

Digital Transmission Examination, Jun '96. Answer 3 questions in 2 hours:-

1. A binary digital communication system transmits equally likely signals $s_i(t)$ for $i=1,2$, during the interval $(0,T)$ where $s_1(t) = At/T$ and $s_2(t) = 0$. On the channel, additive white Gaussian noise with a 2-sided PSD $\eta/2 = 10^{-15}$ Watts/Hz adds to the signal. The signals are detected by a matched filter receiver.

- Show that the optimum detection threshold γ_0 can be expressed as $\gamma_0 = a^2T/6$.
- Show that the average bit-error probability P_b can be expressed as $P_b = Q(\sqrt{[A^2T/(6\eta)]})$.
- If $A = 0.2$ mV determine the maximum bit-rate possible with binary signalling such that $P_b \leq 10^{-3}$.

2(a) Sketch the waveform shapes when the bit sequence '01001110' is transmitted using the following line codes (i) NRZ-M (ii) Manchester and (iii) Miller

(b) A voiced signal in the frequency range 300 to 3300 Hz is sampled at 8000 samples/second and then PCM encoded such that the level of quantisation distortion $\leq 0.25\%$ of the peak-to-peak signal voltage. The resulting bit stream is transmitted using a 16-level pulse waveform. Determine the minimum system bandwidth needed in order to detect symbols without ISI if 50% raised cosine filtering is used.

(d) Instead of using raised cosine filtering a three term zero forcing equaliser is used at the receiver in the system described above. Determine the equaliser coefficients if the sample values of the channel pulse response are $r(\pm 2T) = 0.1$, $r(\pm T) = 0.3$ and $r(0) = 1$.

3. A binary FSK communication system transmits equally likely signals $s_1(t) = A \cos(200\pi t)$ and $s_2(t) = A \cos(2200\pi t)$ during the symbol interval $(0,T)$ where $T=0.01$ seconds. On the channel, AWGN with 2-sided PSD $\eta/2$ of 10^{-4} Watts/Hz adds to the signal. The symbols are coherently detected by a matched filter receiver.

- Prove with full justification that the average noise power at the matched filter output can be expressed as $A^2T\eta/2$.
- If $A=0.5$, determine the average bit-error probability P_b .
- What is the theoretical minimum system bandwidth needed for the transmission of these binary FSK signals without ISI? Comment on the practicality of achieving this bandwidth in a real system.

4. (a) For the bit sequence '11100101110010000011' sketch the in-phase and quadrature waveforms associated with a QPSK modulation scheme. In your sketches use one cycle of carrier per quadrature symbol.

(b) QPSK is used to transmit data through a channel with AWGN of 2-sided PSD $\eta/2 = 0.5 \times 10^{-12}$ Watts/Hz. What is the value of the quadrature -modulated carrier amplitudes required in order to ensure that the symbol error probability does not exceed 10^{-4} if the data-rate is 10k bits/second.

(d) If the bit-error probability is the main performance criterion of a digital communication system, which of the following two modulation schemes would be selected for the transmission over a channel with AWGN. Assume that a Gray code is used for the MPSK symbol-to-bit assignment.

Coherent 8-ary orthogonal FSK with $E_b/\eta = 8$ dB.

Coherent 8-ary PSK with $E_b/\eta = 13$ dB.

Digital Transmission Examination Jun '97. Answer 3 questions in 2 hours:-

1. A binary base-band system transmits two signals with equal probability:

$$s_1(t) = \begin{cases} 2At/T & : 0 \leq t \leq T/2 \\ A & : T/2 \leq t \leq T \end{cases} \quad s_2(t) = 0 : 0 \leq t \leq T$$

AWGN with 2-sided PSD $N_0/2 = 6.09 \times 10^{-10}$ Watts/Hz is added to the signal. If the received signal is detected by a matched filter then

- Show that the optimum detection threshold $\gamma = A^2T/3$.
- Show that the average bit-error probability $P_b = Q(\sqrt{[A^2T/(3N_0)]})$.
- Determine the minimum value of A which ensures that a 1 M bit/second transmission rate can be achieved with $P_b \leq 10^{-4}$.

2. Consider a binary FSK with equally likely waveforms $s_1(t) = A \cos(4850\pi t)$ and $s_2(t) = A \cos(4050\pi t)$, which are coherently detected by a matched filter receiver. The data rate is $R_b = 400$ bits/second. The 2-sided PSD of AWGN on the channel is $N_0/2 = 10^{-6}$ Watts/Hz.

- Show that the average signal energy per bit E_b at the input of the receiver is $E_b = A^2T/2$ Joules.
- Determine the average signal-to-noise power ratio required at the input of the receiver and the signal amplitude A , if P_b is to be 10^{-3} or less.
- What is the theoretical minimum system bandwidth for transmitting these binary fsk signals? Comment on practical means for achieving this bandwidth in a real system.

3. A digital telephone system converts the voice signal with a frequency range of 3kHz to a binary PCM signal and transmits it over a channel that is band-limited to 52 kHz. The overall equivalent transfer function is of the raised cosine type with roll-off $r = 0.6$. The sampling rate is 8000 symbols/second.
- Find the maximum PCM digital signal rate which can be used by this system without introducing ISI.
 - Find the maximum number of bits per quantised sample pulse (as an integer) which satisfies the condition given in (a).
 - Suppose that the channel transfer function is unknown and that a three-term zero-forcing equaliser is used at the receiver in the system described above. Determine the equaliser coefficients if the sample values of the channel pulse response are $r(\pm 2T) = 0.2$, $r(\pm T) = 0.4$ and $r(0) = 1$.
4. (a) Determine the corresponding input binary message sequence of a DPSK modulator when the transmitted carrier phase sequence is '0 0 0 π π 0 π 0 0 π 0' where the starting digit is "1".
- (b) Suppose that the reference carrier signal of the correlation receiver for a BPSK system becomes $s_1'(t) - s_2'(t) = 2A \cos(\omega_c t + \phi)$ as a result of imperfect synchronisation in terms of a phase error ϕ . Assume that two signals $s_1(t) = A \cos(\omega_c t)$ and $s_2(t) = -A \cos(\omega_c t)$ are transmitted with equal probability. Show that, in this case, the average bit-error probability $P_b = Q(\sqrt{(2E_b/N_0)\cos^2\phi})$.
- (c) Comparing with a DPSK system using an optimum receiver, where $P_{b,DPSK} = 0.5 \exp(-E_b/N_0)$ estimate the maximum phase error ϕ that the system in part (b) can tolerate, given that both systems are operating at an error rate of 10^{-6} with the same E_b/N_0 .
5. (a) Briefly describe the advantages and disadvantages of using an M-PSK or M-QAM ($M > 2$) signal constellation instead of BPSK [6 marks]
- (b) For the binary bit-sequence: 11001100101, sketch the in-phase and quadrature waveforms associated with a QPSK modulation scheme. Use one cycle of carrier per quadrature symbol in the sketches [6 marks]
- (d) A given QPSK system with the quadrature-modulated carrier amplitude $A = 0.37$ mV is designed to transmit binary data in the presence of AWGN with a 2-sided power spectral density $N_0/2 = 4.5 \times 10^{-12}$ W/Hz. Determine the transmission data rate to ensure that the symbol error probability P_s is less than or equal to 10^{-4} . [8 marks]

Digital Transmission Examination Jun '98. Answer 3 questions in 2 hours:-

1. Describe using diagrams where appropriate the following aspects of binary PSK modulation

- PSK modulation & its parameters
- The spectrum of the PSK signal
- The channel band-width required
- PSK de-modulation
- Differential encoding & decoding

What are the advantages and disadvantages of using binary PSK instead of binary ASK or binary FSK?

In order to transmit at 4.8 kbits/second over a channel with 3.2 kHz bandwidth using m-phase PSK ($m > 2$) what is the minimum value of m required? Explain how you would obtain your answer and justify any assumptions made.

2. Find the auto-correlation function of the random binary square-wave signal which has (i) infinite duration, (ii) isochronous baud-rate $1/T$ Hz, (iii) +1 Volt for "1" & -1 Volt for "0", and (iv) equi-probable and statistically independent occurrence of "1"s and "0". Hence show that the PSD of the signal is $T \text{sinc}^2(\pi f T)$. How does the PSD change if the signal has 0 Volts instead of -1 Volt for "0"?
- (b) State the Sampling Theorem and define the "Nyquist sampling rate". Define and comment on what is meant by the Nyquist bandwidth of digital signals (periodic & non-periodic). What is the Nyquist bandwidth of the signal in part (a)? Why is it preferable to use a larger than Nyquist bandwidth in practice?

3. Show that the output signal to noise power ratio of a matched filter detector for a digital signal at the sampling (decision) instant is $2E/N_0$ where E is the signal energy and $N_0/2$ is "2-sided" the noise PSD, both at the filter input and assuming AWGN.

Show that matched filtering and correlation detection are equivalent for signals in additive white noise.

Equi-probable binary symbols are transmitted by anti-podal signalling over a channel perturbed by AWGN. The E_b/N_0 ratio at the input to a correlation detector is 10 dB, where E_b is the energy in joules per bit of the anti-podal signalling. Calculate the error-rate at the output of the detector.

What would the error rate be if orthogonal signalling was used? How does it differ from the previous result and why?

A graph of $Q(z)$ is provided.

4. Define what is meant by inter-symbol interference (ISI) in a digital transmission system. What causes ISI and how does it affect digital transmission?

Briefly outline the varying methods which can be used to reduce the effect of ISI.

The impulse response of a channel is given by:

$$y(-2T) = -0.05; \quad y(-T) = 0.2; \quad y(0) = 1.0; \quad y(T) = 0.3; \quad y(2T) = -0.07.$$

Calculate the coefficients of a 3-term zero-forcing transversal filter and sketch the circuit of the equaliser. How can the equaliser be made adaptive?

Digital Transmission Examination Jan '99. Answer 3 questions in 2 hours:-

1. (a) What are the main advantages and disadvantages of digital voice circuits in telephony? [7 marks]
 (b) Explain why companding is used when digitising speech for transmission by pulse-code modulation at 64kb/s. [4 marks]

A μ -law compression circuit with input $x(t)$ and output $y(t)$, each in the range $-1V$ to $+1V$, is characterised by the following formula:

$$y(t) = \begin{cases} 0.19 \log_e(1 + 200x(t)) & : x(t) \geq 0 \\ -0.19 \log_e(1 - 200x(t)) & : x(t) < 0 \end{cases}$$

If eight bits per sample are available and $y(t)$ is quantised uniformly, estimate, in dB, the signal-to-quantisation noise ratio obtained at the expander output when the input is a triangular waveform of maximum value $+1 V$ and minimum value $-1 V$. [9 marks]

2.(a) Define what is meant by

- (i) binary frequency shift keying (bfsk)
 (ii) binary differential phase shift keying (binary dpsk) [2 marks]

Summarise the advantages and disadvantages of each of these techniques when used in telephony for voice-band data transmission. [6 marks]

(b) For each of the techniques in part (a) give a suitable detector, and explain how it operates. [4 marks]

For each detector, state whether it is coherent or non-coherent and explain the significance of this term. [2 marks]

(c) An "8-APK" data transmission system with the constellation diagram shown in figure 1 operates at 1800 bits per second with a carrier frequency of 600 Hz. Sketch the analogue waveform produced for the following binary bit-stream and show how this may be generated using a vector modulator.

1 0 0 0 1 0 0 0 0 1 0 1 0 1 1

The effect of bandlimiting and pulse shaping need not be shown. [6 marks]

3.(a) What are the main requirements for a PCM waveform used for the synchronous transmission of digital signals over the telephone network. Give an example of a commonly used scheme and indicate how and to what extent it meets the main requirements. [8 marks]

(b) A synchronous transmission scheme using cable with 20 dB attenuation per km, and regenerative repeaters every 2 km is affected by additive white Gaussian noise and produces an error probability of 10^{-3} . Bipolar signalling is used with zero detection threshold at the sampling points. By how much must the output power of the repeaters be increased if the error probability is to be reduced to 10^{-5} . If the line were cross-talk limited so that the output power could not be increased, how must the repeater spacing be modified to achieve this error probability reduction? A graph of the complementary error function $Q(z)$ is attached. [12 marks]

4.(a) State the Shannon-Hartley Theorem with an explanation of its significance. Show that, according to this theorem, arbitrarily low bit-error rates cannot be achieved from a digital channel if E_b/N_0 is less than about -1.6 dB where E_b is the energy per bit and N_0 is the one-sided power spectral density of additive white Gaussian noise. [8 marks]

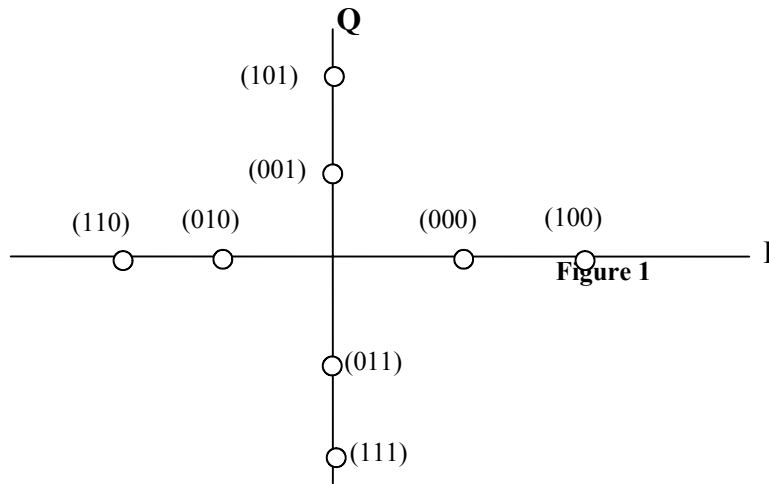
According to Shannon Hartley, what is the maximum bit-rate that can be transmitted and received with arbitrarily small error rates over a 3kHz channel with signalling giving a signal-to-noise ratio of 35 dB? [2 marks]

(b) What is meant by inter-symbol interference in a digital transmission system and how does it arise? [2 marks]

An appropriately shaped symbol centred on $t = 0$ with zero-crossings at $t = \pm T, \pm 2T$, etc. is distorted by the channel such that the received voltages $x(t)$ at $t = 0, \pm T$ and $\pm 2T$ are as follows:

$$x(-2T) = -0.1, \quad x(-T) = -0.2, \quad x(0) = 1, \quad x(T) = 0.1, \quad x(2T) = 0.1$$

Inter-symbol interference is to be reduced around $T=0$ by a 3-term zero-forcing transversal equaliser. Calculate its coefficients and give a diagram of the equaliser. [8 marks]



EE3262 Digital Communications May 2000

Answer THREE questions in two hours.

A graph of the “complementary error function” $Q(z)$ and lists of Fourier Transform properties and trigonometric formulae will be attached to this examination paper.

