CS3282 Digital Communications Examination June'04 (complete)

- Answer Three questions in TWO hours.
- Electronic calculators not capable of storing text may be used.
- Attached: Appendix 1: Graph of complementary error function Q(z) against z.
  Appendix 2: Fourier transform properties
  Appendix 3: Trigonometric formulae

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1(a) What causes 'fading' in a mobile telephone channel and how does it affect the received signal ? **[3 marks]**

Explain the difference between 'flat' fading and 'frequency-selective' fading and state what is meant by the 'coherence bandwidth' of a channel. **[3 marks]**

What factors determine whether the fading is flat or frequency-selective and how does the type of fading we expect influence the design of wireless digital transmitters and receivers? **[2 marks]**

State the purpose of (i) a matched filter and (ii) an adaptive equaliser as may be used in a digital communication receiver. **[4 marks]**

(b) An appropriately shaped symbol with zero-crossings at t = ±T, ±2T, ±3T, etc. relative to the centre is distorted by the frequency-response of a wired channel. It is estimated by averaging over several training symbols that when the received version of the symbol, s(t) say, is normalised to 1 volt at its centre (assumed to occur at t=0), its pulse-shape in the absence of added noise would be as shown in figure 1.

Inter-symbol interference is to be reduced by a 5-term 'zero-forcing' transversal equaliser. Explain how this may be achieved and give a diagram of the equaliser. **[3 marks]**

Given that the zero-forcing equaliser has coefficients C_0, C_1, C_2, C_3, C_4 expressed as a column vector $\underline{c}$, show that the required zero-crossings are achieved if $A\underline{c} = \underline{b}$, where

$$A = \begin{bmatrix} 1 & -0.2 & 0.2 & -0.1 & 0.1 \\ 0.2 & 1 & -0.2 & 0.2 & -0.1 \\ -0.2 & 0.2 & 1 & -0.2 & 0.2 \\ 0.1 & -0.2 & 0.2 & 1 & -0.2 \\ -0.1 & 0.1 & -0.2 & 0.2 & 1 \end{bmatrix}$$

and $\underline{b}$ is a suitably chosen column-vector. Given that the inverse of matrix A is approximately as follows:

$$A^{-1} = \begin{bmatrix} 0.95 & 0.2 & -0.12 & 0 & 0.13 \\ 0 & 0.81 & 0.35 & -0.23 & 0.7 \\ 0.14 & -0.07 & 0.87 & 0.24 & -0.21 \\ -0.09 & 0.14 & -0.06 & 0.88 & 0.34 \\ 0.14 & -0.1 & 0.14 & -0.1 & 0.83 \end{bmatrix}$$

calculate the coefficients C_0, C_1, C_2, C_3 and C_4 **[3 marks]**

Estimate the equaliser's response to s(t) at time 3T seconds after the centre of the reshaped pulse. **[2 marks]**

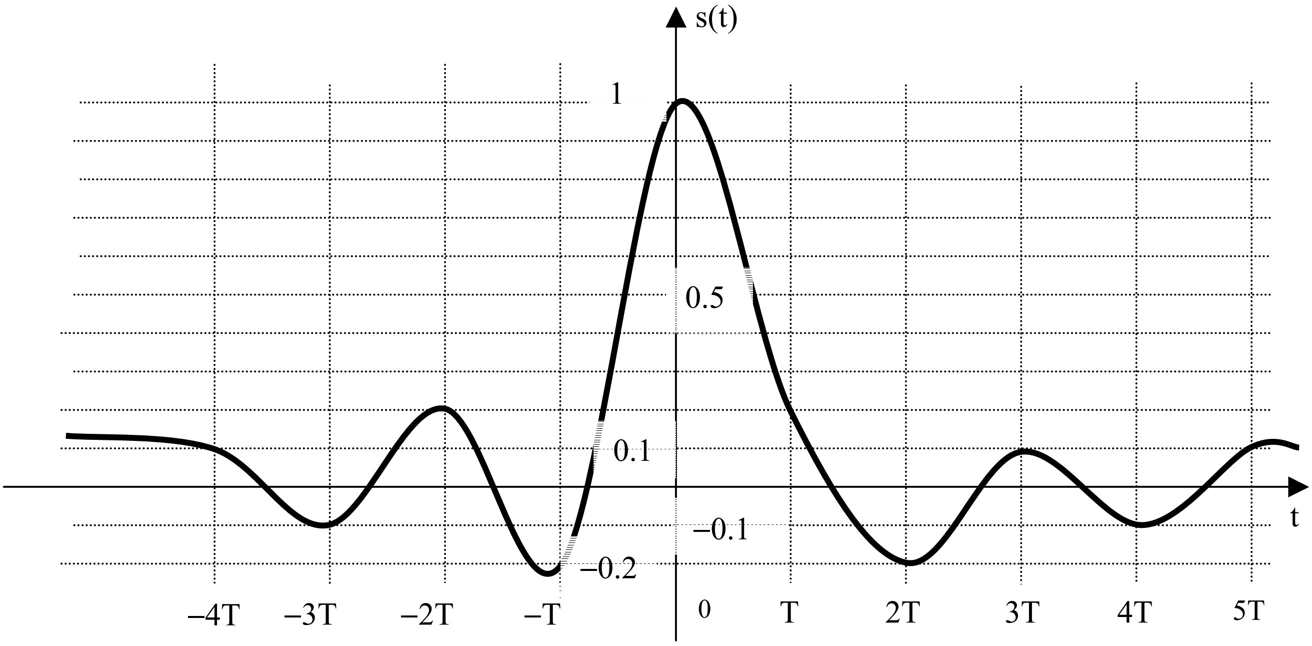


Figure 1 : Graph of distorted pulse received from channel.

2.(a) What are the main requirements for the voltage waveforms that may be used for the synchronous transmission of pulse-code modulated (PCM) digital signals at base-band over wired telephone links. **[2 marks]**

Show how these requirements are fulfilled by:

(i) NRZ-HDB3 coding **[3 marks]**

(ii) Manchester coding: **[3 marks]**

2. (b) What are the requirements for a pulse-shaping filter for minimising inter-symbol interference (ISI) when symbols are transmitted at 1/T symbols/second over a band-limited channel. **[4 marks]**

Using the Fourier transform properties in Appendix B, or otherwise, show that, in theory, ISI is eliminated by shaping each symbol, s(t), transmitted such that when it is received at the threshold decision circuit its Fourier Transform spectrum S(*f*) satisfies:

$$S(f) = 0 \text{ for } |f| > 1/T$$

$$S(f) + S(f - 1/T) = s(0) \quad \text{for } -1/T \le f \le 1/T$$ **[5 marks]**

Sketch the general shape of an "100r% raised cosine" spectrum which satisfies this property, and explain why, in practice, "root raised cosine (RRC)" spectrally shaped signalling is employed. **[3 marks]**

