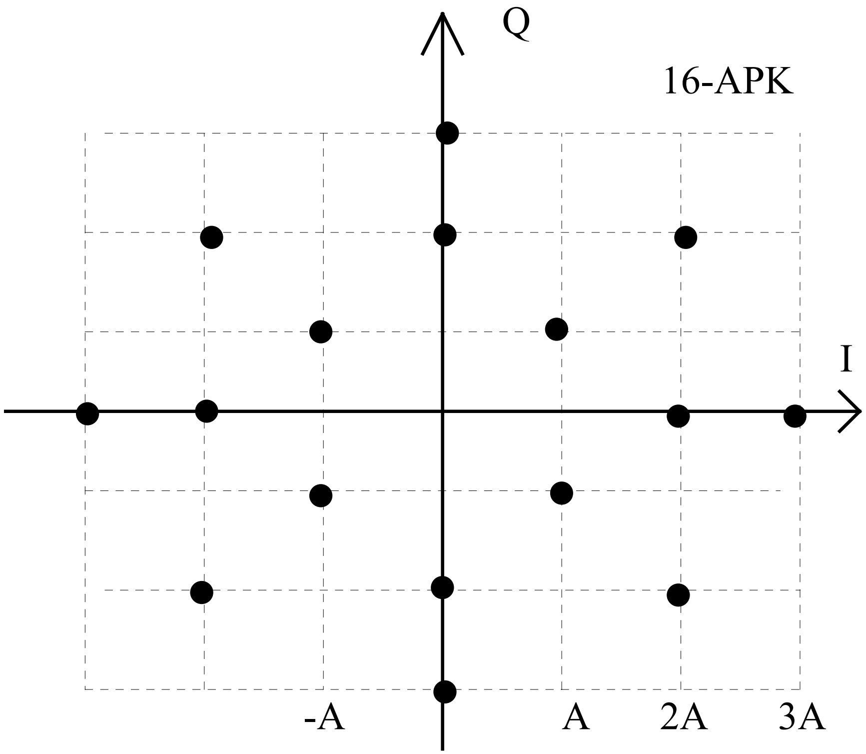
Exercise: Calculate average power of "square 16-QAM" when all symbols equally likely. What is peak voltage?

Solution: As sine-wave of ampl A has power $A^2/2$: avge power is: $4(2A^2/2 + 18A^2/2 + 10A^2/2 + 10A^2/2)/16 = 5A^2$ Watts.

Peak voltage: Volts

- Compare "square 16-QAM" with 16-PSK & 16-APK .
- Avge power & pk voltge of 16-PSK are
  $B^2/2$ Watts & B volts resp.
- If $B^2 = 10A^2$, power for square 16-QAM & 16-PSK will be same.
- Pk voltage for 16-PSK is then B=3.16A
- Pk voltage for square 16-QAM is 4.24A,
- So 16-PSK better in this respect.
- What about sensitivity to noise when avge power equalised by making B=3.16A?
- Min distce between any 2 symbols for square-16 QAM is 2A.
- It is approx. $B\ 2\pi/16 = 6.4A\pi/16 = 1.24A$.
- Symbols closer for 16-PSK, hence effect of noise greater.


Q
16-PSK
I
B


Q
16-APK
I
-A
A
2A
3A

Exercise: Repeat for 16-APK & compare square 16-QAM, 16-PSK & 16-APK.

Replace A in 16-APK diagram by C. Avge power is:

$$4(\ (2C)^2/2\ + (3C)^2/2 + 2C^2/2 + 8C^2/2\ )\ /16\ =\ 2.875C^2\ .$$

	Av power	Pk voltage	Min distance
Square 16-QAM	$5A^2$	4.243A	2A
16-PSK	$B^2/2$	B	$\pi B/8$
16-APK scheme	$2.875C^2$	3C	C

- For same average power (i.e. same E_B/N_0):

  C = 1.32A and B = 3.16A
- For same peak voltage:

  C = 1.414A and B = 4.243A
- Min distce on constellation diag (measure of noise immunity):

	Min distce for same av. power	Min distce for same pk voltage
Square 16-QAM	2A	2A
16-PSK	1.24A	1.667A
16-APK scheme	1.32A	1.414A

Square QAM appears ‘best’ in both circumstances. It has greatest minimum distance in both circumstances.
I am slightly surprised by the ‘peak voltage’ result and expected 16-PSK to be better than square-16-QAM in this case.
Check result please.