- 1(a) With respect to the “TCP/IP” reference model for computer network communications, what is the difference between its transmission control (TCP) and user datagram (UDP) protocols? [3 marks]
 Explain why it is useful to have both these protocols available and for each protocol give an application for which it is particularly suitable. [3 marks]
- 1(b) Why is A-law companding used when digitising speech for transmission at 64 kb/s. and how is it generally implemented? [4 marks]
- 1(c) Let the dynamic range of a digital telephone link be defined as:

$$D_y = 10 \log_{10} \left(\frac{\text{Maximum signal power such that there is no overload}}{\text{Minimum signal power giving at least 20dB SQNR}} \right)$$

Given the following formula for A-law compression applied to $x(t)$ in the range $-V$ to $+V$, where $K=1+\log_e(A)$

$$y(t) = \begin{cases} \frac{A}{K} \left(\frac{x(t)}{V} \right) & : |x(t)| \leq V/A \\ 1 + \frac{1}{K} \log_e \left(\frac{x(t)}{V} \right) & : \frac{V}{A} \leq x(t) \\ - \left[1 + \frac{1}{K} \log_e \left(\frac{x(t)}{V} \right) \right] & : \frac{-V}{A} \geq x(t) \end{cases}$$

derive D_y for a 56 kb/s PCM speech channel when the speech signal is A-law companded with $A=40$, and digitised at 7 bits/sample. You may assume speech waveforms are approximately sinusoidal. [7 marks]
 Compare your result with what would have been obtained without companding. [3 marks]

2(a) Define the auto-correlation function (ACF) and the power spectral density (PSD) of a finite power signal $s(t)$. How are these functions related to each other? [2 marks]
 Assume that the finite power signal $s(t)$ transmits equi-probable NRZ rectangular symbols of voltage $+1$ or -1 and duration T seconds, representing an infinite stream of random binary data. Show that the ACF for this “an-isochronous” random signal is:

$$ACF(\tau) = \begin{cases} (1 - |\tau|/T) & : |\tau| \leq T \\ 0 & : |\tau| > T \end{cases} \quad [7 \text{ marks}]$$

Show that $d^2(\text{ACF}(\tau)) / d\tau^2 = (\delta(\tau-T) - 2\delta(\tau) + \delta(\tau+T)) / T$

Hence or otherwise show that the PSD of $s(t)$ is:

$$\text{PSD}(f) = T^2 \text{sinc}^2(\pi f T) \text{ Watts/Hz} \quad [3 \text{ marks}]$$

2 (b) What are the main requirements for a PCM waveform as used for the synchronous transmission of digital signals over the telephone network. [2 marks]

Show how the following bit-stream:

1 1 0 0 0 0 1 0 1

would be line encoded using:

- (i) NRZ-HDB3 coding [2 marks]
- (ii) Manchester coding: [2 marks]

What are the advantages and disadvantages of Manchester coding as compared with NRZ-HDB3? [2 mark]

3(a) Assuming the Schwartz (or Cauchy-Schartz) inequality:

$$\left| \int_a^b x(u)y(u)du \right|^2 \leq \int_a^b |x(u)|^2 du \int_a^b |y(u)|^2 du$$

show that if binary signalling at $1/T$ Baud with equally likely symbols $s(t)$ and 0 is received from a channel affected by additive white Gaussian noise, a matched filter with impulse-response proportional to $s(T-t)$ will minimise the bit-error rate produced by a threshold detector. [6 marks]

If the one-sided power spectral density of the noise is N_0 Watts/Hz, and the average energy of the signalling component of the received signal is E_b Joules per bit, show that the minimised error probability is $P_b = Q(\sqrt{E_b / N_0})$. [4 marks]

3(b). A receiver receives 2 volt and 0 volt approximately rectangular binary symbols at 500 Baud. The transmission is distorted by additive white Gaussian noise (AWGN) bandlimited from -4 kHz to $+4$ kHz with $\sigma^2 = 0.64$ and with zero mean. An integrate and dump circuit is employed. Estimate the error probability assuming an equal occurrence of logic 1's and zeros and an appropriate threshold. [6 marks]

Compare with what would have been obtained without the "integrate and dump" circuit. [4 marks]

A graph of the "complementary error function" $Q(z)$ against z will be attached.

4. (a) What are the requirements of a pulse shaping filter for minimising inter-symbol interference (ISI) when symbols are transmitted at $1/T$ Baud over a band-limited channel. [4 marks]

(b) Using the sampling theorem as quoted below, or otherwise, show that, in theory, ISI is eliminated by shaping each symbol, $s(t)$, transmitted such that when it is received at the threshold decision circuit its Fourier Transform spectrum $S(f)$ satisfies:

$$S(f) = 0 \text{ for } |f| > 1/T$$

$$S(f) + S(f - 1/T) = s(0) \text{ for } -1/T \leq f \leq 1/T \quad [4 \text{ marks}]$$

Sketch the general shape of an "r% raised cosine" spectrum which satisfies this property, and explain why, in practice, "root raised cosine (RRC)" spectrally shaped signalling is employed. [4 marks]

(c) Show that to achieve a band-width efficiency of 2 bits/second per Hz with r% raised cosine spectrum binary signalling at base-band, r must be zero. Sketch the corresponding time-domain waveform. [3 marks]

(d) Explain how duobinary partial response signalling at 2 bits/second per Hz may be achieved by combining a 0% raised cosine filter with a filter whose frequency response is $H_d(f) = 1 + e^{-2\pi j f T}$. [3 marks]

Calculate and sketch the gain response of the combined filter and explain how it would be realised. [2 marks]

The sampling theorem states that:

If $s(t)$ with Fourier transform spectrum $S(f)$ is sampled at $1/T$ Hz, the resulting spectrum is

$$\frac{1}{T} \sum_{k=-\infty}^{\infty} S(f - k/T)$$

5. Consider how a sequence of bits may be transmitted at 100 bits/second through a band-pass channel centred on 300 Hz using each of the following band-pass modulation techniques.

- (a) Four-level (4-ary) amplitude shift keying
- (b) Binary minimum shift keying
- (c) Quaternary phase shift keying (QPSK)

Assume that 50% raised cosine spectral shaping is applied to the base-band signal before modulation

For each of the three modulation schemes give the following information.

- (i) a brief statement defining the modulation technique. [3 marks]
- (ii) a suitable generation method [3 marks]

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|-------|--|-----------|
| (iii) | a suitable receiver | [6 marks] |
| (iv) | the bandwidth efficiency in bits/second per Hz | [3 marks] |
| (v) | a constellation diagram if appropriate | [2 marks] |
| (vi) | a sketch of the waveform generated for the bit stream: 0 0 1 1 0 1 | [3 marks] |

EE3262 Digital Communications 2001**Answer THREE questions in two hours.**

1. (a) Explain the terms “connection orientated” and “connectionless” as applied to digital communication links and computer networks. **[4 marks]**
- (b) Why is A-law companding used when digitising speech for transmission at 64 kb/s and how is it generally implemented? **[4 marks]**
For acceptable speech quality on a land based PCM digital telephone link, the signal to quantisation noise ratio must be at least 30 dB over a dynamic range of at least 40 dB. Assuming speech waveforms to be approximately sinusoidal, estimate the minimum bit-rate required for uniformly quantised speech sampled at 8 kHz. How would the use of companding affect this minimum bit-rate? **[4 marks]**
- (c) What features of speech signals may be exploited to reduce the bit-rate required for land based and wireless telephony? Explain how the GSM 06.10 (RPE-LTP) speech coding technique achieves acceptable speech quality at the required bit-rate: **[8 marks]**

2. (a) Define the cross-correlation $R_{xy}(\tau)$ between two finite energy signals $x(t)$ and $y(t)$. By considering the formula

$$G(\tau) = \text{Min}_{all \lambda} \int_{-\infty}^{\infty} (x(t) - \lambda y(t + \tau))^2 dt$$

explain how $R_{xy}(\tau)$ measures the similarity between the two signals. **[3 marks]**

Give a lower and upper bound for $R_{xy}(\tau)$ in terms of the energies of $x(t)$ and $y(t)$. **[2 marks]**

Show how the convolution of $x(t)$ and $x(T-t)$ may be expressed as a cross-correlation between $x(t)$ and a copy of itself. **[2 marks]**

Show how a correlation detector may be designed to give similar performance to a matched filter detector. **[3 marks]**

- (b) Explain the terms (i) asynchronous and (ii) synchronous as applied to digital transmission. **[2 marks]**

Describe one commonly used asynchronous transmission technique and one commonly used synchronous transmission technique. In each case, discuss the main applications of the technique and its advantages and disadvantages. **[8 marks]**

3. (a) Binary bipolar signalling at $1/T$ baud with equally likely symbols $s(t)$ and $-s(t)$ is received from a channel affected by additive white Gaussian noise. The two-sided power spectral density of the noise is $N_0/2$ watts/Hz. The receiver employs a filter with frequency-response $H(f)$ followed by a threshold detector sampling the filter output at intervals of T seconds at the end of each symbol. Derive an expression for the noise power at the threshold-detector in terms of $H(f)$ and N_0 , stating any assumptions made. **[2 marks]**
- Calculate the value, $a(T)$, of the filter's response to $s(t)$ at $t = T$, and hence derive an expression for the error probability P_B assuming that the most appropriate threshold is chosen. **[3 marks]**
- Assuming the Cauchy-Schwartz inequality:

$$\left| \int_a^b x(u)y(u)du \right|^2 \leq \int_a^b |x(u)|^2 du \int_a^b |y(u)|^2 du$$

show that to minimise P_B , the impulse-response of the filter must be proportional to $s(T-t)$. **[3 marks]**

In this case, derive an expression for $a(T)$ in terms of the energy of $s(t)$. **[2 marks]**

If the average energy of the signalling component of the received signal is E_b joules per bit, show that the minimised error probability is $P_B = Q(\sqrt{2E_b / N_0})$. **[2 marks]**

- (b) A receiver receives 2 volt and -2 volt NRZ rectangular binary symbols at 200 baud. The transmission is distorted by additive white Gaussian noise bandlimited from -3 kHz to $+3$ kHz with $\sigma^2 = 4$ and with zero mean. An integrate and dump circuit is employed before the threshold-detector. Estimate

the error probability assuming an equal occurrence of ‘one’ and ‘zero’ symbols and an appropriate threshold. **[5 marks]**

Compare your answer with what would have been obtained without the “integrate and dump” circuit. **[3 marks]**

A graph of the “complementary error function” $Q(z)$ against z is attached to this examination paper.

4. (a) State the Shannon-Hartley Theorem with an explanation of its significance. Show that, according to this theorem, arbitrarily low bit-error rates cannot be achieved from a digital channel if E_b/N_0 is less than about -1.6 dB where E_b is the energy per bit and $N_0/2$ is the two-sided power spectral density of additive white Gaussian noise. **[8 marks]**

According to the Shannon-Hartley Theorem, what is the maximum bit-rate that can be transmitted and received with arbitrarily small error-rates over a 4 kHz channel with signalling giving a signal-to-noise ratio of 30 dB? **[2 marks]**

- (b) What is meant by inter-symbol interference in a digital transmission system and how does it arise? **[2 marks]**

An appropriately shaped symbol centred on $t = 0$ with zero-crossings at $t = \pm T$,

$\pm 2T$, etc. is distorted by the frequency response of the channel such that the received voltages $x(t)$ at $t = 0, \pm T$ and $\pm 2T$ are expected to be as follows in the absence of channel noise :

$$x(-2T) = -0.1, \quad x(-T) = 0.1, \quad x(0) = 1, \quad x(T) = -0.1, \quad x(2T) = 0.1$$

Inter-symbol interference is to be reduced by a 3-term “zero-forcing” transversal equaliser.

Explain how this may be achieved and give a diagram of the equaliser. **[3 marks]**

Given that the zero-forcing equaliser has coefficients C_0, C_1 and C_2 , expressed as a column-vector \underline{c} , show that the required zero-crossings are achieved if $A\underline{c} = \underline{b}$ where

$$A = \begin{bmatrix} 1 & 0.1 & -0.1 \\ -0.1 & 1 & 0.1 \\ 0.1 & -0.1 & 1 \end{bmatrix}$$

and \underline{b} is a suitably chosen column-vector. **[2 marks]**

Given that the inverse of matrix A is approximately as follows:

$$A^{-1} = \begin{bmatrix} 0.98 & -0.087 & 0.107 \\ 0.107 & 0.98 & -0.087 \\ -0.087 & 0.107 & 0.98 \end{bmatrix}$$

calculate the coefficients C_0, C_1 , and C_2 . **[1 mark]**

Give an expression for the frequency response of the equaliser and comment on how it will affect any additive white Gaussian channel noise. **[2 marks]**

5. (a) Define what is meant by binary PSK modulation and explain, with the aid of a simple diagram the operation of a suitable modulator which incorporates pulse-shaping. **[3 marks]**

Name the parameters which must be known in advance or decided upon by the designer in order to adapt the modulator to data transmission over a given channel. **[2 marks]**

Sketch the general form of the spectrum of a binary PSK signal assuming a random data pattern. **[1 mark]**

Estimate the channel band-width required for a given bit-rate $1/T$ assuming binary PSK with 100% raised cosine pulse shaping. Give the minimum possible bandwidth and mention the difficulties of achieving this minimum bandwidth in practice. **[2 marks]**

With the aid of a simple diagram, explain how binary PSK demodulation may be achieved. **[2 marks]**

Briefly discuss the advantages of differential binary PSK encoding and decoding **[2 marks]**

- (b) What are the advantages and disadvantages of using binary PSK instead of binary ASK or binary FSK? **[4 marks]**

- (c) What is the maximum bit-rate that can be achieved using binary PSK with 3 kHz bandwidth?

Suggest a suitable modulation and pulse-shaping scheme (not necessarily binary PSK) for transmitting data at 4 kbits/second over a channel with 3 kHz bandwidth? Justify any assumptions made in deciding upon this suggestion. **[4 marks]**

Digicoms examination June 2002: Answer 3 questions out of 5.

1.(a) Compared to analogue techniques, what do you consider to be the four main advantages and the most important disadvantage of digital voice transmission in wired and wireless telephony? Why is digital voice transmission used for mobile telephony and exchange to exchange transmissions but not widely for wired links into the home or office? [8]

(b) What features of speech signals are exploited by

- (i) the G711 64 kb/s standard coder for wired telephony
- (ii) the GSM 06.10 (RPE-LTP) 13 kb/s coder for mobile telephony

to achieve acceptable speech quality at the required bit-rate.

[6 marks]

(c) A speech signal is digitised at 13 kb/s. How could this bit-stream be efficiently transmitted over a channel of 6 kHz bandwidth centred on 10 kHz? According to the Shannon-Hartley Law, what signal-to-noise ratio would be required to ensure that arbitrarily low bit-error rates are achievable for this transmission.

[6 marks]

2. (a) Define the auto-correlation function (ACF) and the power spectral density (PSD) of a finite power signal $s(t)$. How are these functions related to each other?

[2 marks]

Assume that the finite power signal $s(t)$ transmits equi-probable NRZ rectangular symbols of voltage +1 or -1 and duration T seconds, representing an infinite stream of random binary data. Show that the ACF for this "an-isochronous" random signal is:

$$ACF(\tau) = \begin{cases} (1 - |\tau|/T) & : |\tau| \leq T \\ 0 & : |\tau| > T \end{cases} \quad [7 \text{ marks}]$$

Show that

$$\frac{d^2}{d\tau^2} ACF(\tau) = \frac{1}{T} (\delta(\tau - T) - 2\delta(\tau) + \delta(\tau + T))$$

Hence or otherwise show that the PSD of $s(t)$ is:

$$PSD(f) = T \text{sinc}^2(\pi f T) \text{ Watts/Hz} \quad [3 \text{ marks}]$$

2 (b) Explain the terms: (i) asynchronous and (ii) synchronous as applied to digital transmission. [2 marks]

Describe one commonly used asynchronous transmission technique and one commonly used synchronous transmission technique. In each case, discuss the main applications of the technique and its advantages and disadvantages. [8 mks]

3 (a) Assuming the Cauchy-Schartz inequality:

$$\left| \int_a^b x(u)y(u)du \right|^2 \leq \int_a^b |x(u)|^2 du \int_a^b |y(u)|^2 du$$

show that if binary signalling at $1/T$ Baud with equally likely symbols $s_1(t)$ and $s_2(t)$ is received from a channel affected by additive white Gaussian noise, a matched filter with impulse-response proportional to $s_1(T-t) - s_2(T-t)$ with an appropriate choice of threshold will minimise the bit-error rate produced by a threshold detector. [8 marks]

If the one-sided power spectral density of the noise is N_0 Watts/Hz, and the average energy of the received difference signal $s_1(t) - s_2(t)$ is E_d Joules, show that the minimised error probability is

$$P_B = Q(\sqrt{E_d / (2N_0)}) \quad [6 \text{ marks}]$$

(b) A receiver receives 2 volt and -2 volt NRZ rectangular binary symbols. The transmission is distorted by additive white Gaussian noise with two-sided PSD $N_0/2 = 1 \times 10^{-3}$ Watts/Hz. If the received signal is detected with the aid of a matched filter what is the maximum bit-rate that can be sent with a bit-error rate of less than 1 bit in 1000. [6 mks]

A graph of the "complementary error function" $Q(z)$ against z is attached to this examination paper.

4. (a) What are the requirements for a pulse-shaping filter for minimising inter-symbol interference (ISI) when symbols are transmitted at $1/T$ Baud over a band-limited channel. [4 marks]

(b) Using the sampling theorem as quoted below, or otherwise, show that, in theory, ISI is eliminated by shaping each symbol, $s(t)$, transmitted such that when it is received at the threshold decision circuit its Fourier Transform spectrum $S(f)$ satisfies:

$$S(f) = 0 \text{ for } |f| > 1/T \\ S(f) + S(f - 1/T) = s(0) \text{ for } -1/T \leq f \leq 1/T \quad [4 \text{ marks}]$$

Sketch the general shape of an "100% raised cosine" spectrum which satisfies this property, and explain why, in practice, "root raised cosine (RRC)" spectrally shaped signalling is employed. [4 marks]

(c) What is the function of an equaliser in a digital receiver?