3(a) Assuming the Schwartz (or Cauchy-Schartz) inequality:

$$\left|\int_a^b x(u)y(u)du\right|^2 \le \int_a^b |x(u)|^2 du \int_a^b |y(u)|^2 du$$

show that if unipolar binary signalling is used such that equally likely symbols s(t) and 0, each of duration T seconds, are received without inter-symbol interference but with additive white Gaussian noise (AWGN) of zero mean , a matched filter with impulse-response proportional to s(T-t) will minimise the bit-error probability produced by a threshold-detector. **[7 marks]**

If the two-sided power spectral density (PSD) of the noise is N_0 /2 watts/Hz, and the average energy of the signalling component of the received signal is E_b Joules per bit, show that the minimised error probability is $P_b = Q(\sqrt{E_b / N_0})$. **[5 marks]**

3(b). A receiver receives equi-probable 2 volt and 0 volt rectangular NRZ binary symbols at 500 symbols per second without inter-symbol interference but with AWGN of two-sided PSD $N_0/2 = 80 \times 10^{-6}$ watts/Hz and zero mean. An integrate and dump averaging circuit and optimum threshold-detector is employed to detect the bit-stream. Estimate the bit-error probability. **[4 marks]**

Estimate the bit-error probability that could be obtained without the "integrate and dump" averaging circuit if the received signal is passed through an ideal low-pass filter with cut-off frequency 4 kHz assuming that the low-pass filter does not significantly change the rectangular pulse-shape at the sampling points **[4 marks]**

A graph of the error function Q(z) against z is attached.

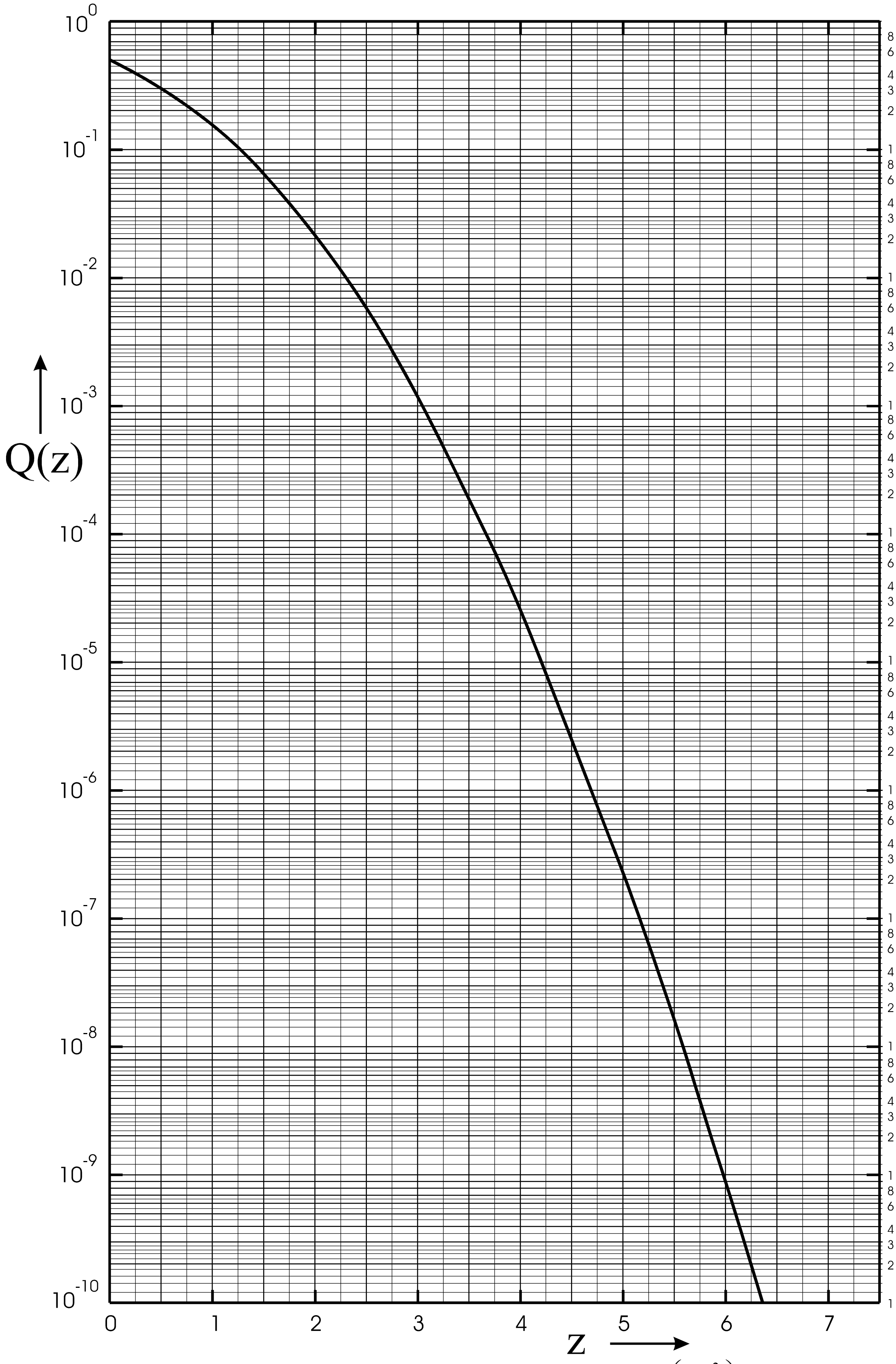
4 (a) State what is meant by a vector-modulator and explain how such a modulator can be used to generate a quaternary phase shift keyed (QPSK) signal from a given bit-stream. Give a constellation diagram for the modulation scheme. **[6 marks]**
(b) Show how a QPSK transmission may be demodulated. **[4 marks]**
(c) What are the advantages and disadvantages of using QPSK instead of ASK or FSK? **[2 marks]**
(d) State the maximum bit-rate that can be achieved using QPSK with 3 kHz bandwidth and explain why this maximum would be difficult to achieve in practice. If the carrier is 3 kHz, and the bit-rate is 2 kb/s, sketch the QPSK waveform obtained for the bit-stream 001110 without pulse shaping **[4 marks]**
(e) If the channel is affected by additive white Gaussian noise at the receiver and the bit-error rate is measured as 1 error in 10^{10} bits, by how much can the transmitter power be reduced before the bit-error rate exceeds 1 in 10^5. **[4 marks]**

5 (a) What features of speech signals are exploited by
(i) the G711 64 kb/s standard coder for wired telephony
(ii) the GSM 06.10 (RPE-LTP) 13 kb/s coder for mobile telephony
to achieve acceptable speech quality at the required bit rate **[8 marks]**

(b) Show how a speech signal digitised at 64 kb/s may be efficiently transmitted over a channel of 32 kHz bandwidth centred on 100 Hz using an 8-level modulation scheme such as 8-APK with RRC pulse shaping? **[3 marks]**

Give a labelled constellation diagram for the modulation scheme and explain why Gray coding is used in such multi-level digital modulation schemes. **[4 marks]**

According to the Shannon-Hartley Law, what signal-to-noise ratio would be required to ensure that arbitrarily low bit-error rates are achievable for this speech transmission. **[5 marks]**



Graph of Complementary Error function, $Q(z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$

Appendix 2: Properties of the Fourier Transform

Property	Signal x(t)	Fourier Transform X(*f*)
Transform & inverse:	$x(t) = \int_{-\infty}^{\infty} X(f)e^{2\pi jft}\,df$	$X(f) = \int_{-\infty}^{\infty} x(t)e^{-2\pi jft}\,dt$

Similarly let the Fourier transforms of y(t), $y_1(t)$ & $y_2(t)$ be Y(*f*), $Y_1(f)$ & $Y_2(f)$ respectively.