CCITT modem standards:-

Version	Bits per second	Modulation	Protocol
Bell 103	300	FSK	Async
Bell 202	1200	FSK	Async
V22	1200/600	QPSK/FSK	Async/sync
V26bis	2400	QPSK	Sync
V27	4800/2400	8-DQPSK	sync
V29	9,600	16-APK	sync
V32	9,600	32-QAM/16-QAM	sync
V33	14,400	32-QAM	sync
V34	33,600	>1024-QAM	sync
V90	56,000	>1024-QAM	sync

Problems:

9.1: A digital system transmits a 0Hz to 2MHz video signal sampled at 8MHz with a 16 bit analogue to digital converter. The signal is transmitted by 16-QAM with raised cosine spectrum pulse shaping with roll-off factor α (or r) equal to 0.5. What is the transmission bandwidth needed.

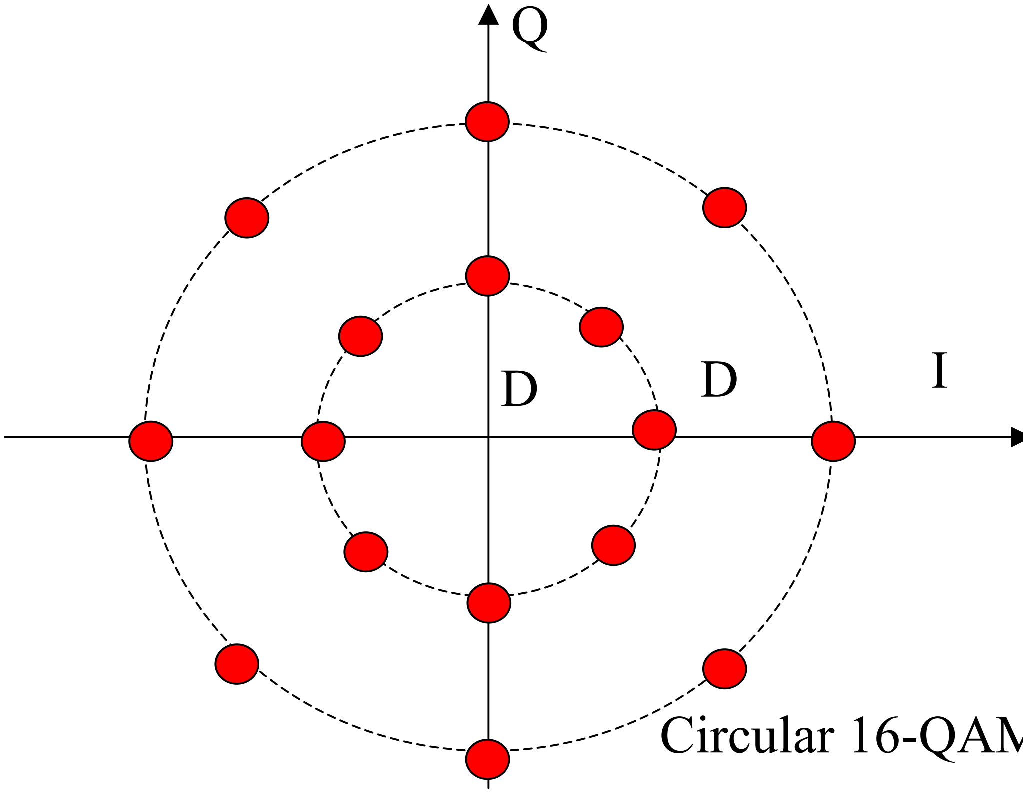
Solution: Bit-rate = 128Mb/s. Four bits/symbol. This would give 4 bits/sec/Hz if α=0, but as α = 0.5 the bandwidth efficiency is reduced by $(1+\alpha)$ = 1.5. This is because of the spectrum of the shaped pulse. See previous notes. It becomes 2.66 bits/sec per Hz. Therefore required bandwidth is 48 MHz. This is considerably greater than the analogue bandwidth.

9.2: An 8-ary ASK scheme uses a root-raised cosine spectrum filter in both transmitter and receiver, with α=0.33. What bandwidth is required for 64 kb/s?

9.3: What are the advantages and disadvantages of coherent detection as opposed to non-coherent detection for 8-FSK?

9.4: Devise a suitable constellation and symbol assignment for (a) 8-APK, (b) 32-QAM.

9.5: Compare the following 'circular-16' QAM scheme with the three schemes mentioned in the notes.



Circular 16-QAM