Explain how some of the effects of channel distortion may be reduced by a 3-term "zero-forcing" transversal equaliser and give a diagram of such an equaliser with coefficients C_0 , C_1 , and C_2 . [4 marks]

Give an expression for the frequency response of the equaliser in terms of the three coefficients and comment on how the equaliser will affect any additive white Gaussian channel noise. [4 marks]

The sampling theorem states that:

If $s(t)$ with Fourier transform spectrum $S(f)$ is sampled at $1/T$ Hz, the resulting spectrum is

$$\frac{1}{T} \sum_{k=-\infty}^{\infty} S(f - k/T)$$

- 5 (a) State what is meant by a vector-modulator and explain how such a modulator can be used to generate a quaternary phase shift keyed (QPSK) signal from a given bit-stream. Give a constellation diagram for the modulation scheme. **[6]**
 (b) Show how a QPSK transmission may be demodulated. **[4 marks]**
 (c) What are the advantages and disadvantages of using QPSK instead of ASK or FSK? **[2 marks]**
 (d) State the maximum bit-rate that can be achieved using QPSK with 3 kHz bandwidth and explain why this maximum would be difficult to achieve in practice. If the carrier is 3 kHz, and the bit-rate is 2 kb/s, sketch the QPSK waveform obtained without pulse shaping **[4 marks]**
 (e) If the channel is affected by additive white Gaussian noise at the receiver and the bit error rate is measured as 1 error in 10^{10} bits, by how much can the transmitter power be reduced before the error rate exceeds 1 in 10^5 . **[4 marks]**

EE3262 Digital Communications Examination June'03 (complete)

Answer Three questions in TWO hours.

Attached: Appendix 1: Graph of complementary error function $Q(z)$ against z .

Appendix 2: Fourier transform properties

Appendix 3: Trigonometric formulae

1. (a) What is the purpose of the 'physical layer' in a digital communication system? **[1 mark]**
 Draw a block-diagram of a digital transmitter and corresponding receiver which is valid for the physical layer transmission of bit-streams over wired or wireless channels. **[3 marks]**
 How may the channel and the receiver circuitry be expected to distort the transmitted wave-forms? **[2 marks]**
 Succinctly state the purpose of (i) a pulse shaping filter (ii) a matched filter and (iii) an adaptive equaliser as may be used in this system. **[4 marks]**
 Explain why a Nyquist frequency-response is required between appropriate points in the block-diagram and indicate where these points occur. **[2 marks]**

- (b) An appropriately shaped symbol with zero-crossings at $t = \pm T, \pm 2T, \pm 3T$, etc. relative to the centre is distorted by the frequency-response of a wired channel. It is estimated by averaging over several training symbols that when the received symbol, $x(t)$ say, is normalised to 1 volt at its centre (assumed to occur at $t=0$), its voltages at $t = \pm T$ and $\pm 2T$ relative to the centre would be as follows in the absence of channel noise:

$$x(-2T) = -0.1, \quad x(-T) = 0.2, \quad x(0) = 1.0, \quad x(T) = -0.3, \quad x(2T) = 0.2$$

Inter-symbol interference is to be reduced by a 3-term 'zero-forcing' transversal equaliser. Explain how this may be achieved and give a diagram of the equaliser. **[3 marks]**

Given that the zero-forcing equaliser has coefficients C_0, C_1, C_2 , expressed as a column vector \underline{c} , show that the required zero-crossings are achieved if $A\underline{c} = \underline{b}$, where

$$A = \begin{bmatrix} 1 & 0.2 & -0.1 \\ -0.3 & 1 & 0.2 \\ 0.2 & -0.3 & 1 \end{bmatrix}$$

and \underline{b} is a suitably chosen column-vector. Given that the inverse of matrix A is approximately as follows:

$$A^{-1} = \begin{bmatrix} 0.93 & -0.15 & 0.12 \\ 0.30 & 0.90 & -0.15 \\ -0.10 & 0.30 & 0.93 \end{bmatrix}$$

calculate the coefficients C_0, C_1 , and C_2 . **[3 marks]**

Give an expression for the frequency-response of the equaliser and comment on how it will affect a stream of similarly shaped symbols at $1/T$ baud received with additive white Gaussian noise.

[2 marks]

2.(a) State Parseval's Theorem for a signal $s(t)$ of finite energy and, using the Fourier transform properties attached, or otherwise, derive an expression for the Fourier transform of $s^*(4T-t)$ in terms of the Fourier transform $S(f)$ of $s(t)$. The 'star' denotes complex conjugate. **[3 marks]**
 Show that if $s(t)$ is passed through a filter with frequency-response $H(f)$, the energy of the output signal will be:

$$\int_{-\infty}^{\infty} |S(f)|^2 |H(f)|^2 df \quad \text{[1 mark]}$$

Define the '2-sided' power spectral density (PSD) of a power signal $n(t)$, and derive an expression for the power of the output when $n(t)$ is passed through a filter with frequency-response $H(f)$. **[3 marks]**

(b) A single symbol equal to $s(t)$ for logic "1" or zero for all t for "0" is transmitted over an ideal channel to represent a single bit. This channel may be assumed to have unity gain and zero phase-shift at all frequencies. The symbol is received with additive white Gaussian noise of '2-sided' PSD $N_0/2$ watts/Hz. To distinguish $s(t)$ from zero, the receiver employs a filter $H(f)$ with impulse-response $s(4T-t)$ and passes its output to a threshold detector. Show that, in principle, the input to the threshold detector at time $t = 4T$ is:

$$a(4T) = \begin{cases} E + n_0 & \text{for logic "1"} \\ n_0 & \text{for logic "0"} \end{cases} \quad \text{where } E = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

and n_0 is a Gaussian random variable of zero mean and variance $EN_0/2$. **[7 marks]**
 Assuming that a "1" or a "0" is equally likely, show that with an appropriate choice of threshold, the probability of making a correct decision is $1 - P_B$ where:

$$P_B = Q\left(\sqrt{\frac{E}{2N_0}}\right)$$

and $Q(z)$ is the "complementary error function" as plotted in the graph attached. **[2 marks]**

(c) Show that if the detector in part (b) is to be realisable in practice, $s(t)$ must be zero for $t > 4T$. **[1 mark]**

If the detector is to be used to detect a stream of bits at $1/T$ baud rather than a single bit, what property must the shape of $s(t)$ have? **[1 mark]**

With $s(t)$ centred on $t=0$, why is it advantageous to design $H(f)$ to have the impulse-response $s(4T-t)$ rather than $s(T-t)$ and make the decision at $t = 4T$ rather than at $t = T$? **[2 marks]**

3. (a) Define what is meant by the 'dynamic range' of a speech quantiser. Why is pseudo-logarithmic companding (A- or mu-law) used when speech is digitised for transmission by pulse-code modulation and how does it increase the dynamic range, as compared with uniform quantisation, without increasing the bit-rate? **[7 marks]**

(b) Digitised speech, uniformly sampled at 8 kHz, is transmitted over a 42 kHz bandwidth channel using QPSK with 50% RRC symbols. Calculate the number of bits per speech sample that may be transmitted and hence estimate the maximum achievable SQNR assuming speech to be approximately sinusoidal. Calculate the bandwidth efficiency. **[7 marks]**

(c) If the signal power of any transmission along the 42 kHz bandwidth channel must be restricted such that the channel signal-to-noise ratio is 10 dB, according to the Shannon-Hartley Law, what is the maximum SQNR achievable with a bit-error rate that can be made arbitrarily low? If the minimum acceptable SQNR is 30 dB, what is the speech dynamic range at the channel capacity? **[6 marks]**

4. (a) Explain the term asynchronous as applied to digital transmission. Describe one commonly used asynchronous technique and its main applications. **[6 marks]**

(b) What are the main requirements for PCM wave-forms used for the synchronous transmission of digital signals over the telephone network. **[2 marks]**

Show how the stream of twelve bits: 1 1 1 0 0 0 0 0 0 1 would be transmitted at base-band using NRZ-HDB3 coding and show how the first six of these bits would be transmitted by Manchester coding **[4 marks]**

What are the advantages and disadvantages of Manchester coding as compared to NRZ-HDB3? **[1 mark]**

(c) What is a regenerative repeater and why are such repeaters used in wired telephony? **[1 mark]**

A synchronous transmission scheme uses cable with 20 dB attenuation per km and identical regenerative repeaters every 2 km. Each repeater is affected by the same level of additive white Gaussian noise and has been set to transmit at the same power level. It is observed that each repeater increases the error-rate by 1 bit in approximately 1000. Calculate the increase in power level that would be required at the output of each repeater to reduce the increase in error-rate per repeater to 1 bit in 10^9 . **[2 marks]**

If the output power of the repeaters could not be increased for practical reasons, how must the repeater spacing be modified to achieve the same error-rate reduction at each repeater? **[2 marks]**

If the total transmission distance is 34 km, how will the overall error-rate be affected by the change in repeater spacing. **[2 marks]**

5.(a) Define what is meant by:

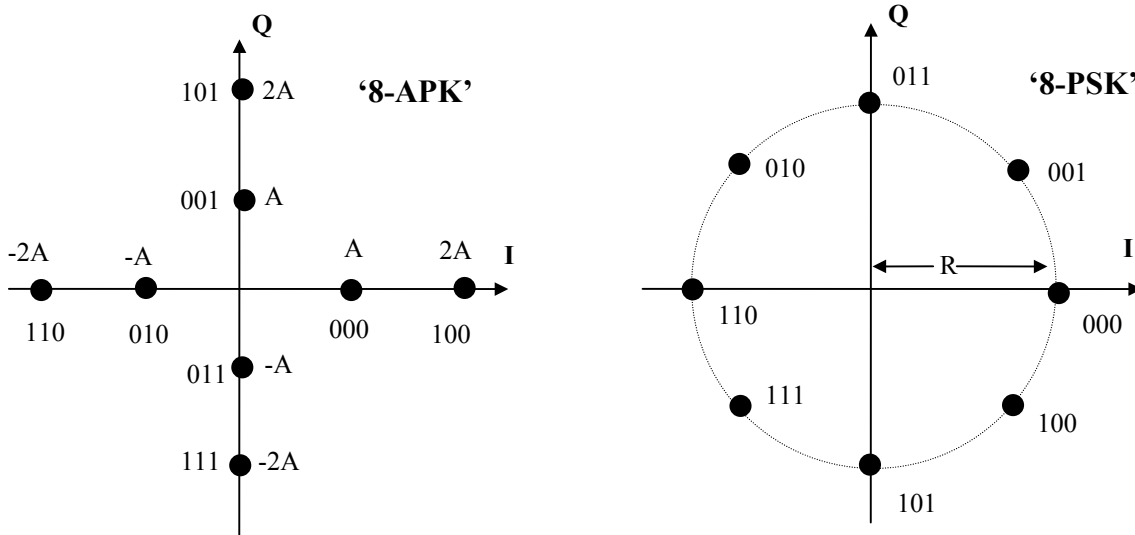
(i) binary frequency shift keying (binary FSK) **[2 marks]**

(ii) Gaussian minimum shift keying (GMSK) **[2 marks]**

What are the main advantages of GMSK that led to its adoption for second generation GSM mobile telephones? Show how GMSK may be generated. **[4 marks]**

(b) Why is Gray coding used in multi-level digital modulation schemes? **[2 marks]**

The constellation diagrams for an ‘8-APK’ modulator and an ‘8-PSK’ modulator are shown below:



Assuming all symbols are equally likely, calculate the average energy per bit, E_b , for each of these modulation schemes when the symbol rate is 1 kbaud. Show that if $R \approx 1.58A$, the average energy per bit is the same for both schemes. **[3 marks]**

Symbol detection is achieved by choosing the nearest of the eight symbols on the constellation diagram to a received symbol and therefore the minimum distance between symbols is taken as a measure of noise immunity. Which of the two schemes has the greater noise immunity for a given E_b/N_0 ? If the ratio E_b/N_0 were unimportant but the peak voltage had to be limited to 1 volt in each case to avoid amplifier non-linearity, which scheme would then have the greater noise immunity? **[3 marks]**

Sketch the waveform produced by each of the schemes above when the 3 kb/s bit-stream is as follows:

0 0 0 1 1 0 0 1 1 1 0 1

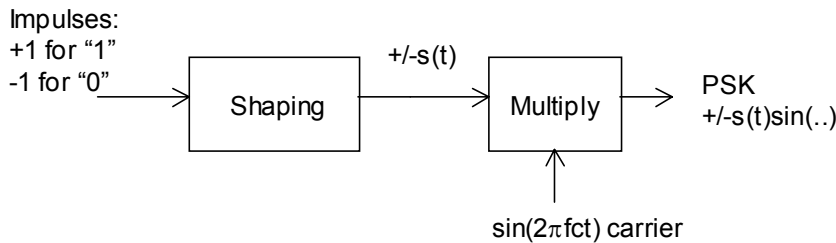
and the carrier frequency is 2 kHz. The effect of band-limiting and pulse-shaping need not be shown.

[4 marks]

Solution methods

Digital Transmission Jun '98. Solution Methods (with additional notes):-

1. (i) Binary PSK modulation (with pulse shaping) & its parameters:-



Transmit a sinusoidal carrier with phase changes determined by the data.

Parameters we need to know are:

- the carrier frequency f_c ,
- the signalling rate $1/T$ baud,
- spectral efficiency (bits/second per Hz)
- channel bandwidth required
- pulse shaping (e.g. 50% raised cosine spectral shaping).
- power of transmission
- time-domain envelope

other parameters may be mentioned also. Note inter-dependency of parameters: spectral efficiency depends on required bandwidth which depends on pulse shaping.

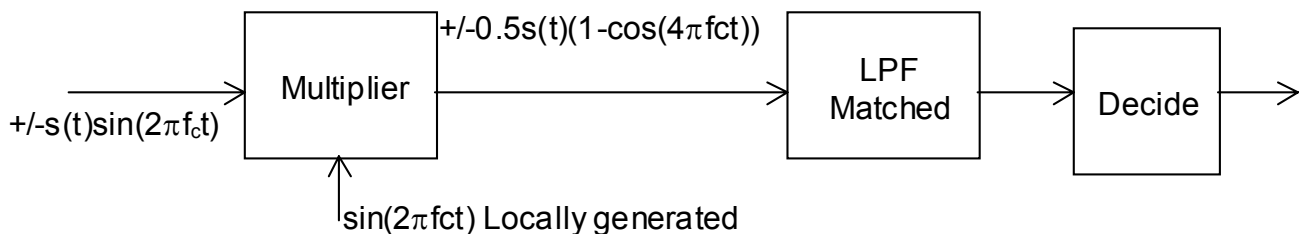
(i) Spectrum of psk is a double side-band suppressed carrier spectrum as for ASK.

The difference between psk & ask is that with psk the modulating signal is anti-podal ($\pm V$) whereas with ask it is unipolar ($+V$ & 0). The difference does not affect spectral occupancy, though it does affect efficiency. Remember that the bandwidth is doubled by the modulation.

The pulse shaping avoids abrupt changes in phase and thus reduces higher frequency components of the spectrum and increases the "sinc"-like decay rate of the time-domain signal that results from the bandwidth limiting. This reduces ISI.

(iii) If signalling rate is $1/T$ and pulse shaping is 100% minimum base-band is $(1/(2T))(1+r)$ Hz. For 50% raised cosine spectral pulse shaping this is $3/(4T)$ Hz at base-band. After modulation this becomes $(1/T)(1+r)$.

(iv) Coherent Detector for binary PSK (with pulse shaping):-



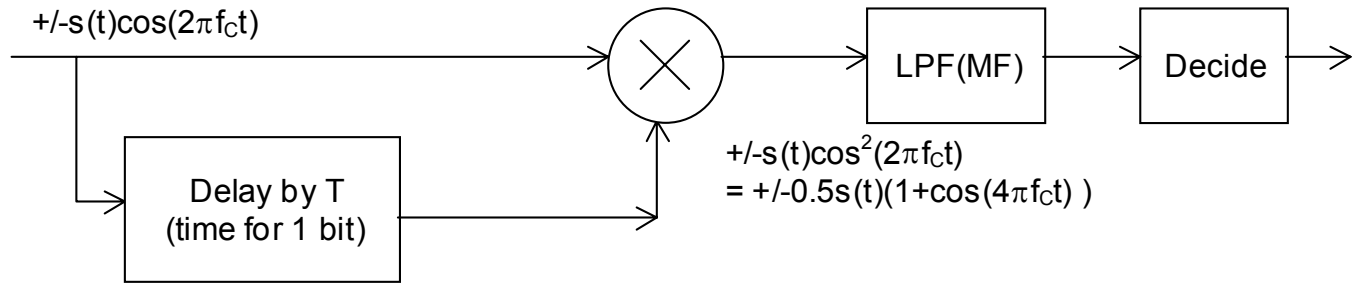
The low-pass filter LPF allows its output to change at the data-rate and eliminates higher frequencies resulting from the $\pm 0.5s(t)\cos(4\pi f_c t)$. If the LPF frequency response is matched to the pulse shape $s(t)$ it will be low-pass as required and optimal for the detection of the polarity of $s(t)$ when the transmission is contaminated by AWGN.