Rect & sinc	$\text{rect}_A(t) = \text{rect}(t/A)$	$A\,\text{sinc}_{1/A}(f) = A\,\text{sinc}(Af)$
	$\text{sinc}_A(t) = \text{sinc}(t/A)$	$A\,\text{rect}_{1/A}(f) = A\,\text{rect}(Af)$
Delay	$y(t-\tau)$	$e^{-2\pi j\tau}Y(f)$
Frequency shift	$e^{2\pi jFt}\,y(t)$	$Y(f-F)$
Amplitude scaling:	$Ay(t)$	$AY(f)$
Time-reversal:	$y(-t)$	$Y(-f)$
Superposition:	$Ay_1(t)+By_2(t)$	$AY_1(f)+BY_2(f)$
Constant:	A	$A\delta(f)$
Impulse:	$A\delta(t)$	A
Gaussian	$(\sqrt{\pi}/\alpha)\exp(-\pi^2 t^2/\alpha^2)$	$\exp(-\alpha^2 f^2)$
Time-scaling:	$y(At)$	$(1/\lvert A\rvert)\,Y(f/A)$
Differentiation:	$d^m\{y(t)\}/dt^m$	$(2\pi jf)^m\,Y(f)$
Product:	$y_1(t)y_2(t)$	$Y_1(f)\otimes Y_2(f) = \int_{-\infty}^{\infty} Y_1(\theta)Y_2(f-\theta)\,d\theta$
Convolution:	$\int_{-\infty}^{\infty} y_1(\tau)y_2(t-\tau)\,d\tau$	$Y_1(f)\,Y_2(f)$
Cross-correlation:	$\int_{-\infty}^{\infty} y_1(\tau)y_2(t+\tau)\,d\tau$	$Y_1(f)\,Y_2^*(f)$
Auto-correlation:	$\int_{-\infty}^{\infty} y(\tau)y(t+\tau)\,d\tau$	$\lvert Y(f)\rvert^2$
Repeat:	$\text{repeat}_P\{y(t)\}$	$(1/P)\text{sample}_{1/P}\{Y(f)\}$
Sample:	$\text{sample}_T\{y(t)\}$	$(1/T)\text{repeat}_{1/T}\{Y(f)\}$

Properties for real signals:-

For all real signals:	$x^*(t)=x(t)$	$X(-f) = X^*(f)$ i.e. $\lvert X(-f)\rvert = \lvert X(f)\rvert$ & $\phi(-f) = -\phi(f)$
Real and even:	$x(t) = x(-t)$	X(*f*) is purely real & $X(-f) = X(f)$
Real and odd:	$x(t) = -x(-t)$	X(*f*) is purely imaginary & $X(-f) = -X(f)$

Formulae:-

$$\text{sinc}(x) = \begin{cases} \sin(\pi x)/(\pi x) & : x \neq 0 \\ 1 & : x = 0 \end{cases} \qquad \text{rect}(x) = \begin{cases} 1 & : |x| < 0.5 \\ 0.5 & : |x| = 0.5 \\ 0 & : |x| > 0.5 \end{cases} \qquad \text{sinc}_A(x) = \text{sinc}(x/A) \qquad \text{rect}_A(x) = \text{rect}(x/A)$$

$$\text{repeat}_P\{x(t)\} = \sum_{n=-\infty}^{\infty} x(t-nP) \qquad \text{sample}_T\{x(t)\} = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT) \quad = \quad x(t)\text{repeat}_T\{\delta(t)\}$$

Fourier series for $\text{repeat}_P\{x(t)\}$:-

$$\sum_{k=-\infty}^{\infty} C_k e^{2\pi jkt/P} = \sum_{k=0}^{\infty} M_k \cos((2\pi k/P)t + \theta_k) = A_0 + \sum_{k=1}^{\infty}\left(A_k \cos((2\pi k/P)t) + B_k \sin((2\pi k/P)t)\right)$$

$$C_k = (1/P)X(k/P) \;;\quad M_0 = |C_0|;\; M_k = 2|C_k|;\; \theta_k = \arg(C_k);\; A_0 = C_0;\, A_k = 2\,\text{Re}\{C_k\};\, B_k = 2\,\text{Im}(C_k\}$$

Appendix 3: Trigonometric formulae

$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$
$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$
$\tan(A \pm B) = (\tan(A) \pm \tan(B)) / (1 \mp \tan(A)\tan(B))$

$\sin(2A) = 2\sin(A)\cos(A)$
$\cos(2A) = 2\cos^2(A) - 1 = 1 - 2\sin^2(A)$
$\tan(2A) = 2\tan(A) / (1 - \tan^2(A))$

$2\cos(A)\cos(B) = \cos(A + B) + \cos(A - B)$
$2\sin(A)\cos(B) = \sin(A + B) + \sin(A - B)$
$2\sin(A)\sin(B) = \cos(A - B) - \cos(A + B)$

$\cos(A) + \cos(B) = 2\cos((A + B)/2)\cos((A - B)/2)$
$\sin(A) + \cos(B) = \sin(A) + \sin(B+\pi/2)$
$\sin(A) + \sin(B) = 2\sin((A + B)/2)\cos((A - B)/2)$

$\cos(\theta) = (e^{j\theta} + e^{-j\theta}) / 2$
$\sin(\theta) = (e^{j\theta} - e^{-j\theta}) / (2j)$

$\lambda\cos(\theta) + \mu\sin(\theta) = R\cos(\theta + \phi)$ where $R^2 = \lambda^2 + \mu^2$ and $\phi = \tan^{-1}(\mu/\lambda) + \{\pi \text{ if } \lambda<0\}$

CS3282 Solution methods June 2004

1.(a) Fading is due to the effect of multi-path propagation since the radio transmission from a given base-station or a given mobile user will be reflected by buildings and thus reach its intended target by a multitude of different routes. In fact, direct "line-of-sight" paths between base-stations and mobiles are rare in large cities with tall buildings. The different routes introduce different phase shifts, and so when the reflected signals reach their target some of them will add in phase and reinforce each other, and some will be out of phase and can cancel each other out. The cancellation is referred to as "fading", and it will be frequency dependent, i.e. the same phase differences will cause reinforcement at some frequencies and cancellation at other frequencies. Further, if the mobile is moving, the nature of the fading will change with time, often very rapidly.

The effect of multi-path propagation is to make the gain and phase response of the radio channel non-flat and highly variable. The channel acts like a filter whose gain and phase response vary.