The local carrier must be generated accurately to be of the correct frequency and exactly in phase with the carrier being received. It must do this from the data itself. One method is to square the incoming signal and then divide the frequency of the resulting signal by two.

(vi) Differential PSK

Coherently detected PSK has the problem of maintaining long term carrier phase relationship at the receiver. Minor frequency shifts arising from frequency division multiplexing and de-multiplexing appear as continually changing phase shifts whose effect is accumulated over time. Instead, differential PSK (DPSK) may be used where the phase shift of the carrier with respect to the previous bit transmitted indicates the current bit: say 0 degree shift for "1" and 180 degree for "0". (In practice 90 degree and 270 degree phase shifts are often preferred so that phase changes occur at the data rate even when a long succession of "0"s or "1"s are sent).

Differential detection of DPSK: Considering the case where the phase shifts are 0 degrees and 180 degrees to indicate “0” or “1” respectively the following detection techniques may be used. Instead of generating a local carrier, the detector takes the previous symbol as the required carrier segment. Whether this makes the technique classifiable as “coherent” or “non-coherent” is a matter for discussion. There is certainly some penalty to be paid for this form of detection of differentially encoded PSK as compared with a fully coherent technique, but the penalty is not believed to be great. We assume that pulse shaping has been applied to produce shaping $s(t)$.



(If 90° and 270° rather than 0 and 180° phases are used, introduce a 90° phase shift after the delay).

The matched filter “LPF(MF)” acts as a LPF in getting rid of the $\pm 0.5s(t)\cos(4\pi f_c t)$ component and is optimal for the detection of the polarity of $s(t)$ when the transmission is contaminated by AWGN.

Advantages of PSK:

- (i) PSK has superior performance for a given signal-to-noise ratio than FSK or ASK. The same error-rate is achieved by PSK on a channel having a worse signal-to-noise ratio (by up to about 3 dB), than for FSK (except where MSK frequency spacing is used). This advantage applies to differential PSK.
- (ii) PSK (and DPSK) is less susceptible than FSK to the effects of group-delay distortion.
- (iii) Differential PSK has the additional advantage of being insensitive to inversion and relatively insensitive to phase changes introduced by the channel, for example, by carrier mismatches in FDM demodulation

Disadvantage of PSK:

- (i) Not constant envelope when band-limited by channel
- (ii) Receiver quite complicated even at low bit-rates. Receiver normally coherent with the need to derive a local carrier.

Remember that FSK has constant envelope therefore not sensitive to amplitude (gain) variations in the channel. FSK has particular advantages at low data rates in the simplicity of its transmitter and receiver at low data rates.

FSK is less bandwidth efficient than ASK or PSK except when MSK frequency spacing is used. Bit error rate performance in AWGN worse than for PSK and requires higher SNR for same BER. We lose the advantage of simplicity for FSK at higher data rates, but there are new advantages to be gained especially with MSK and orthogonal FSK as used for DAB and digital TV.

With binary PSK, the best bandwidth efficiency we can hope to achieve is 1 bit/second per Hz.

With QPSK (“4-ary” PSK), say with phases $0, 90^\circ, 180^\circ, 270^\circ$, we can achieve up to 2 bits/second per Hz with the same signalling rate of up to 1 symbol/second per Hz..

With “8-ary” PSK, say with $0, 45^\circ, 90^\circ, 135^\circ, 180^\circ, 225^\circ, 270^\circ$ & 305° we could achieve up to 3 bits/second per Hz with the same signalling rate .

In this case we need to achieve only $4.8/3.2 = 1.5$ bits/second per Hz .

Therefore let’s use QPSK (“4-ary” PSK) with 3.2 symbols/second.

At 1 symbol/second per Hz, this would only need 3.2 kHz, so we can have some nice pulse shaping to reduce ISI.

The use of 100% raised cosine spectral shaping would require a band-width of $3.2(1+r)$ kHz.

If $3.2(1+r) = 4.8$, then $r = 0.5$. Therefore let’s use 50% raised cosine pulse shaping to make good use of the available bandwidth.

Solution to Qn2 (Jun'98). (a) Auto-correlation function (ACF) and “2-sided” PSD of a power signal $x(t)$:

$$ACF(\tau) = \lim_{D \rightarrow \infty} \frac{1}{D} \int_{-D/2}^{D/2} x(t)x(t + \tau)dt$$

$$PSD(f) = \lim_{D \rightarrow \infty} \frac{1}{D} |X_D(f)|^2$$

PSD(f) is the Fourier transform of ACF(τ).

There are two common methods of finding ACF(τ) for this type of problem. The first is more mathematical and the second is graphical . The first is as follows:-

Let $D = 2mT$ and $x(t) = \sum_{k=-\infty}^{\infty} r_k(t)$ with $r_k(t) = \begin{cases} \pm 1 : (k - 0.5)T \leq t \leq (k + 0.5)T \\ 0 : otherwise \end{cases}$

This expresses $x(t)$ as the sum of individual pulses whose values are randomly distributed between ± 1 . We have $2m$ such pulses in the range $-mT \leq t \leq mT$.

$$ACF(\tau) = \lim_{m \rightarrow \infty} \frac{1}{2mT} \int_{-mT}^{mT} \sum_{k=-\infty}^{\infty} r_k(t) \sum_{\ell=-\infty}^{\infty} r_{\ell}(t + \tau) dt$$

$$= \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=m}^m \sum_{\ell=-m}^m \int_{-\infty}^{\infty} r_k(t) r_{\ell}(t + \tau) dt$$

When $k \neq \ell$, $r_k(t)$ and $r_{\ell}(t)$ will either have no overlap, (therefore $r_k(t) r_{\ell}(t) = 0$ for all t) or any overlap will be randomly $+1$ or -1 producing an average of $r_k(t) r_{\ell}(t)$ close to zero, so that in the limit, the contribution of $r_k(t) r_{\ell}(t)$ is zero.

Therefore $ACF(\tau) = \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=m}^m \int_{-\infty}^{\infty} r_k(t) r_k(t + \tau) dt$

Now $\int_{-\infty}^{\infty} r_k(t) r_k(t + \tau) dt = \begin{cases} T - |\tau| : |\tau| \leq T \\ 0 : |\tau| > T \end{cases}$

Therefore $ACF(\tau) = \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=m}^m (T - |\tau|) = \lim_{m \rightarrow \infty} \frac{1}{2mT} (2m)(T - |\tau|)$ when $|\tau| \leq T$

$$= \frac{T - |\tau|}{T} \lim_{m \rightarrow \infty} \frac{1}{2m} (2m) = \frac{T - |\tau|}{T}$$
 when $|\tau| \leq T$

This means that $ACF(\tau) = \begin{cases} 1 - |\tau|/T : |\tau| \leq T \\ 0 : |\tau| > T \end{cases}$ as required.

Let's try it the more graphical way:-

Deal with two simple parts first. Because squaring ± 1 pulses produces 1, $(x(t))^2 = 1$ for all t . Therefore:

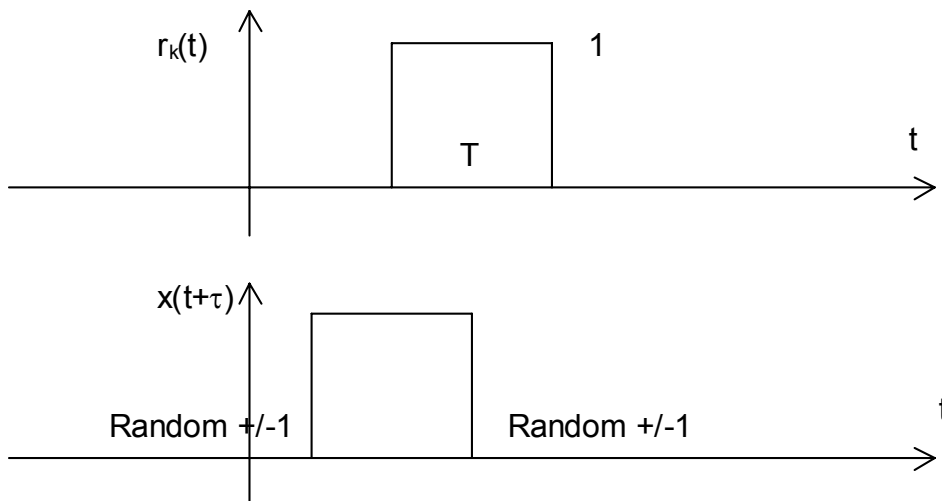
$$ACF(0) = \lim_{D \rightarrow \infty} \frac{1}{D} \int_{-D/2}^{D/2} x(t)^2 dt = \lim_{D \rightarrow \infty} \frac{1}{D} \int_{-D/2}^{D/2} 1^2 dt = 1$$

Also, for any value of positive $\tau > T$, $ACF(\tau)$ must be zero because then for all values of t , $x(t)x(t+\tau)$ is the product of two uncorrelated random variables (chosen from $+1$ & -1). This is because $\tau > T$ and $x(t)$ and $x(t+\tau)$ are in different random pulses. So the product is just as likely to be $+1$ as it is to be -1 . The average of $x(t)x(t+\tau)$ over all t must therefore be zero when $\tau > T$.

A similar situation arises when τ is negative and $< -T$. Again $ACF(t) = 0$ when $\tau < -T$

So now we have to consider the more complicated case where $ACF(\tau)$ when $-T < \tau < T$.

Signal $x(t)$ has an infinite number of random ± 1 valued pulses, but let's look at just one of the $+1$ ones, and let $r_k(t)$ consist of just this one pulse with zero elsewhere. Consider the product of $r_k(t)$ with $x(t+\tau)$



Over the T seconds of $r_k(t)$, the product $r_k(t)x(t+\tau)$ has two parts:

- (a) the overlap of width $T-|\tau|$ where the product is always equal to 1
- (b) the non-overlap where the product will be a random variable equally likely to be +1 or -1.

(The width is $T-\tau$ when $\tau > 0$ (as in diagram) and $T-(-\tau)$ when $\tau < 0$ (not shown). Hence the width formula $T-|\tau|$ works for both cases.)

If we now take $r_k(t)$ to be -1 pulse from $x(t)$ rather than a +1 one, the height of $x(t)x(t+\tau)$ will still be 1 with width $T-|\tau|$. For each $r_k(t)$, the value of the integral of $r_k(t)x(t+\tau)$ over all time will be $(T-|\tau|)$ plus a random component with zero mean. The auto-correlation can now be written as:

$$\begin{aligned} \text{ACF}(\tau) &= \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=m}^m \int_{-\infty}^{\infty} r_k(t)x(t+\tau)dt \\ &= \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=m}^m [(T-|\tau|) + \text{random component}] \\ &= \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=m}^m (T-|\tau|) + 0 = 1-|\tau|/T \end{aligned}$$

Taking the Fourier transform of $\text{ACF}(\tau)$ directly is hard. Instead, let $x(t) = \delta(t+T) - 2\delta(t) + \delta(t-T)$.

Then if $y(t) = \int_{-\infty}^t x(t)dt$ and $z(t) = \int_{-\infty}^t y(t)dt$, the required $\text{ACF}(\tau)$ is $z(\tau)$.

Also, if the FT of $x(t)$ is $X(f)$ then the FT of $y(t) = \int_{-\infty}^t x(t)dt$ is $Y(f) = (1/(2\pi jf)) X(f)$.

Further, the FT of $z(t) = \int_{-\infty}^t y(t)dt$ is $(1/(2\pi jf))Y(f) = (1/(2\pi jf))^2 X(f) = -(1/(4\pi^2 f^2)) X(f)$

Now the FT of $x(t) = \delta(t+T) - 2\delta(t) + \delta(t-T)$ is $X(f) = e^{2j\pi fT} - 2 + e^{-2j\pi fT} = -2(1 - \cos(2\pi fT))$

Therefore FT of $z(t)$ is $(1/(2\pi^2 f^2))(1 - \cos(2\pi fT)) = (1/(2\pi^2 f^2))(2\sin^2(\pi fT))$ by rule: $\cos(2\theta) = 1 - 2\sin^2(\theta)$.
 $= T^2 \sin^2(\pi fT) / (\pi^2 f^2 T^2) = T^2 \text{sinc}^2(\pi fT)$

Therefore $\text{PSD}(f) = \text{FT of } z(\tau)$ is $T^2 \text{sinc}^2(\pi fT)$

For 0 Volts representing symbol zero, $\text{ACF}(0) = 0.5$ since half of the time $(x(t))^2 = 1$ and the other half $(x(t))^2 = 0$.

When $|\tau| > T$, $\text{ACF}(\tau) = \text{Average of } 0 \times 0, 0 \times 1, 1 \times 0 \text{ \& } 1 \times 1 = 0.25$.

When $|\tau| \leq T$, $\text{ACF}(\tau) = 0.25 + 0.25(1 - |\tau|/T)$

$\text{PSD}(f) = 0.25 (\delta(f) + T^2 \text{sinc}^2(\pi fT))$

2(b) For an analogue signal with "1-sided" band-width B Hz, the sampling rate must be greater than 2B samples per second. The Nyquist sampling rate for such an analogue signal is 2B samples/second.

The Nyquist bandwidth of a digital communication signal is half the signalling rate i.e. half the Baud rate.

Nyquist bandwidth of signal in part (a) is $(1/T)/2 = 1/(2T)$ Hz.

It is preferable to use a larger than Nyquist bandwidth in practice because of

- (i) the need to minimise the degree to which ISI occurs due to timing inaccuracies causing us to sample not quite at the time-domain zero-crossings.
- (ii) the need to allow for non-ideal filter characteristics.

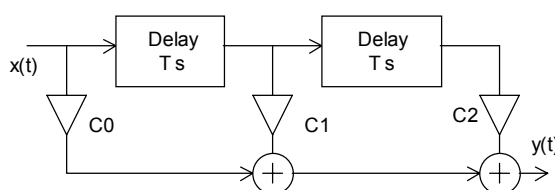
Solution to Qn 3 (Jun'98): First part based on bookwork.

For anti-podal signalling, Error Prob = 3.35×10^{-6}

For orthogonal (unipolar) signalling: Error Prob = 7.65×10^{-4}

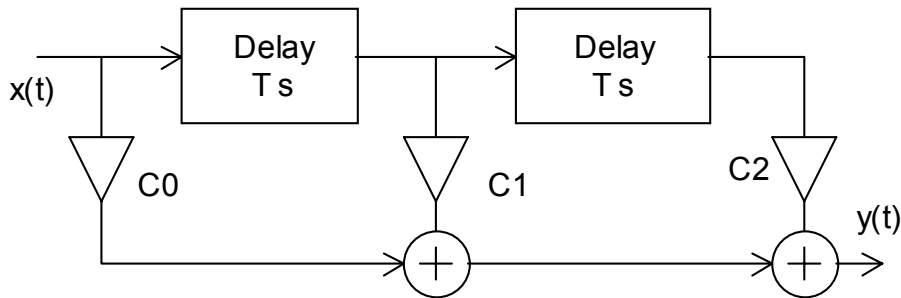
This is higher because of the lower signal energy per bit.

Solution to Qn 4 (Jun'98): Methods for overcoming ISI: pulse shaping, line coding, partial response signalling, equalisation.



$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0.2 & -0.05 \\ 0.3 & 1 & 0.2 \\ -0.7 & 0.3 & 1 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix}$$

$$C_0 = -0.2478 \quad C_1 = 1.1465 \quad C_2 = -0.3613$$



$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -.2 & -.1 \\ .1 & 1 & -.2 \\ .1 & .1 & 1 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix}$$

Solutions to EE3262: Digital Communications Jan 1999

1(a) The main points are:-

Advantages:

- (1) Ease of multiplexing. TDM equipment simpler than FDM etc.
- (2) Ease of signalling as control signals are digital.
- (3) Use of modern technology for DSP etc.
- (4) Integration of transmission and switching.
- (5) Use of links with lower signal to noise ratios possible.
- (6) Regeneration possible
- (7) Easier to accommodate other services, e.g. data video, music
- (8) Performance easily monitored
- (9) Ease of encryption

Disadvantages:

- (1) Increased bandwidth needed
- (2) Need for a/d and d/a conversion
- (3) Need for time-synchronisation
- (4) Topologically restricted multiplexing because of timing delays
- (5) Need to co-exist with existing analogue facilities (e.g. links to home).

1(b) Companding is used to extend the dynamic range of a digitised signal by varying the quantisation step with signal level. Higher sample values are encoded with higher step-sizes, and lower sample values with smaller step sizes; the aim being to make the expectation of the quantisation error to some extent proportional to the signal level. Companding therefore tries to make the signal-to quantisation noise independent of signal level. This is only possible for amplitudes above a certain minimum value.

With uniform quantisation of 8 kHz sampled speech at 8 bits per sample, the dynamic range of input signal levels which give acceptable SQNR is too small for telephony. The maximum SQNR is approximately $6 \times 8 = 48$ dB. and a SQNR of less than about $6 \times 5 = 30$ dB. will be obtained for amplitudes which use only the least significant 5 bits and are therefore 18 dB below the maximum. If 30 dB is taken as the minimum acceptable SQNR, the dynamic range available between minimum input signal level to achieve an acceptable SQNR and maximum allowed level (avoiding overflow) is only 18 dB. The use of A or Mu-law companding extends this range by about 24 dB.

1 (c) Effective step size $\Delta e = \Delta / (dy(t)/dx(t))$

where Δ is the step for $y(t)$ which is $2/256 = 2^{-7}$ as the range is -1 to +1 V which is uniformly quantised with 265 steps.

$$dy(t)/dx(t) = \begin{cases} 38/(1 + 200x(t)) & : x(t) > 0 \\ 38/(1 - 200x(t)) & : x(t) < 0 \end{cases}$$

$$\text{Quantisation noise power} = (1/12) \int_{-1}^1 p(x) \Delta_e^2(x) dx$$

where $p(x)$ is pdf of input signal.

For a triangular signal of amplitude 1, $p(x) = 0.5$ for all x .

Therefore, the quantisation noise power is:

$$\begin{aligned} & 0.5(1+200x)^2 dx + 0.5(1-200x)^2 dx \\ &= (0.5\Delta^2/(12 \times 38^2)) \left(\int_0^1 [x+200x^2+13333x^3] + \int_{-1}^0 [x-200x^2+13333x^3] \right) \\ &= 4.767 \times 10^{-5} \end{aligned}$$

Signal power (triangular wave, +/-1 V) = 1/3

Therefore, SQNR = $1/(3 \times 4.767 \times 10^{-5}) = 6992 = 38.44 \text{ dB}$

Solution to qn 2 (Jan'99):

(a) fsk: Transmit constant amplitude sine wave segments of different frequencies – say f_0 Hz for logic '0' and f_1 Hz for logic '1'.

psk: transmit a sinusoidal carrier of fixed frequency with phase changes determined by the binary data. Differential psk transmits one phase change

(e.g. +90 degrees) for a change of binary digit (i.e. from 0 to 1, or 1 to 0), and a different phase change (e.g. –90 degrees) for no change (0 following 0, or 1 following 1).

Advantage of fsk:

Constant envelope.

Simplicity of transmitter and receiver at low data rates.

Disadvantages of fsk:

(i) More susceptible to the effects of group-delay characteristics of telephone lines than psk.

(ii) Requires higher SNR for same BER.

(iii) A zero-crossing detector (or a detector of similar complexity) must be used for fsk at higher data rates (say 1200 bits/s and above) since only one or two

complete cycles can be transmitted per bit, and this is not enough to allow simple detectors to operate. This is probably as complicated as (or more complicated

than) a psk detector operating at the same data rate. Hence we lose the advantage of simplicity for fsk at higher data rates.

Advantages of dpsk:

(i) psk has superior performance for a given signal-to-noise ratio. The same error rate is achieved by psk on a channel having a worse signal-to-noise ratio (by up to about 3 dB), than for fsk. This advantage applies to differential dpsk.

(ii) psk (and dpsk) is less susceptible than fsk to the effects of group delay distortion.

(ii) Differential dpsk has the additional advantage of being insensitive to inversion and relatively insensitive to phase changes introduced by the channel, for example, by carrier mismatches in FDM demodulation

Disadvantage of psk:

Not constant envelope when bandlimited by channel

Receiver quite complicated even at low bit-rates. Receiver normally coherent with the need to derive a local carrier.

For fsk, any of 4 methods may be chosen from lecture notes: discriminator type, phase-locked loop, zero-crossing detector and a coherent detector.

A discriminator type:-

For each of the two frequencies, f_0 and f_1 say, there is a bandpass filter and an envelope detector (diode & smoothing capacitor say followed by low-pass filter). At the appropriate sampling points, the two outputs are compared, the larger indicating the frequency of the current symbol being transmitted. This is a non-coherent detector.

Differential detection of DPSK:-

Assume an increase of 90 degrees signifies "1" and –90 degrees signifies "0".

The 90 deg phase increase in the diagram means that the symbols will be in phase producing $A^2 \cos^2(\omega_c t)$ or $A^2 \sin^2(\omega_c t)$ when a 0 is received.

They will be 180 degrees out of phase producing $-A^2 \cos^2(\omega_c t)$ or $-A^2 \sin^2(\omega_c t)$ when a 1 is transmitted.

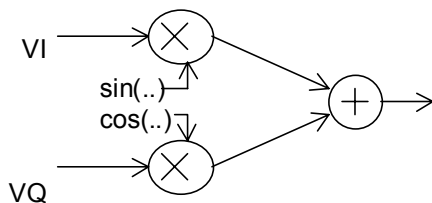
Since $\cos^2(\omega_c t) = 0.5(1 + \cos(2\omega_c t))$ and $\sin^2(\omega_c t) = 0.5(1 - \cos(2\omega_c t))$, the low-pass filter can remove the component at $2\omega_c$. We are left with A^2 when 0 is received and $-A^2$ when 1 is received. Hence we can decide according to the sign of the LPF output. This is non coherent

2(b) contin Coherent detectors operate by mixing the incoming data stream with a locally generated carrier signal. This carrier signal must be of exactly the same frequency and in phase with the carrier of the received signal. Non-coherent detectors do not require such a phase matched carrier and operate by determining the amplitude of the envelope carried by the carrier at appropriate sampling points. The phase relationship between the symbol stream and the carrier becomes unimportant with non-coherent detection. A given symbol, or component of noise will affect the detector output in the same way, regardless of its phase relationship with the carrier. A better performance is obtained with coherent detectors when noise is present, because only half the noise power will be detected by the "in-phase" detector, the other half being seen by a quadrature detector. Hence for the same BER a 3dB worse SNR can be tolerated by a coherent detector than by a non-coherent detector.

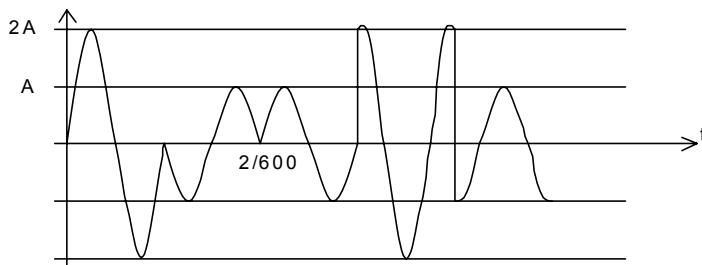
2(c) Assume carrier phase is zero at $t=0$ (start of first symbol). There are 3 bits per symbol, therefore 600 symbols/second. One symbol per cycle:

Symbol allocation table:-

Symbol:	000	001	010	011	100	101	110	111
VI:	+A	0	-A	0	2A	0	-2A	0
VQ:	0	A	0	-A	0	2A	0	-2A



NB VI multiplies "sin" here as in some textbooks.



Solution to Qn3 (Jan'99) .

(a) Data must be sent continuously as a stream of pulses synchronised to a clock waveform. A synchronising code (say 101010) is sent at the start of the transmission, and thereafter, the receiver clock must be kept synchronised from the transmission itself. Pulses must be bandlimited with zeros at sampling points to eliminate inter-symbol interference.

Further requirements:

- (i) the average data level must be zero as the dc component of the transmitted signal is usually lost,
- (ii) the signal must have a frequency component at the clock rate to allow synchronisation at the receiver.

Example: HDB3: uses ternary coding (alternate +V, -V, for logic 1) to keep the average signal voltage at zero. An incorrect polarity 1 (ignored for coding purposes at the receiver) is transmitted in place of any 4th consecutive zero to ensure that there is always a signal present for clock synchronisation even when a long string of zeros are being transmitted.

3.(b) Assume symbols at the sampling points are +0.5 V and -0.5 V. Therefore threshold for decision are at 0 volts. Error probability is 0.001. From table of (zero mean, unit variance) probabilities, or the graph of Q(z), if $Q(z) = 0.001$, then $z = 3.07$ (approx). i.e. if the probability of the normalised Gaussian variable being greater than z is 0.001, then z must be equal to 3.07.

Now assume that the actual noise (after matched filtering etc) has variance σ^2 . Then the probability of the actual noise being greater than 3.07σ is 0.001. To produce an error, the actual noise must exceed the voltage 0.5 V, ie to make -V appear as +V.

Therefore, $3.07\sigma = 0.5$ and this means that the standard deviation of the actual noise must be $\sigma = 0.163$.

From table or graph again, if the error probability must be reduced to $Q(z) = 0.00001$, then z must become equal to 4.25 (approx).

We know that $\sigma = 0.163$.

Therefore $4.25\sigma = 0.69$ must now be the lowest noise value that will cause an error. Therefore the symbol height must be raised from 0.5 to 0.69.

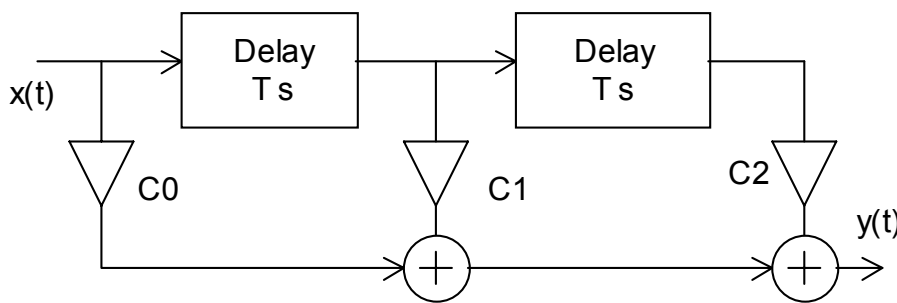
In terms of power, this represents an increase of $20 \log_{10}(1.38/1) = 2.8$ dB.

Or we could reduce repeater spacing by $(2/20) * 2.8 = 0.28$ km.

Solution to Qn4 (Jan'99):

(a) See lecture notes for theory; For $S/N = 35$ dB & $B = 3$ kHz, $C \approx 0.33B \times 35 = 35$ kb/s.

(b) See lecture notes for theory



$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & -.2 & -.1 \\ .1 & 1 & -.2 \\ .1 & .1 & 1 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix}$$

$C_0 = 0.1804$; $C_1 = 0.9592$; $C_2 = -0.114$;

Solutions to EE3262: Digital Communications May 2000

1. (a) Above the "TCP/IP internet" layer is the "TCP/IP transport" layer which is similar to the "OSI transport" layer, except that there are two different end-to-end protocols possible:-

- (i) TCP: (Transmission control protocol) which is designed to be a reliable error free protocol with acknowledgement and retransmission facilities. The data to be sent is fragmented into suitably sized messages (data blocks) which are passed down one by one to the internet layer for transmission. If a suitable acknowledgement is not received for each message within a given time the appropriate message block is retransmitted.
- (ii) UDP (User Datagram Protocol); A "fire and forget" protocol which sends messages as for TCP but does not have acknowledgement/retransmission facilities.

TCP allows error free transmission of data and is suitable for word-processor documents, email and other text and images where real time transmission is not important.

The delay can be very variable and the TCP/IP protocol increases this delay especially when packets are lost and have to be retransmitted.

UDP is not error free but is faster and more suitable for real time data such as speech over IP. Packets received in error and lost packets are not retransmitted and must be corrected or recreated at the receiver.

(b) Main points:-

A μ -law companding is used to increase the dynamic range over which the signal-to-quantisation noise ratio is acceptable to the telephone user.

This is achieved by increasing the quantisation step size as the signal amplitudes increase.

With uniform quantisation, the quantisation noise remains constant in power regardless of the signal power. If this noise power is made small enough to be unobtrusive for a quiet speaker, it is unnecessarily small for a louder speaker. Further, the cost of making the noise unobtrusive for a quiet speaker is to make the step size Δ between adjacent quantisation levels small, and having a small value of Δ restricts the largest amplitude that can be quantised without overflow, especially when the number of quantisation levels is limited.

With 8 bits per sample, there are 256 quantisation levels which is not enough to avoid overflow for larger signals when the uniform step-size is small enough for quieter signals.

More compact answer:-

By a rule of thumb, assuming sinusoidal properties, the signal to quantisation noise ratio (SQNR) for the largest possible sinusoid quantised with 8 bits/sample is $8 \times 6 + 1.8 = 49.8$ dB. If the minimum acceptable SQNR is 30 dB, as is often assumed in telephony, the dynamic range of signals that can be quantised with sufficient accuracy and without overflow is $49.8 - 30 = 19.8$ dB. This dynamic range is not wide enough in practice and companding offers a way of increasing the dynamic range by increasing the quantisation step size as the signal amplitudes increase.

Eight-bit 'A-law' companding is generally implemented by uniformly quantising the signal with 12-bits per sample and then using a look up table to convert the 12-bit uniform samples to 8-bit A-law samples.

1 (b) Max input signal power without overflow = $V^2/2$.

Let minimum input sinusoidal amplitude giving a SQNR ≤ 10 dB be x .

Then minimum input power giving a SQNR ≤ 10 dB = $x^2/2$.

Assume x in linear region, i.e. that $x \leq V/A$.

Then SQNR = $(x^2/2) / (\Delta_e^2 / 12)$ where Δ_e is the "effective step size".

Now $\Delta_e = \Delta / (dy/dx)$ where Δ = step-size used to uniformly quantise $y(t)$.

As there are 2^6 levels to quantise the range of $y(t)$ from -1 to $+1$, $\Delta = 2^{-6}$.

$$K = 1 + \log_e(A) = 1 + \log_e(40) = 4.7$$

$$dy/dx = A/(KV) = 40 / (4.7V) \quad \text{for } -V/A \leq x(t) \leq V/A$$

Therefore $\Delta_e = (4.7/40)\Delta V = 1.836 \times 10^{-3} V$ for $-V/A \leq x(t) \leq V/A$ with $A = 40$.

Now if x is to be the minimum possible such that SQNR ≤ 20 dB, it follows that:

$$(x^2/2) / (\Delta_e^2 / 12) = 10^2$$

$$(x^2/2) = 10^2 (\Delta_e^2 / 12) = 2.81 \times 10^{-5} V^2$$

Note that $x = 7.495 \times 10^{-3} V$ which is clearly in the linear range $-V/40$ to $V/40$.

The SQNR will not get worse than 20 dB as $x(t)$ is made larger than this value.

Now we can calculate the dynamic range:

$$Dy = \frac{2.81 \times 10^{-5} V^2}{(1.83 \times 10^{-3} V)^2 / 12} = 1780 = 32.5 \text{ dB.}$$

For uniform quantisation, the dynamic range is $6 \times 7 + 1.8$ dB - 20 dB = 23.5 dB.

Hence we have gained 9 dB in dynamic range by employing A law companding.

Question 2 (May 2000) Solution:

(a) "Power" or "finite power" signal has finite average power and therefore infinite energy.

Auto-correlation function:

$$R_x(\tau) = \lim_{D \rightarrow \infty} \frac{1}{D} \int_{-D/2}^{D/2} x(t)x(t+\tau)dt$$

Two sided power spectral density:

$$PSD(f) = \lim_{D \rightarrow \infty} \frac{1}{D} |X_D(f)|^2$$

If $x(t)$ is passed thro' an ideal band-pass filter with cut-off frequencies f_L and f_U Hz, the power of the resulting signal is obtained by integrating PSD(f) between $-f_U$ to $-f_L$ and f_L to f_U . Units of PSD(f) are Watts (relative to 1 Ohm) per Hz.

For a "finite power" signal, '2-sided' PSD(f) is the Fourier transform of $R_x(t)$

Let $D = 2mT$ and $x(t) = \sum_{k=-\infty}^{\infty} r_k(t)$ with $r_k(t) = \begin{cases} \pm 1 : (k - 0.5)T \leq t \leq (k + 0.5)T \\ 0 : otherwise \end{cases}$

This expresses $x(t)$ as the sum of individual pulses whose values are randomly distributed between ± 1 . We have $2m$ such pulses in the range $-mT \leq t \leq mT$.

$$ACF(\tau) = \lim_{m \rightarrow \infty} \frac{1}{2mT} \int_{-mT}^{mT} \sum_{k=-\infty}^{\infty} r_k(t) \sum_{\ell=-\infty}^{\infty} r_{\ell}(t + \tau) dt$$

$$= \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=m}^m \sum_{\ell=-m}^m \int_{-\infty}^{\infty} r_k(t) r_{\ell}(t + \tau) dt$$

When $k \neq \ell$, $r_k(t)$ and $r_{\ell}(t)$ will either have no overlap, (therefore $r_k(t) r_{\ell}(t) = 0$ for all t) or any overlap will be randomly $+1$ or -1 producing an average of $r_k(t) r_{\ell}(t)$ close to zero, so that in the limit, the contribution of $r_k(t) r_{\ell}(t)$ is zero.