If we restrict transmissions to a relatively narrow band of frequencies, the gain and phase differences across this narrow band may not be too serious, and the main problem will be "flat fading", i.e. the gain and phase-delay across the whole band being affected more or less equally due to cancellation. However, if we wish to increase the bit-rate of our transmissions by using a wider radio-frequency band, "frequency selective" fading will become a problem where some frequencies within the wider band will be severely attenuated while others are enhanced.

The "coherence bandwidth", B_C Hz say, of the radio channel is the largest bandwidth we can use without having to worry about frequency selective fading. For a channel whose bandwidth is less than B_C Hz, any fading may be considered "flat fading".
A typical value for a city of B_C = 30 kHz would allow an analogue mobile telephone system with 30 kHz channels to work without an equaliser, whereas a 900 MHz GSM system with 200 kHz bandwidth channels would definitely require equalisation.

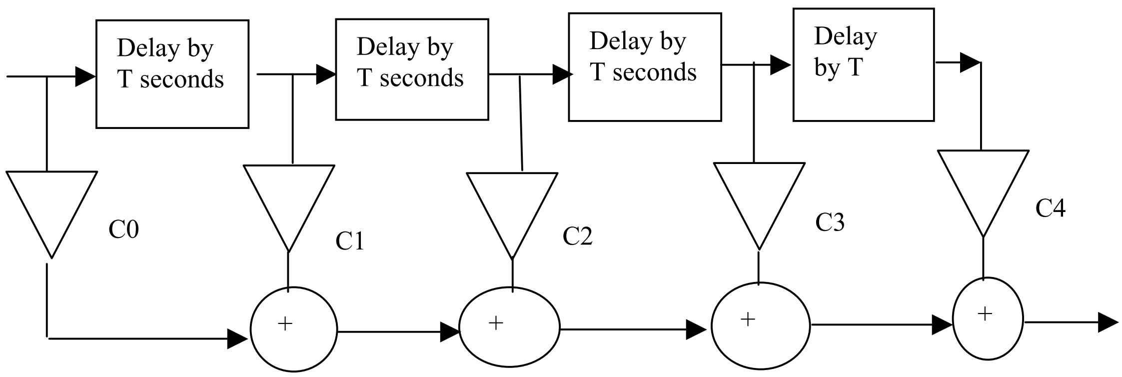
Fading affects the phase-response as well as the gain-response of a radio channel. A non-linear phase response means that the phase delay is not the same for all frequencies so that some frequency components of a signal will be delayed by different amounts of time than others. Such a phase-response can seriously alter the shape of a signalling pulse and thus cause "inter-symbol interference" (ISI) when it runs into the next pulse to arrive. Reductions in the gain-response due to fading cause a smaller signal to be received, and therefore noise picked up by the receiving antenna will have a greater effect and produce more bit-errors.

(i) A matched filter is tuned to the pulse-shape being transmitted and minimises the effect of AWGN on the bit-error probability when a threshold-detector isused.

(ii) An "equaliser" is a filter or frequency-selective amplifier whose frequency-response is, in principle, the inverse of that of the channel over the selected frequency band, so that the effects of frequency selective fading are reversed and the channel then appears to have a constant gain and phase-delay across all frequencies.

1 (b) The <u>channel equaliser</u> aims to cancel out the gain and phase distortion introduced by the filtering effect of the channel which will have changed the shape of the signal launched into it. It tries to correct the "100 r % RC" shapes as seen at the detector. A zero-crossing equaliser is a simple way of doing this. More sophisticated adaptive FIR digital filtering schemes are much more successful for radio communications.

The zero-forcing equaliser is as follows:



When the input is a single signalling pulse of the appropriate shape but distorted by the channel's frequency response, and it is centred on t=0 as detected by the timing circuitry, the output must be forced to be zero at t= T, -T, 2T & -2T. The filter is assumed to delay the signal by 2T. So the main lobe centred on t=0 becomes delayed to t=2T. The output at t=2T is arbitrary and may be taken to be 1. The output at t= 0, T, 2T, 3T & 4T is as follows:

$0 = y(0) = x(0)C_0 + x(-T)C_1 + x(-2T) C_2 + x(-3T) C_3 + x(-4T) C_4$
$0 = y(T) = x(T)C_0 + x(0)C_1 + x(-T) C_2 + x(-2T) C_3 + x(-3T) C_4$
$1 = y(2T) = x(2T)C_0 + x(T)C_1 + x(0) C_2 + x(0) C_3 + x(-2T) C_4$
$0 = y(3T) = x(3T)C_0 + x(2T)C_1 + x(T) C_2 + x(T) C_3 + x(-T) C_4$
$0 = y(4T) = x(4T)C_0 + x(3T)C_1 + x(2T) C_2 + x(T) C_3 + x(0) C_4$

Given x(-2T), x(T), x(0), x(T) & x(2T), as read from the graph, values of the five coeffs may be found.

Expressed in matrix form this becomes $\underline{y} = A\,\underline{c}$ where

$$A = \begin{bmatrix} 1 & -0.2 & 0.2 & -0.1 & 0.1 \\ 0.2 & 1 & -0.2 & 0.2 & -0.1 \\ -0.2 & 0.2 & 1 & -0.2 & 0.2 \\ 0.1 & -0.2 & 0.2 & 1 & -0.2 \\ -0.1 & 0.1 & -0.2 & 0.2 & 1 \end{bmatrix}$$

and $\underline{y} = [\ 0\ \ 0\ \ 1\ \ 0\ \ 0\ ]^T$

It follows that $\underline{c} = A^{-1}\,\underline{y}$ with A^{-1} as given in the question.
Therefore $C_0 = -0.12$, $C_1 = 0.35$ $C_2 = 0.87$ $C_3 = -0.06$ $C_4 = 0.14$

The equaliser has the difference equation:

$y(nT) = x(nT)C_0 + x((n-1)T)C_1 + x((n-2)T) C_2 + x((n-3)T) C_3 + x((n-4T)) C_4$

Therefore
$y(5T) = x(5T)C_0 + x(4T)C_1 + x(3T) C_2 + x(2T) C_3 + x(T) C_4$
$\quad = 0.1 * (0.12) + (-0.1)*0.35 + 0.1*0.87 + (-0.2)*(-0.06) + 0.2 * 0.14$
= ….

Not asked:-

$H((f)) = C_0 + C_1\exp(-2\pi j f T) + C_2\exp(-4\pi j f T) + C_3\exp(-6\pi j f T) + C_4\exp(-8\pi j f T)$
This will make the channel noise no longer white, introduces correlation in the noise from sampling point to sampling point, increases the noise at the sampling points and thus affects the assumptions used to design the matches filter. This is a disadvantage of this simple equalisation technique.