Therefore $ACF(\tau) = \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=m}^m \int_{-\infty}^{\infty} r_k(t) r_k(t + \tau) dt$

Now $\int_{-\infty}^{\infty} r_k(t) r_k(t + \tau) dt = \begin{cases} T - |\tau| : |\tau| \leq T \\ 0 : |\tau| > T \end{cases}$

Therefore $ACF(\tau) = \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=m}^m (T - |\tau|) = \lim_{m \rightarrow \infty} \frac{1}{2mT} (2m)(T - |\tau|)$ when $|\tau| \leq T$

$$= \frac{T - |\tau|}{T} \lim_{m \rightarrow \infty} \frac{1}{2m} (2m) = \frac{T - |\tau|}{T}$$
 when $|\tau| \leq T$

This means that $ACF(\tau) = \begin{cases} 1 - |\tau|/T : |\tau| \leq T \\ 0 : |\tau| > T \end{cases}$ as required.

Let $x(t) = \delta(t+T) - 2\delta(t) + \delta(t-T)$.

Then if $y(t) = \int_{-\infty}^t x(t) dt$ and $z(t) = \int_{-\infty}^t y(t) dt$, the required $ACF(\tau)$ is $z(\tau)$.

Also, if the FT of $x(t)$ is $X(f)$ then the FT of $y(t) = \int_{-\infty}^t x(t) dt$ is $Y(f) = (1/(2\pi j f)) X(f)$.

Further, the FT of $z(t) = \int_{-\infty}^t y(t) dt$ is $(1/(2\pi j f)) Y(f) = (1/(2\pi j f))^2 X(f) = -(1/(4\pi^2 f^2)) X(f)$

Now the FT of $x(t) = \delta(t+T) - 2\delta(t) + \delta(t-T)$ is $X(f) = e^{2j\pi f T} - 2 + e^{-2j\pi f T} = -2(1 - \cos(2\pi f T))$

Therefore FT of $z(t)$ is $(1/(2\pi^2 f^2))(1 - \cos(2\pi f T)) = (1/(2\pi^2 f^2))(2\sin^2(\pi f T))$ by rule: $\cos(2\theta) = 1 - 2\sin^2(\theta)$.

$$= T^2 \sin^2(\pi f T) / (\pi^2 f^2 T^2) = T^2 \text{sinc}^2(\pi f T)$$

Therefore PDS(f) = FT of $z(\tau)$ is $T^2 \text{sinc}^2(\pi f T)$

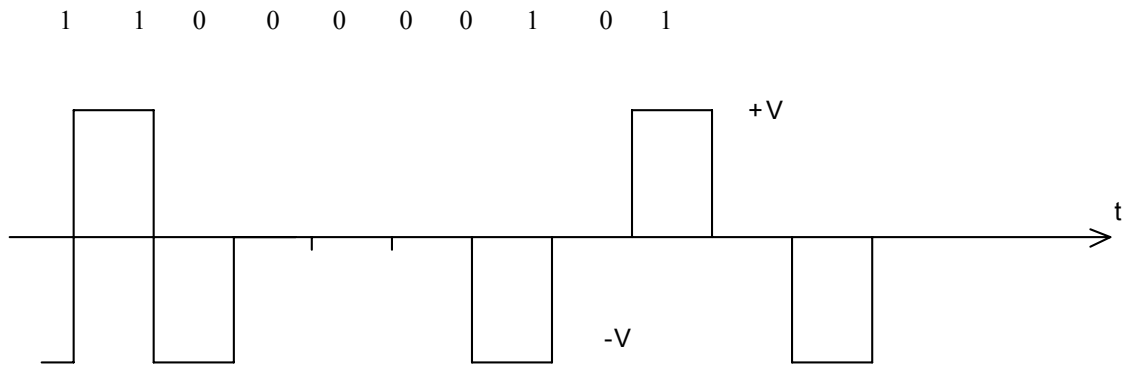
- (b) When considering how to synchronously transmit PCM information, 2 factors must be borne in mind:-
- (1) The DC component of a signal is normally lost over wire lines, because of AC coupling, the use of transformers, etc. We would therefore like to keep the average voltage level zero for data transmission.
 - (2) For synchronous transmission, we need to ensure that the signal always has a frequency component at the signalling rate (or an exact multiple or sub-multiple of the signalling rate) to allow synchronisation at the receiver.

Using AMI is fine when, say, 1 0 1 0 1 0 1 0 ... is transmitted.

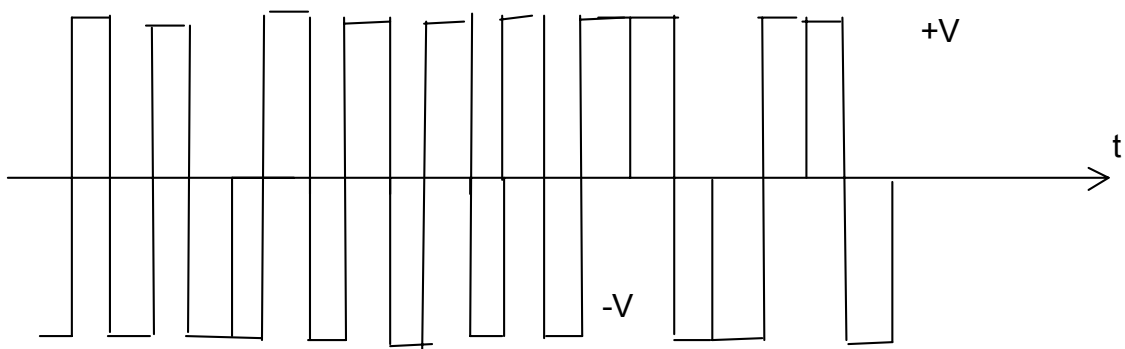
However there are still problems when 0 0 0 0 0 0 ... 0 is transmitted. In this case, the receiver can lose synchronisation. A commonly used solution is known as HDB3 coding.

HDB3 coding: (high density bipolar, order 3):

This scheme uses ternary coding to send binary coded data, as described above, but with an incorrect "1" in place of any 4th consecutive zero



The incorrect "1" is included for clock synchronisation. It is taken to be a data bit of "0" at the receiver. Bi-phase-L, better known as "Manchester coding", and represents a "one" by a pulse of width T/2 positioned during the first half bit-interval. A zero has a pulse of width T/2 in the 2nd half interval.



Manchester coding has the advantage of absolutely guaranteeing zero dc level and is easy for the receiver clock to synchronise itself to. Its disadvantage in comparison to HDB3 is that the bandwidth requirement is considerably higher.

Solution to Question 3 (May 2000)

(a) If symbol $s_i(t)$ with FT $S_i(f)$ is passed through a filter with frequency-response $H(f)$, output waveform produced has FT spectrum $S_i(f)H(f)$ and by the inverse FT is as follows:

$$a_i(t) = \int_{-\infty}^{\infty} H(f)S_i(f)e^{j2\pi ft} df$$

If white Gaussian noise (WGN) with one-sided PSD N_0 Watts/Hz over 0 to B Hz is applied to the same filter, the power of the output $n_0(t)$ (also Gaussian noise) is:

$$Noise\ power = 0.5N_0 \int_{-B}^B |H(f)|^2 df \approx 0.5N_0 \int_{-\infty}^{\infty} |H(f)|^2 df$$

when the bandwidth B of the noise is much wider than the bandwidth of the filter $H(f)$.

Let the noise variance be σ^2 before the matched filter and σ_0^2 after it.

It may be assumed that σ_0^2 is equal to the power of $n_0(t)$.

Assuming a threshold $a_i(T)/2$, error probability is $Q(z)$ with $z = [a_i(T)/2] / \sigma_0$.

We would like to make $[a_i(T)/2] / \sigma_0$ as large as possible.

Let $F = [a_i(T)]^2 / \sigma_0^2 =$

$$\frac{\left| \int_{-\infty}^{\infty} H(f) S_i(f) e^{j2\pi ft} df \right|^2}{0.5 N_0 \int_{-\infty}^{\infty} |H(f)|^2 df}$$

Schwartz's inequality for complex valued functions $x(u)$ and $y(u)$ states that:

$$\left| \int x(u)y(u)du \right|^2 \leq \int |x(u)|^2 du \int |y(u)|^2 du$$

Equality applies if $x(u) = k y^*(u)$

Taking u as f , $x(u)$ as $H(f)$ and $y(u)$ as $S_i(f)e^{2\pi jft}$ this inequality gives:

$$F \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |S_i(f) e^{2\pi jft}|^2 df}{0.5 N_0 \int_{-\infty}^{\infty} |H(f)|^2 df}$$

$$\therefore F \leq \frac{1}{0.5 N_0} \int_{-\infty}^{\infty} |S_i(f) e^{2\pi jft}|^2 df = \frac{1}{0.5 N_0} \int_{-\infty}^{\infty} |S_i(f)|^2 df$$

For equality, i.e. to maximise F we must have $H(f) = k S_i^*(f) e^{-2\pi jfT}$ for some k . We call this the “matched filter”. $H(f)$ is the FT of $h(t)$, the filter’s impulse response.

If the FT of $s_i(t)$ is $S_i(f)$, then the FT of $s_i(-t)$ is $S_i^*(f)$.

Also, if the FT of $s_i(t)$ is $S_i(f)$, the Fourier transform of $s_i(t-T)$ is $S_i(f) e^{-2\pi jfT}$.

Therefore $h(t)$ is equal to the symbol $s_i(t)$ modified in the following 3 ways:

(i) reversed in time, (ii) delayed by T seconds, and (iii) multiplied by any constant k .

$$\begin{aligned} \text{By Parseval's theorem, maximum } F &= (2/N_0) \int_{-\infty}^{\infty} (s_i(t))^2 dt = (2/N_0) \text{ times energy of } s_i(t) \\ &= (2/N_0)(2E_b) = 4E_b/N_0 \end{aligned}$$

since average energy per bit, E_b , is average of energies of equi-probable $s_i(t)$ symbols and zero valued symbols, and is therefore equal to half the energy of $s_i(t)$.

Therefore, error probability = $Q([a_i(T)/2] / \sigma_0) = Q(\sqrt{F}/2) = Q(\sqrt{E_b/N_0})$ as required.

3(b) We demonstrate two solutions: one for those who thoroughly understand part (a), and one for those who prefer a direct approach based on earlier lectures.

Direct approach:

Consider the output from the “integrate and dump” circuit. The noise has variance:

$$\sigma_0^2 = \sigma^2 T / (2B) = 0.64 \times 0.002 / 8000 = 16 \times 10^{-8} \quad \text{Therefore } \sigma_0 = 4 \times 10^{-4}$$

The 2 Volt and 0 Volt pulses become 0.004 and 0 Volt pulses and the decision threshold will become 0.002 Volts.

The error probability is $\text{Prob}("0") \times \text{Prob}(n_0(T) > 0.002) + \text{Prob}("1") \times \text{Prob}(n_0(T) < (0 - 0.002))$

$$= 0.5 \times Q(0.002 / \sigma_0) + 0.5 \times Q(0.002 / \sigma_0) = Q(5) \approx 2.4 \times 10^{-7}$$

The error-rate is one bit in about 4,000,000. (About one character wrong in 100 page document, or about one serious speech sample error in a G711 64 kb/s transmission about every 100 seconds, assuming errors in the least significant 4 bits are not serious).

Without the integrate & dump circuit, the noise would have variance $\sigma^2 = 0.64$ and the decision would be on the basis of 0 Volt and 2 Volt pulses. The threshold would be 1 Volt and the error probability would be the probability of a noise sample (whose variance is 0.64) exceeding 1 Volts when “0” is transmitted or being less than -1 Volts when a “1” is transmitted.

The error probability without the integrate & dump circuit is therefore $Q(1/0.8) = Q(1.25) \approx 10^{-1}$.

This is an error-rate of one bit in 10 (Almost all characters wrong, or about 1 in 3 serious speech sample errors at 8 x 8k b/s per second with G711.)

Approach based on part (a):

With integrate & dump filter which is matched filter for rectangular signalling,

$$N_0 = 0.64 / 4000 = 0.16 \times 10^{-3}$$

$$E_b = 0.5 (\text{Integral of 4 from 0 to } T) = 2T = 0.004.$$

Therefore error prob = $Q(\sqrt{(0.004/0.00016)}) = Q(\sqrt{(40/1.6)}) = Q(\sqrt{25}) = Q(5)$.

Hence etc.

Question 4 Solution (May 2000):

- (a) Because of the channel’s finite bandwidth, the response to any symbol will not be time-limited. It will continue ringing forever, though dying away quite rapidly in amplitude. Inter-symbol interference (ISI) can occur due to the ringing of one symbol into the next.

Three main requirements for the pulse shaping filter are:

1. The shaping filter must produce pulses whose spectra fall within the band width of the channel.

2. To avoid ISI occurring at the receiver’s threshold detector by ensuring that the transmitting filter shapes the symbols so that zero-crossings at the output of the receiving filter (i.e. at the threshold detector) occur T seconds, $2T$ seconds, and so on after (and before) the centre of the symbol. So when we sample at $t=0, T, 2T$, etc. we only see the centre of one symbol, all the other symbols being zero at those instants.

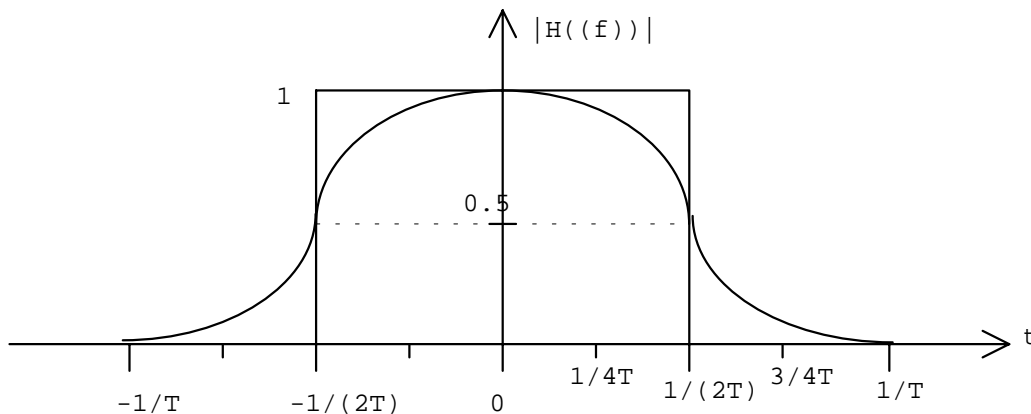
3. To shape the pulses such that the ringing dies away in amplitude as quickly as possible, so that timing “jitter” or timing error at the receiver, causing samples to be taken not exactly at the zero crossing times, does not cause very serious error.

If we combine the transmitting filter, the channel and the receiving filter into a single frequency response $H(f)$ say, then requirement is to make $H(f)$:

- (i) a “Nyquist frequency response” i.e. a frequency response with the required time-domain zero-crossings
- (ii) a frequency response whose inverse FT (i.e. the corresponding impulse response) dies away as quickly as possible.

(b) If a time-domain waveform $s(t)$ say has the required zero crossings, sampling it at $t=0, \pm T, \pm 2T, \pm 3T, \dots$ gives a discrete time signal $\{s[n]\}$ which is equal to $s(0)$ at $n=0$ and zero for $n \neq 0$.

Its DTFT is $S(e^{j\Omega}) = s(0)$ for all Ω and by the sampling theorem, this is equal to the sum $\sum_{k=-\infty}^{\infty} S(f - k/T)$ with $f = \Omega T / (2\pi)$. If $S(f)$ is band-limited between $f = \pm 1/T$ Hz, only $S(f)$ and $S(f-1/T)$ contribute to $S(e^{j\Omega})$ in the range $0 \leq f \leq T$. Hence $S(f) + S(f-1/T) = s(0)$.



We need to make the overall frequency response $H(f)$ satisfy Nyquist’s first criterion by being a raised cosine spectrum. With a bandlimiting pulse shaping filter at the transmitter and its matched filter equivalent at the receiver, assuming the frequency response of the channel is cancelled out by an equaliser, $H(f)$ will have magnitude spectrum equal to the square of that of the pulse-shaping filter. Hence we make the magnitude spectrum of the pulse shaping filter RRC.

4 (c) To achieve 2 bit/s per Hz, the bandwidth of the pulse must be from $-1/(2T)$ to $1/(2T)$ Hz.

$S(f)$ must be zero for $|f| > 1/(2T)$. Therefore $r=0$.

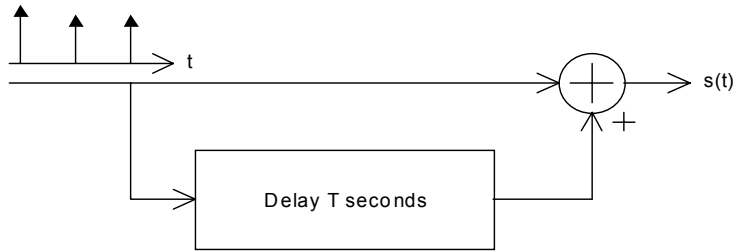
{ To satisfy: $S(f) + S(f-1/T) = \text{constant}$ for $-1/T < f < 1/T$, this means that $S(f)$ must be constant over $-1/(2T) < f < 1/(2T)$. Interesting generalisation not needed here }

Candidates should know by now that the inverse FT of a $-1/(2T)$ to $1/(2T)$ Hz brick-wall frequency response (i.e. a 0% RC spectrum) is $1/T \text{ sinc}(\pi t/T)$ with zero crossings at $\pm T, \pm 2T$, etc. and be able to give a rough sketch of this, without having to calculate it.

4 (d)

Duobinary partial response signalling is a technique that deliberately introduces inter-symbol interference in such a way that it can be removed at the receiver. This can reduce the bandwidth of the transmission or the sharpness of the required pulse shaping filter at the band edges.

Introduce before the transmitting pulse-shaping filter the following filter:

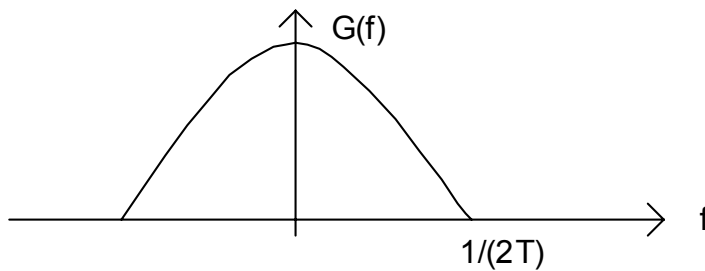
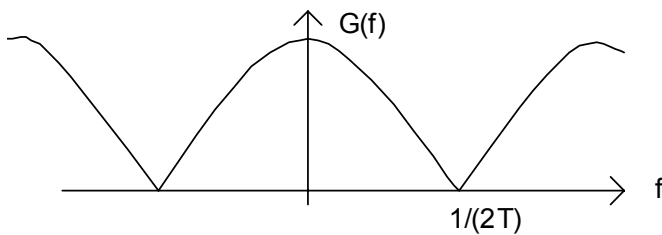


Since delaying a signal by T seconds multiplies its spectrum by $e^{-2j\pi fT}$, the spectrum of this filter is

$$H_p(f) = (1 + e^{-2j\pi fT}) = e^{-j\pi fT} (e^{j\pi fT} + e^{-j\pi fT}) = e^{-j\pi fT} (2 \cos(\pi fT))$$

The gain response is $G(f) = 2 \cos(\pi fT)$ as sketched.

Combining this with the 0% RC filter gives the gain response:



The filter may be realised as an FIR digital filter with multipliers equal to samples of the truncated infinite impulse response. Truncating to FIR incurs distortion, but as serious as with a “sinc” function since the PR filtering makes it die away faster.

Question 5 (May 2000) Solution

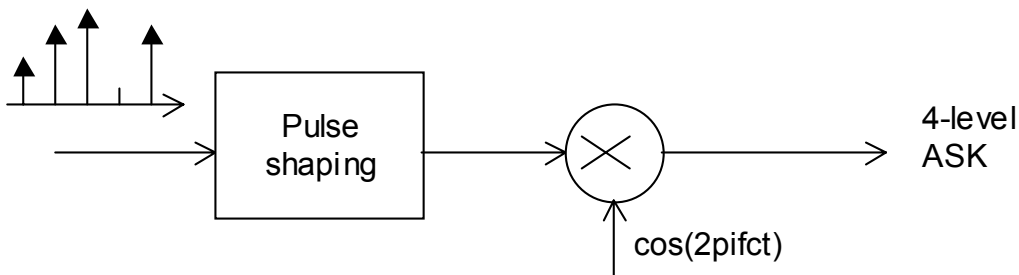
(a) ASK: (i) Combine pairs of bits to be represented by four symbols labelled as:

$$S_0: 00; \quad S_1: 01 \quad S_2: 11 \quad S_3: 10$$

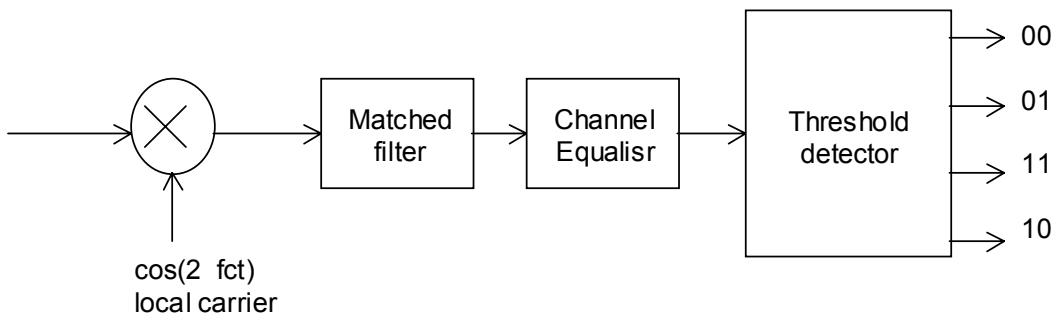
Note the Gray coding to make the symbol error rate approximately equal to the bit error rate.

Represent each symbol by a pulse of height say 0, 1, 2, & 3 Volts respectively obtained by exciting a pulse-shaping filter with impulses of value 0, 1, 2 & 3 Volts. Modulate as shown below.

(ii)

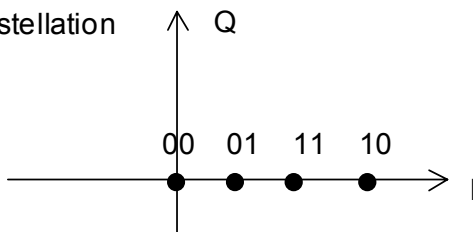


(iii) Coherent receiver



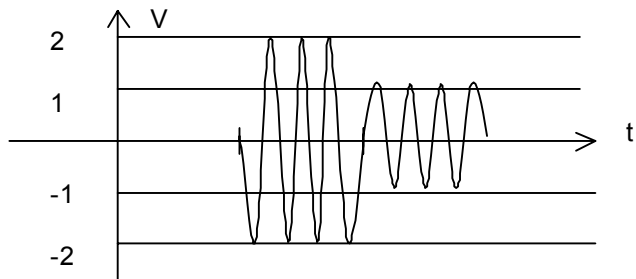
- (iv) Min bandwidth with 50% RC pulses:
 At base-band: $1.5/(2T)$ Hz with $T = 0.01$ seconds.
 After DSB modulation, bandwidth is: $3/(2T)$ Hz = 150 Hz
- (v) 1.5 b/s per Hz

(vi) Constellation



(vi) 0 0 1 1 0 1 becomes the sequence of symbols: $S_0 S_2 S_1$

Question 5 Solution: (continued)



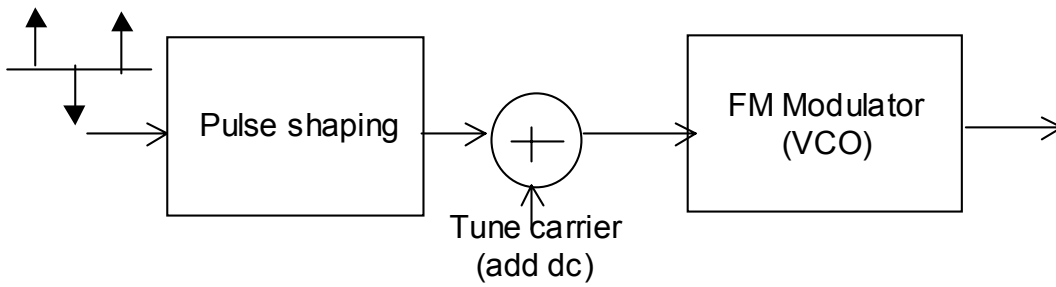
(b) Binary minimum shift keying:

(i) This is binary FSK with frequency separation $1/(2T)$ Hz = 50 Hz.

Let $f_0 = f_c - 25$ Hz and $f_1 = f_c + 25$ Hz. with $f_c = 300$ Hz.

The principle is to transmit a constant amplitude sine wave whose frequency varies between the frequencies assigned to each symbol. For binary signalling there would be two frequencies, f_0 and f_1 say. This narrow spacing makes MSK very efficient in its spectral utilisation, but the price to be paid is increased complexity in the generation and detection process.

(ii) Generation methods:

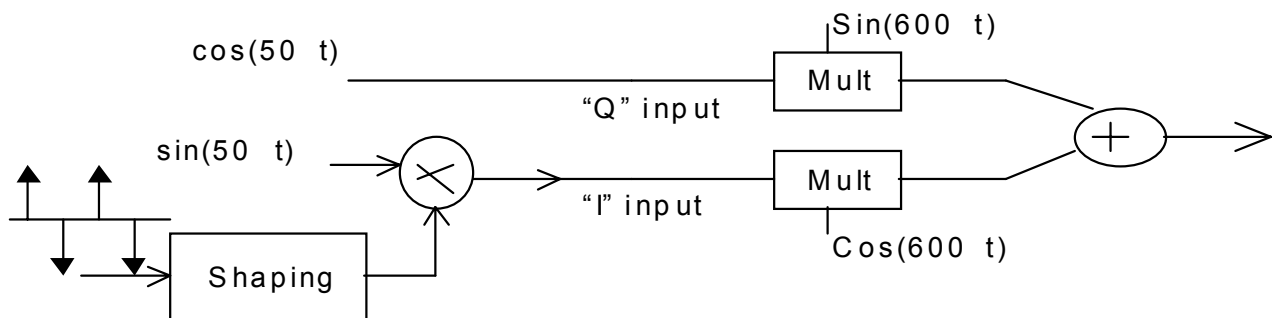


This is “Voltage controlled oscillator(VCO)” or FM modulator method. It involves simply applying a base-band pulse shape to control the frequency of the output. Although a rectangular pulse shape is easiest to visualise, it is better to have a more smoothly changing pulse. The result is a transition between the two frequencies which is gradual rather than sudden and therefore without phase discontinuity. This is a “continuous phase form of FSK i.e. CPFSK.

An alternative way of generating FSK with $f_0 = 257$ Hz and $f_1 = 325$ Hz is to use a “vector modulator” which is illustrated below.

The principle is to apply $\cos(50\pi t)$ to the Q input and $\pm\sin(50\pi t)$ to the I input. The sign determines the symbol: logic “1” or “0”. The shaping filter smooths the changes between +1 and -1 as applied to the multiplier.

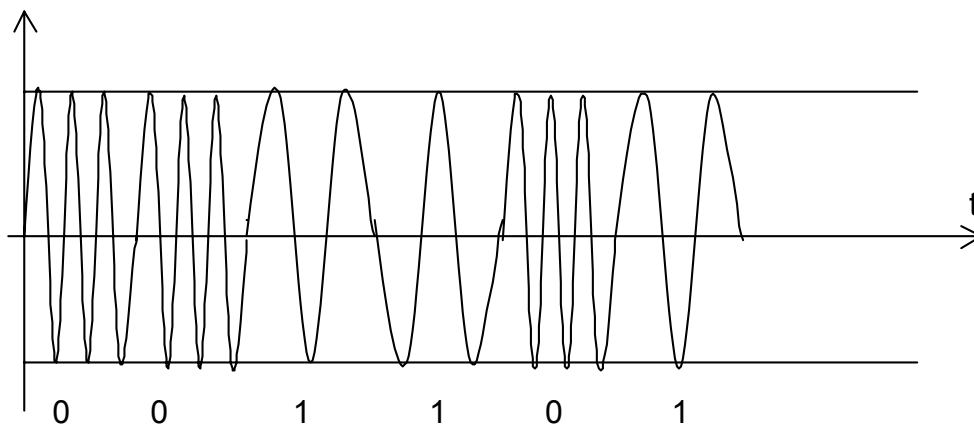
(iii) Detection must be coherent. Similar to technique used for coherent ASK detection. We must have carrier sine-



waves of frequencies f_0 and f_1 locally generated at receiver which match exactly in frequency and phase the FSK symbols being received. The incoming signal is multiplied by both sine waves and the two signals which result are low-pass filtered. A comparator then has to decide which frequency f_0 or f_1 produced the larger output, & that determines the symbol.

- (iv) Min bandwidth with 50% RC: about 100 Hz
- (v) Approaching 1 bit/sec per Hz
- (vi) Not really appropriate.

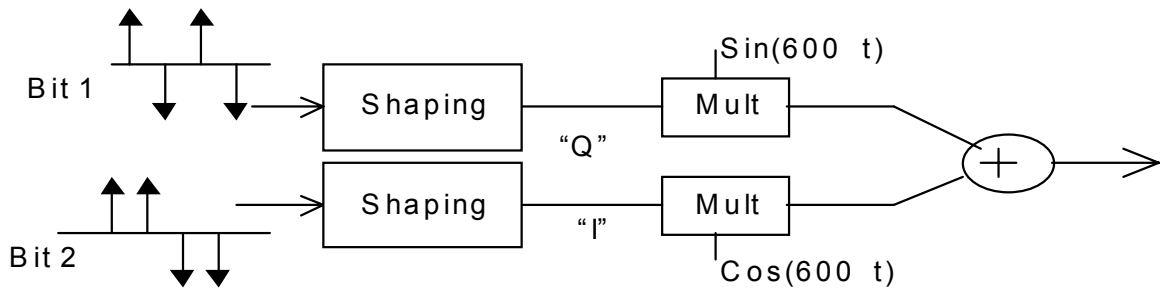
(vi)



(c) QPSK:

(i) Transmits two bits at once using phase shifts which are typically 45° for 00 say, 135° for 01, -135° for 10 and -45° for 11.

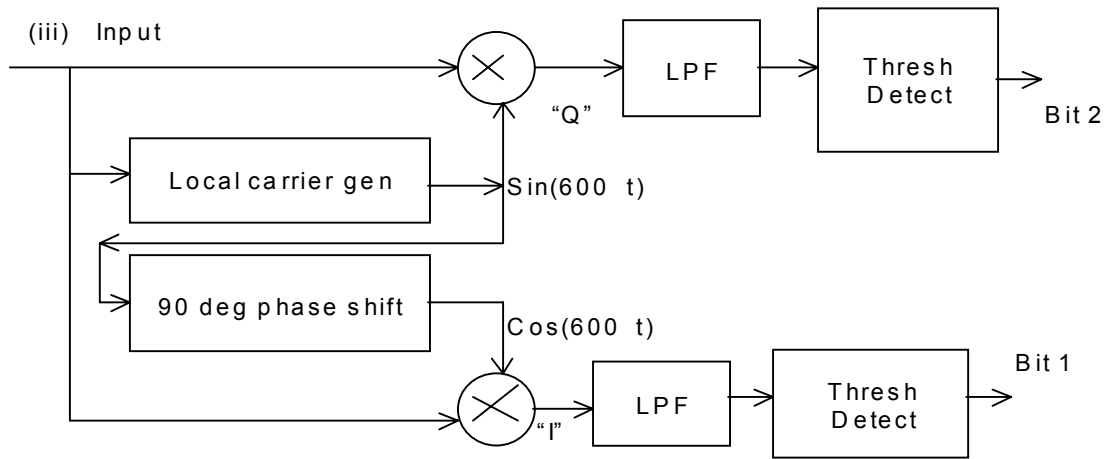
(ii)



Generated as for ASK but with I and Q carriers and with bipolar shaped pulses ($\pm V$) rather than unipolar ($+V$ & 0). If inputs to I and Q of the vector modulator are $+V$ & $+V$ respectively at $t = nT$ the result will be a 45 degree shift of the carrier. $+V -V$ gives -45 degrees and so on.