2 a

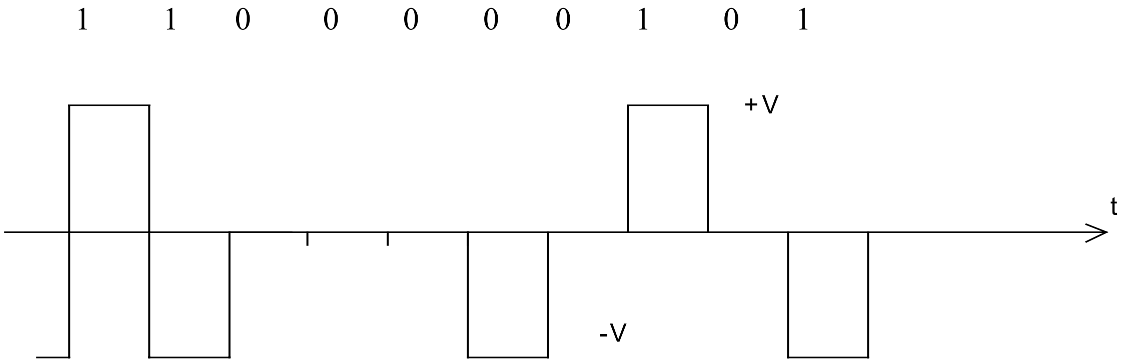
When considering how to synchronously transmit PCM information, two factors must be borne in mind:-

(1) The DC component of a signal is normally lost over wire lines, because of AC coupling, the use of transformers, etc. We would therefore like to keep the average voltage level zero for data transmission.
(2) For synchronous transmission, we need to ensure that the signal always has a frequency component at the signalling rate (or an exact multiple or sub-multiple of the signalling rate) to allow synchronisation at the receiver.

Using AMI is fine when, say, 1 0 1 0 1 0 1 0 ... is transmitted.
However there are still problems when 0 0 0 0 0 0 .... 0 is transmitted. In this case, the receiver can lose synchronisation. A commonly used solution is known as HDB3 coding.

<u>HDB3 coding: (high density bipolar, order 3):</u>

This scheme uses ternary coding to send binary coded data, as described above, but with an
Two possible choices of synchronous techniques (only one needed in answer):
HDB3 coding: (high density bipolar, order 3): This synchronous scheme uses ternary coding to send binary coded data, but with an incorrect "1" in place of any 4th consecutive. zero



The incorrect "1" is included for clock synchronisation. It is taken to be a data bit of “0” at the receiver.

2(a) continued

Bi-phase-L, better known as “Manchester coding”, and represents a “one” by a pulse of width T/2 positioned during the first half bit-interval. A zero has a pulse of width T/2 in the 2nd half interval. Manchester coding has the advantage of absolutely guaranteeing zero dc level and is easy for the receiver clock to synchronise itself to. Its disadvantage in comparison to HDB3 is that the bandwidth requirement is considerably higher.

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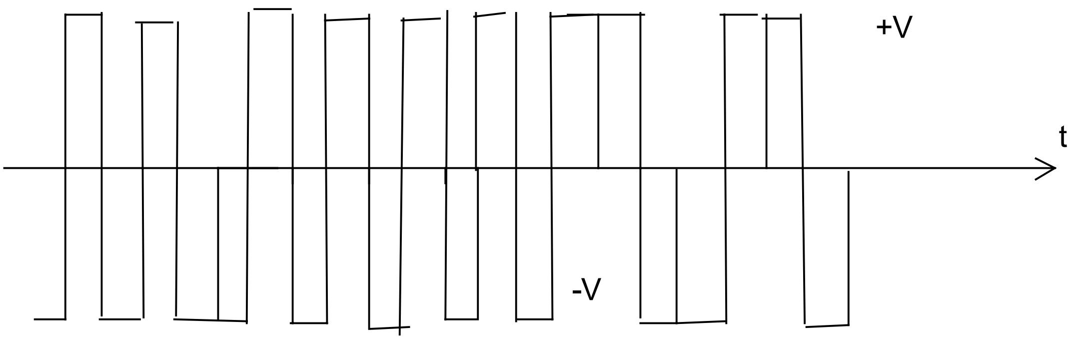


(1)

Synchronous techniques require a clock to be generated at the receiver exactly synchronised with the data. But the data rate can be much higher and continuous. Suitable for high speed telephone links.

2 (b)

Because of the channel's finite bandwidth, the response to any symbol will not be time-limited. It will continue ringing forever, though dying away quite rapidly in amplitude. Inter-symbol interference (ISI) can occur due to the ringing of one symbol into the next.

Rectangular symbols are not suitable for transmitting data over band-limited channels. Any shaped pulse which is time-limited will require infinite bandwidth . A symbol with finite bandwidth must have infinite time-duration.

In practice, a pulse must be non-zero for more than T seconds when the signaling rate is 1/T Baud to achieve approximate bandlimiting. Each symbol will run into the previous and next symbol.
"Pulse shaping" means that we carefully choose the time-domain shapes of the symbols such that zero-crossings occur at $t=\pm T$, $\pm 2T$, etc… when they are detected at the decision block within the receiver. So at the centre of the next pulse, the current one will be zero and not affect the measurement at the decision block.

A convenient way of generating symbols with the time-domain shape we require is to generate an impulse of the appropriate strength for each symbol and then to shape this impulse by passing it through a "transmitting" filter. The impulse-response of the transmitting filter is the symbol shape we wish to launch into the channel.

To achieve zero ISI the frequency response of the pulse shaping filter must be such that the combined effect of all filtering, and the channel (with equalisation) has the frequency response on a "Nyquist" filter. A "100r %" raised cosine frequency response is commonly used to achieve this Nyquist frequency response (giving the correctly positioned zero-crossings). In order to make the overall frequency response,(from impulses at the transmitter to the detector within the receiver) into a 100% RC response the transmitter shaping filter must be "100r % root-RC (RRC)" frequency response.

Three main requirements for the pulse shaping filter are:

1. The shaping filter must produce pulses whose spectra fall within the band width of the channel.

2. To avoid ISI occurring at the receiver's threshold detector by ensuring that the transmitting filter shapes the symbols so that zero-crossings at the output of the receiving filter (i.e. at the threshold detector) occur T seconds, 2T seconds, and so on after (and before) the centre of the symbol. So when we sample at t=0, T, 2T, etc. we only see the centre of one symbol, all the other symbols being zero at those instants.

3. To shape the pulses such that the ringing dies away in amplitude as quickly as possible, so that timing "jitter" or timing error at the receiver, causing samples to be taken not exactly at the zero crossing times, does not cause very serious error.

2(b) continued

If we combine the transmitting filter, the channel and the receiving filter into a single frequency response H((*f*)) say, then requirement is to make H((*f*)):

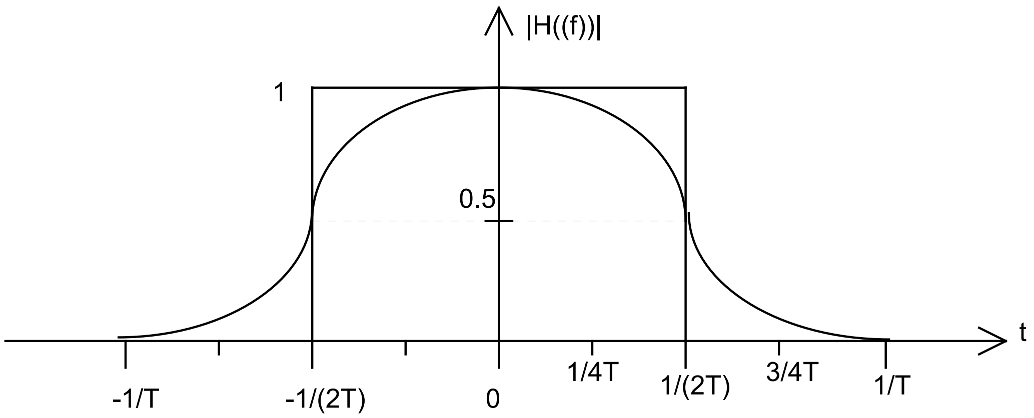
(i) a "Nyquist frequency response" i.e. a frequency response with the required time-domain zero-crossings

(ii) a frequency response whose inverse FT (i.e. the corresponding impulse response) dies away as quickly as possible.

If a time-domain waveform s(t) say has the required zero crossings, sampling it at t= 0, ±T, ±2T, ±3T, … gives a discrete time signal {s[n]} which is equal to s(0) at n=0 and zero for n≠0.
Its DTFT is $S(e^{j\Omega}) = s(0)$ for all Ω and by the sampling theorem, this is equal to the sum

$\sum_{k=-\infty}^{\infty} S((f - k/T))$ with f = ΩT/(2π) .

If S((f)) is band-limited between $f = \pm 1/T$ Hz, only S((*f*)) and S((*f*-1/T)) contribute to $S(e^{j\Omega})$ in the range 0 ≤ f ≤ T. Hence S((*f*)) + S((*f*-1/T)) = s(0).



We need to make the overall frequency response H((*f*)) satisfy Nyquist's first criterion by being a raised cosine spectrum. With a bandlimiting pulse shaping filter at the transmitter and its matched filter equivalent at the receiver, assuming the frequency response of the channel is cancelled out by an equaliser, H((*f*)) will have magnitude spectrum equal to the square of that of the pulse-shaping filter. Hence we make the magnitude spectrum of the pulse shaping filter RRC.

3(a) If symbol $s_i(t)$ with FT $S_i((f))$ is passed through a filter with frequency-response $H((f))$, output waveform produced has FT spectrum $S_i((f))H((f))$ and by the inverse FT is as follows:

$$a_i(t) = \int_{-\infty}^{\infty} H((f))S_i((f))e^{j2\pi ft}\,df$$

If white Gaussian noise (WGN) with 2-sided PSD N_0 /2 watts/Hz is applied to the same filter, the power of the output $n_0(t)$ (coloured Gaussian noise) is:

$$\approx \quad 0.5N_0\int_{-\infty}^{\infty} |H((f))|^2\,df$$

The bandwidth of the noise is assumed infinite or at least much wider than the bandwidth of the filter $H((f))$.
Assuming a threshold $a_i(T)/2$.) error probability is Q(z) with $z = [a_i(T)/2] / \sigma_0$.
We would like to make $[a_i(T)/2] / \sigma_0$ as large as possible.
Let $F = [a_i(T)]^2 / \sigma_0^2$ =

$$\frac{\left|\int_{-\infty}^{\infty} H((f))\,S_i((f))\,e^{2\pi jft}\,df\right|^2}{0.5N_0\int_{-\infty}^{\infty}|H((f))|^2\,df}$$

Schwartz's inequality for complex valued functions x(u) and y(u) states that:

$$\left|\int x(u)y(u)du\right|^2 \le \int|\mathrm{x(u)}|^2\,du\int|\mathrm{y(u)}|^2\,du$$

Equality applies if x(u) = k y*(u)

Taking u as f, x(u) as $H((f))$ and y(u) as $S_i((f))e^{2\pi jft}$ this inequality gives:

$$F \le \frac{\int_{-\infty}^{\infty}|H((f))|^2\,df\int_{-\infty}^{\infty}|S_i((f))e^{2\pi jft}|^2\,df}{0.5N_0\int_{-\infty}^{\infty}|H((f))|^2\,df}$$

$$\therefore \quad F \le \frac{1}{0.5N_0}\int_{-\infty}^{\infty}\left|S_i((f))e^{2\pi jft}\right|^2 df \;=\; \frac{1}{0.5N_0}\int_{-\infty}^{\infty}|S_i((f))|^2\,df$$

For equality, i.e. to maximise F we must have $H((f)) = k\,S_i^*((f))e^{-2\pi jfT}$ for some k. We call this the "matched filter". $H((f))$ is the FT of h(t), the filter's impulse response.
If the FT of $s_i(t)$ is $S_i((f))$, then the FT of $s_i(-t)$ is $S_i^*((f))$.
Also, if the FT of $s_i(t)$ is $S_i((f))$, the Fourier transform of $s_i(t-T)$ is $S_i((f))e^{-2j\pi fT}$.
Therefore h(t) is equal to the symbol $s_i(t)$ modified in the following 3 ways:
(i) reversed in time,
(ii) delayed by T seconds, and
(iii) multiplied by any constant k.

By Parseval's theorem, maximum $F = (2/N_0)\int_{-\infty}^{\infty}(s_i(t))^2\,dt = (2/N_0)\,times\ energy\ of\ s_i(t)$

$= (2/N_0)(2E_b) = 4E_b/N_0$

since average energy per bit, E_b , is average of energies of equi-probable $s_i(t)$ symbols and zero valued symbols, and is therefore equal to half the energy of $s_i(t)$.
Therefore, error probability = $Q([a_i(T)/2] / \sigma_0) = Q((\sqrt{(F)} / 2) = Q(\sqrt{(E_b/N_0)})$ as required.

3(b) We demonstrate two solutions: one for those who thoroughly understand part (a), and one for those who prefer a direct approach based on earlier lectures.

Direct approach:

Consider the output from the "integrate and dump" circuit. The noise has variance:
$\sigma_0^2 = \sigma^2$ T / (2B) = 0.64 x 0.002 / 8000 = 16 x 10^{-8} Therefore σ_0 = 4 x 10^{-4}.
The 2 Volt and 0 Volt pulses become 0.004 and 0 Volt pulses and the decision threshold will become 0.002 Volts.
The error probability is

Prob ("0") x Prob(n_0(T)>0.002) + Prob("1") x Prob(n_0(T) < (0 - 0.002))
= 0.5 x Q(0.002 / σ_0) + 0.5 x Q(0.002 / σ_0) = Q(5) ≈ 2.4 x 10^{-7}

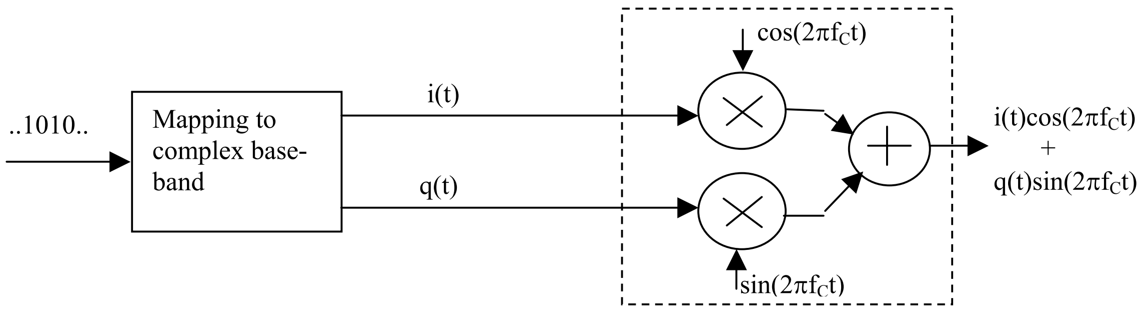
The error-rate is one bit in about 4,000,000. (About one character wrong in 100 page document, or about one serious speech sample error in a G711 64 kb/s transmission about every 100 seconds, assuming errors in the least significant 4 bits are not serious).
Without the integrate & dump circuit, the noise would have variance σ^2 = 0.64 and the decision would be on the basis of 0 Volt and 2 Volt pulses. The threshold would be 1 Volt and the error probability would be the probability of a noise sample (whose variance is 0.64) exceeding 1 Volts when "0" is transmitted or being less than –1 Volts when a "1" is transmitted.
The error probability without the integrate & dump circuit is therefore Q(1/0.8) = Q(1.25) ≈ 10^{-1}.
This is an error-rate of one bit in 10 (Almost all characters wrong, or about 1 in 3 serious speech sample errors at 8 x 8k b/s per second with G711.)

Approach based on part (a):

With integrate & dump filter which is matched filter for rectangular signalling,
N_0 = 0.64 / 4000 = 0.16 x 10^{-3}
E_b = 0.5(Integral of 4 from 0 to T) = 2T = 0.004.
Therefore error prob = Q(√(0.004/0.00016) = Q(√(40/1.6)) = Q(√25) = Q(5).
Hence etc.

Question 4

(a) An "analogue vector-modulation" is as follows:



The signal i(t) is the "in-phase" component of a "complex base-band signal" and q(t) is the "quadrature" component.

The complex base-band signal is said to be complex valued and equal to i(t) + jq(t).

The block shown with a dashed edge is a "vector modulator" multiplying i(t) by the "in-phase carrier component" $\cos(2\pi f_C t)$, q(t) by the "quadrature carrier component" $\sin(2\pi f_C t)$, and adding the two resulting signals together.

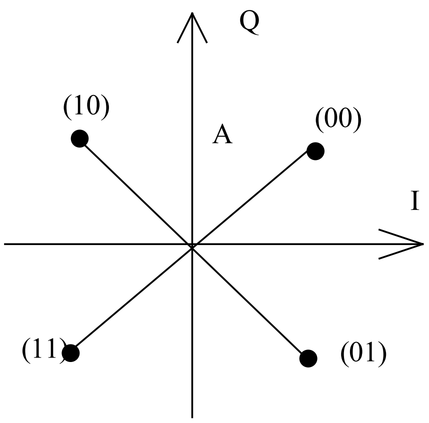
QPSK is achieved by taking alternate bits to produce ±A amplitude bipolar pulses on i(t) and q(t) alternately. A possible approach is summarised by the table below:

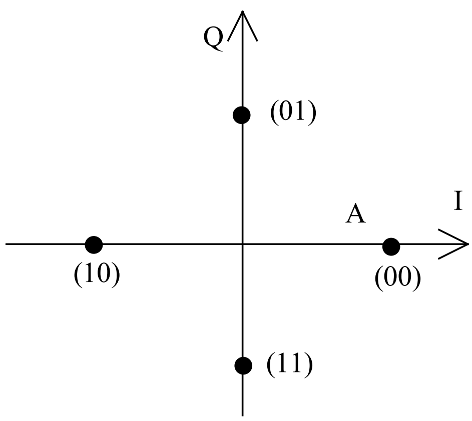
Pair	i(t)	q(t)
0 0	-A	-A
0 1	-A	A
1 0	A	-A
1 1	A	A

This may be summarised as a constellation diagram. In practice, suitably shaped pulse of amplitude A or -A would be used.

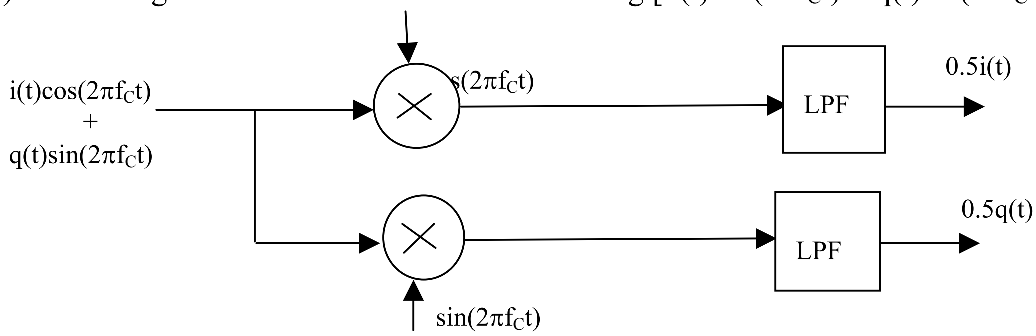
An alternative of (A, 0), (0 , A), (-A , 0), (0, -A) is acceptable.

Constellations for QPSK (either will do):-





4 (b) An "analogue vector-demodulator" for receiving [$i(t)\cos(2\pi f_C t) + q(t)\sin(2\pi f_C t)$] is shown below:



The blocks labelled "LPF" remove the unwanted high frequency components at $2f_C$ Hz..
Since

$$[\, i(t)\cos(2\pi f_C t) + q(t)\sin(2\pi f_C t)]\cos(2\pi f_C t) = 0.5\,[\, i(t) + i(t)\cos(4\pi f_C t) + q(t)\sin(4\pi f_C t)]$$

and

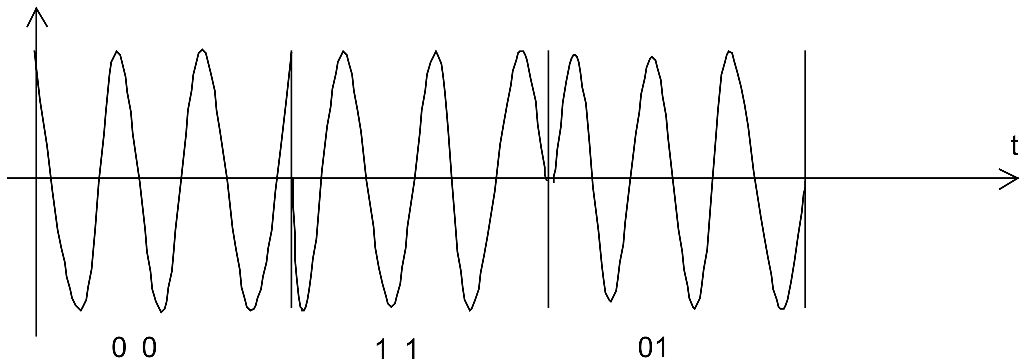
$$[\, i(t)\cos(2\pi f_C t) + q(t)\sin(2\pi f_C t)]\sin(2\pi f_C t) = 0.5\,[\, i(t)\sin(4\pi f_C t) + q(t) - q(t)\sin(4\pi f_C t)]$$

it can be seen how this works.
The vector demodulator is capable of receiving "two signals for the price of one".
It is therefore capable of receiving the i(t) and q(t) channels transmitted by QPSK. Looking up the received amplitudes in the table above gets us back to the bit pairs.

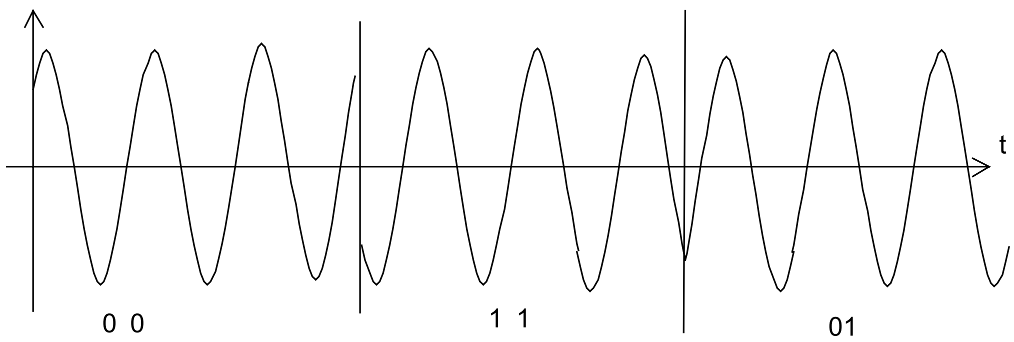
5 (c) Over ASK, constant envelope, reduction in BER in same SNR conditions.
Over FSK, higher bandwidth efficiency generally except when comparing with MSK.

5 (d) Max bit-rate 2 b/s per Hz with 0% pulse shaping.
Difficult to achieve because of sinc time-domain shaped pulse which dies away slowly, needs high order pulse shaping filter to approximate it and because of this, high susceptibility to symbol timing errors.

For second constellation:



For first constellation:-



4 (e) If BER is 1 in 10^{10}, $P_B = 10^{-9}$

If $Q(a(T) / (2\sigma_0)) = 10^{-9}$, then $a(T) / 2\sigma_0 = 6.0$ (from Q(z) graph)

If we decrease a(T) to $a_1(T)$ say, such that $Q(a_1(T) / 2\sigma_0) = 10^{-5}$, then

$a_1(T) / 2\sigma_0 = 4.25$ (from graph)

Therefore $a(T) / a_1(T) = 6 / 4.25 = 1.41$.

In dB the increase is 20 log10 (1.41) = 3 dB.

Therefore decreasing the power by 3 dB will increase the BER from 1 in 10^{10} to 1 in 10^5.

5(a)

Some of the characteristics of speech which may be exploited to reduce the bit-rate:-

(i) Non-uniform amplitude distributions: e.g. lower amplitude sample values are more common than higher ones.

(ii) Sample-to-sample correlation: e.g. adjacent samples are usually fairly close in value.

(iii) Short term periodicity due to vocal tract resonances.

(iv) Long term (pitch-interval to pitch-interval) quasi-periodicity in voiced speech.: characteristic waveform repeated after each pitch-interval whose duration is determined by the rate of vibration of the vocal cords (for voiced speech) which determines the pitch of the voice. Unvoiced speech (consonants) does not have this correlation, but tends to be at lower levels than voiced speech.

(v) Inactivity factors: pauses in speech (60 % duration per speakers)

(vi) Perceptual effects: e.g. ability to band-limit speech to 4kHz without loss of intelligibility and non-sensitivity of hearing to phase,

G711 (64 kb/s pcm) band-limits to 4 kHz. With its a-law companding it quantises the lower level samples more accurately than the higher ones. This tends to lower the quantisation noise when the signal is quiet, and allows it to increase in amplitude when the speech gets louder. Hence it exploits perception. It is a “waveform coding” approach.

RPE-LTP is a “parametric” encoding technique which exploits all the factors apart possibly from (v). It exploits (ii) and (iii) particularly by parametrising the vocal tract resonances and measuring the fundamental frequency of voiced speech portions at suitable intervals of time.

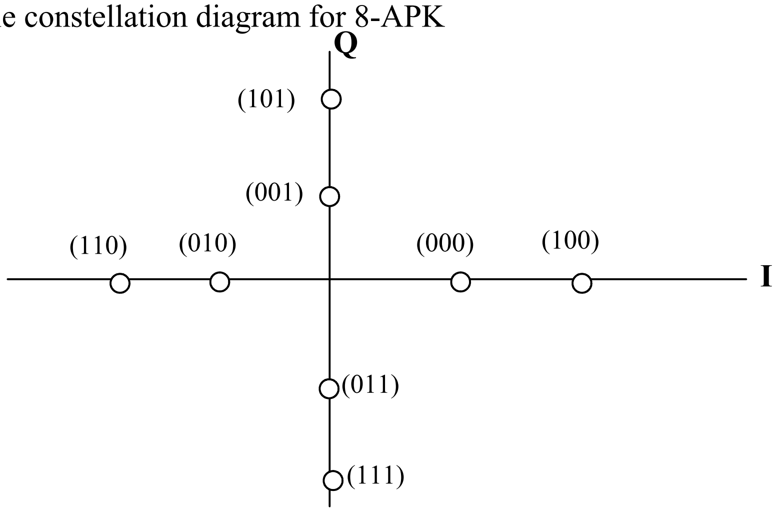
1 (c) Various answers possible: a nice design exercise. Must be multi-level and allow some bandwidth for pulse shaping. e.g. 8-level QAM (gives 3 bits per symbol) and a Baud rate of 13/4 = 3.25 kBaud.

This allows pulse shaping with 100r% RC spectra such that r (64k Hz/ 3) = 32 kHz

This gives r = 0.5, i.e. we can use 50 % RRC pulse shaping.

Gray coding makes the symbol error rate approx equal to the bit error rate when the noise is no so high that noise takes us to non adjacent symbols. Adjacent symbols differ in just one bit with Gray code.

A suitable constellation diagram for 8-APK



Hartley Shannon tells us that theoretical maximum bit-rate that can be transmitted with an arbitrarily small bit-error rate (BER), with a given average signal power, over a channel with bandwidth B Hz which is affected by AWGN is $C = B\log_2\left(1+\frac{S}{N}\right) bits/second$

where S/N is the mean-square signal to noise ratio (not in dB), and the logarithm is to the base 2. To avoid calculating logs to the base 2,

$$C = \frac{1}{\log_{10} 2} B \log_{10}\left(1 + \frac{S}{N}\right) \approx 3.32\, B \log_{10}\left(1 + \frac{S}{N}\right)$$

This means that if S/N >>1, the channel capacity C ≈ 0.332 times B times the SNR in dB.

If C = 64kb/s, B = 32 kHz, if follows that the S/N must be $10^{0.602} - 1 = 2.9$

Our approximation gives us about 6 dB. Interesting.