(iv) Min bandwidth: 150 Hz

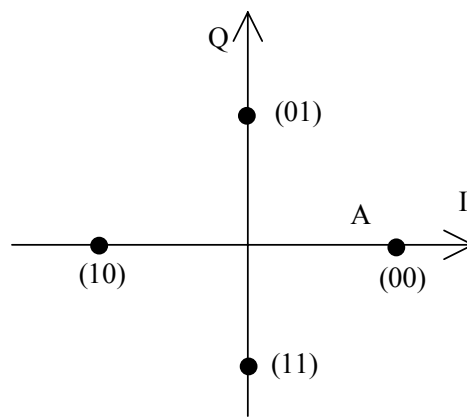
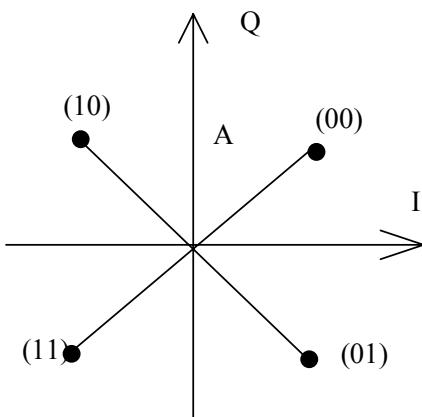
(iii) Input



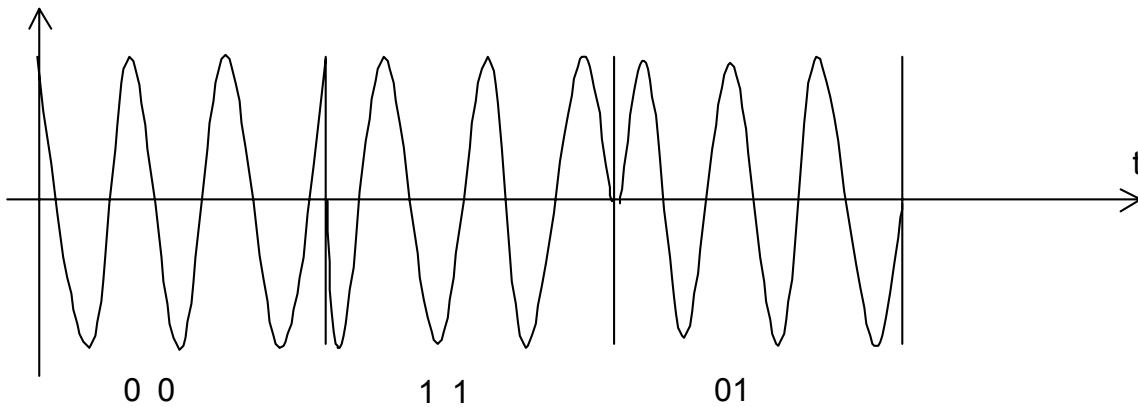
(v) 1.5 bits/second per Hz

Question 5 concluded:

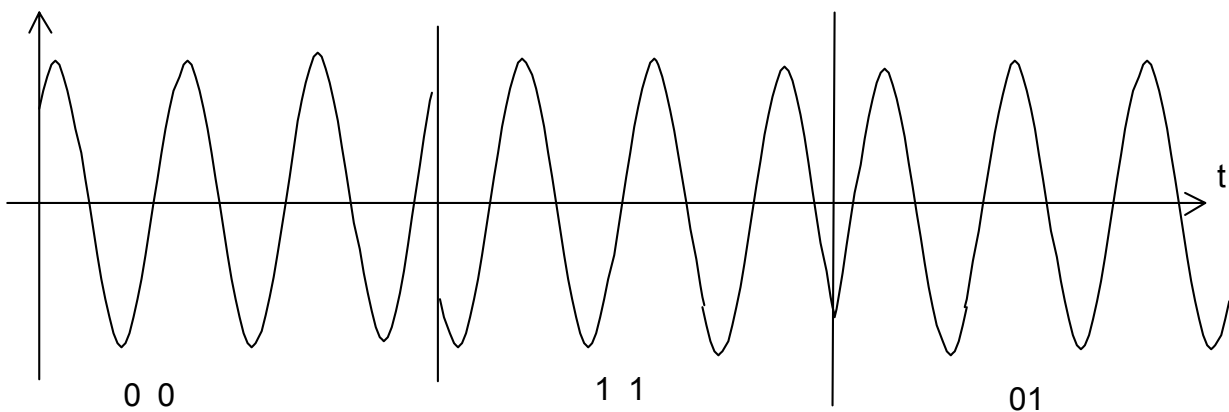
(vi) Constellations for QPSK (either will do):-



(vii) For second constellation:



For first constellation:-



Digital Communications 2001 Examination 2001 solution methods

1 (a) Telephony is traditionally a “connection-oriented” service in that one telephone is set up to be connected to another and remains connected until the call is terminated. [1]

In a connection-oriented digital link, bits or collections of bits will arrive in the order they were sent. [1]

Computer networks, however, are essentially “connection-less”. Digital information is sent in batches or “packets” which may be conveyed by several or many different routes to the receiver. [1]

Each packet may take a different route with different delay, error probability and blocking probability. Packets may not arrive in the order they were sent. Connectionless links may be thought of as being modelled on the postal service rather than the old-fashioned telephone service. Packets of information are like letters, each with an address. However, the packets are sent over the same kinds of channels as are used for telephone calls. [1]

Reference: A.S. Tanenbaum “Computer Networks, 3rd Ed” pp23-24

(b) A μ -law companding is used to increase the dynamic range over which the signal-to-quantisation noise ratio is acceptable to the telephone user. This is achieved by increasing the quantisation step size as the signal amplitudes increase. With uniform quantisation, the quantisation noise remains constant in power regardless of the signal power. If this noise power is made small enough to be unobtrusive for a quiet speaker, it is unnecessarily small for a louder speaker. Further, the cost of making the noise unobtrusive for a quiet speaker is to make the step size Δ between adjacent quantisation levels small, and having a small value of Δ restricts the largest amplitude that can be quantised without overflow, especially when the number of quantisation levels is limited.

With 8 bits per sample, there are 256 quantisation levels which is not enough to avoid overflow for larger signals when the uniform step-size is small enough for quieter signals.

More compact answer:-

By a rule of thumb, assuming sinusoidal properties, the signal to quantisation noise ratio (SQNR) for the largest possible sinusoid quantised with 8 bits/sample is $8 \times 6 + 1.8 = 49.8$ dB. If the minimum acceptable SQNR is 30 dB, as is often assumed in telephony, the dynamic range of signals that can be quantised with sufficient accuracy and without overflow is $49.8 - 30 = 19.8$ dB. This dynamic range is not wide enough in practice and companding offers a way of increasing the dynamic range by increasing the quantisation step size as the signal amplitudes increase.

Eight-bit 'A-law' companding is generally implemented by uniformly quantising the signal with 12-bits per sample and then using a look up table to convert the 12-bit uniform samples to 8-bit A-law samples.

Dynamic range is:

$$10 \log_{10} \left\{ \frac{\text{Max possible signal power (no overflow)}}{\text{Min. power which gives acceptable SQNR}} \right\} \text{ dB.}$$

Let no. of bits be n and step size be Δ .

Max. amplitude is $2^{n-1} \Delta$.

Max power = $(2^{n-1} \Delta)^2 / 2$

Let min amplitude giving acceptable SQNR = A .

SQNR = $10 \log_{10} (A^2/2 / [\Delta^2 / 12]) = 30$

$A^2/2 / [\Delta^2 / 12] = 10^3 = 1000$

Min power = $A^2/2 = 1000[\Delta^2 / 12]$

$$10 \log_{10} \left(\frac{(2^{n-1} \Delta)^2 / 2}{1000[\Delta^2 / 12]} \right) \geq 40$$

$$\frac{2^{2n} / 8}{1000/12} \geq 10,000$$

$4^n \geq 1.5 \times 10^7$ $n = 12$. bits per sample.

Therefore required bit rate is $8k \times 12 = 9600$ bits/second

A law would reduce requirement by 4 bits per sample; i.e. to 64 b/s.

(c) Some of the characteristics of speech which may be exploited:-

- (i) Non-uniform amplitude distributions: e.g. lower amplitude sample values are more common than higher ones.
- (ii) Sample-to-sample correlation: e.g. adjacent samples are usually fairly close in value.
- (iii) Short term periodicity due to vocal tract resonances.
- (iv) Long term (pitch-interval to pitch-interval) quasi-periodicity in voiced speech.: characteristic waveform repeated after each pitch-interval whose duration is determined by the rate of vibration of the vocal cords (for voiced speech) which determines the pitch of the voice. Unvoiced speech (consonants) does not have this correlation, but tends to be at lower levels than voiced speech.
- (v) Inactivity factors: pauses in speech (60 % duration per speakers)
- (vi) Perceptual effects: e.g. ability to band-limit speech to 4kHz without loss of intelligibility and non-sensitivity of hearing to phase, [4]

G711 (64 kb/s pcm) band-limits to 4 kHz. With its a-law companding it quantises the lower level samples more accurately than the higher ones. This tends to lower the quantisation noise when the signal is quiet, and allows it to increase in amplitude when the speech gets louder. Hence it exploits perception. It is a "waveform coding" approach. [2]

RPE-LTP is a "parametric" encoding technique which exploits all the factors apart possibly from (v). It exploits (ii) and (iii) particularly by parametrising the vocal tract resonances and measuring the fundamental frequency of voiced speech portions at suitable intervals of time. [2]

2 (a) The cross-correlation function between two real finite energy signals $x(t)$ and $y(t)$:

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)y(t + \tau)dt$$

Measures similarity between shapes of these signals when $y(t)$ is advanced by τ seconds (i.e. delayed by $-\tau$ seconds).

Let

$$G(\tau) = \underset{\text{all } \lambda}{\text{Min}} \int_{-\infty}^{\infty} (x(t) - \lambda y(t + \tau))^2 dt$$

For any τ , $G(\tau)$ is a measure of how close we can make $\lambda y(t+\tau)$ to $x(t)$ by a suitable choice of +ve or -ve λ . The smaller $G(\tau)$ the better the match between the shapes.

As λ can amplify or attenuate $y(t)$ to best advantage, only the shape of $y(t)$ matters.

$G(\tau)$ is the minimum possible value of

$$I_{\tau}(\lambda) = \int_{-\infty}^{\infty} (x(t))^2 dt + \lambda^2 \int_{-\infty}^{\infty} (y(t + \tau))^2 dt - 2\lambda \int_{-\infty}^{\infty} x(t)y(t + \tau)dt$$

over all possible values of λ . Now:

$$I_{\tau}(\lambda) = E_x + \lambda^2 E_y - 2\lambda R_{xy}(\tau)$$

where E_x is the energy of $x(t)$ and E_y is the energy of $y(t+\tau)$ which is identical to the energy of $y(t)$.

Setting $d I_{\tau}(\lambda) / d\lambda = 0$ gives: $2\lambda E_y - 2R_{xy}(\tau) = 0$

$$\text{Therefore } \lambda = R_{xy}(\tau) / E_y$$

This must be a minimum as $d^2 I_{\tau}(\lambda) / d\lambda^2 = 2E_y$ which is +ve.

Substituting for λ we obtain:

$$G(\tau) = E_x - (R_{xy}(\tau))^2 / E_y$$

The larger $(R_{xy}(\tau))^2$ the smaller $G(\tau)$ and the closer the similarity between $x(t)$ & the shape of $y(t)$.

We note that $G(\tau)$ is always positive by its original definition. Therefore,

$$0 \leq E_x - (R_{xy}(\tau))^2 / E_y$$

$$\text{i.e. } (R_{xy}(\tau))^2 \geq E_x E_y \quad \text{Therefore:}$$

$$-\sqrt{(E_x E_y)} \leq R_{xy}(\tau) \leq \sqrt{(E_x E_y)} \quad \text{for all } \tau$$

If we are looking for similarity, $R_{xy}(\tau) = \pm\sqrt{[E_x E_y]}$ is best and $R_{xy}(\tau)=0$ is worst.

If the maximum value of $(R_{xy}(\tau))^2$ is only slightly less than $E_x E_y$, we say that $x(t)$ and $y(t)$ are quite strongly correlated.

If the maximum value of $(R_{xy}(\tau))^2$ is close to zero, $x(t)$ and $y(t)$ are weakly correlated. If $\max\{(R_{xy}(\tau))^2\}$ is zero, i.e. if there is no value of τ for which $(R_{xy}(\tau))^2$ is non-zero, then we say that $x(t)$ and $y(t)$ are uncorrelated.

Convolution:

Let $x_n(t)$ be the n th symbol of a digital communication signal at $1/T$ baud. Consider the detection of $x_0(t)$, i.e. the pulse with maximum energy between $t=0$ and $t=T$, using a matched filter. Refer to $x_0(t)$ as $x(t)$ for convenience.

The matched filter has impulse response $x(T-t)$ and its response to $x(t)$ is the convolution of $x(t)$ with $x(T-t)$ i.e.:

$$\begin{aligned} x(t) \otimes x(T-t) &= \int_{-\infty}^{\infty} x(\tau)x(T - (t - \tau)) d\tau \\ &= \int_{-\infty}^{\infty} x(\tau)x(\tau - [t - T]) d\tau = R_{xx}(T - t) \end{aligned}$$

At $t = T$, the sampling point for $x_0(t)$, the matched filter response equals $R_{xx}(0)$.

$$R_{xx}(0) = \int_{-\infty}^{\infty} x(\tau)x(\tau)d\tau = \int_{-rT}^{rT} x(\tau)x(\tau)d\tau$$

when $x(t)$ is approximated by a finite duration signal from $-rT$ to $+rT$.

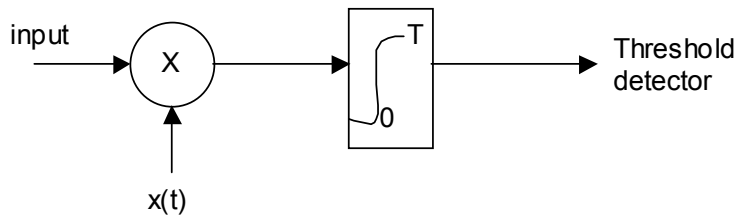
If $x(t)$ were time limited to $0 \leq t \leq T$, we would simply have:

$$R_{xx}(0) = \int_0^T x(\tau)x(\tau)d\tau$$

Better students may refer to the following point:

Normally we must integrate over more intervals of T , say from $-rT$ to $+rT$ (implemented with a delay of rT). As well as dealing with $x_0(t)$ here, we also have the ringing of the other symbols $x_n(t)$ for $n=\pm 1, \pm 2, \pm 3, \dots$ within the signal being filtered (or integrated). However, we are safe in the knowledge that $R_{xx}(0)$ due to $x_0(t)$ will not be affected by the matched filter output (ie $R_{x_n x}(0)$) corresponding to any earlier or later symbol $x_n(t)$ for $n=\pm 1, \pm 2, \pm 3, \dots$. This is the effect of pulse shaping etc. which forces the output of the matched filter (or correlation detector) to be zero for all other symbols at $t=T$ in this case.

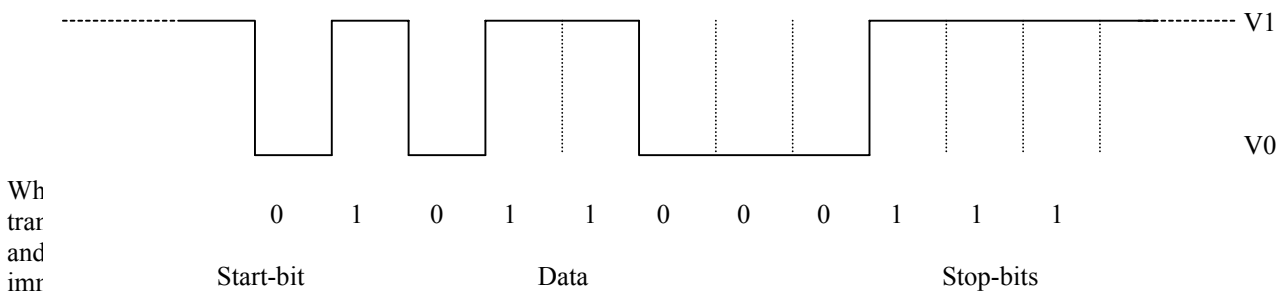
The cross correlation may be implemented as the basis of a correlation detector as follows (for 0-T pulse):



Easier to implement than a matched filter especially when implemented digitally. Gives same value at decision point as we have seen.

2 (b)

Asynchronous transmission (low data rates): Transmitter and receiver clocks are only approximately matched. Data is sent in short words, say 8 bits long, with synchronising start and stop bits. Clocks resynchronise at each start-bit. Consider the transmission of 8-bit ASCII characters according to the well known RS232 protocol:



- Synchronous transmission: Continuous, at high data rate. Synchronising code (say 10101010) sent at start of transmission, and thereafter, the receiver clock must be kept synchronised from the transmission itself. Note that:
- (1) The DC component of a signal is normally lost over wire lines, because of AC coupling, the use of transformers, etc. We would therefore like to keep the average voltage level zero for data transmission.
 - (2) For synchronous transmission, we need to ensure that the signal always has a frequency component at the signalling rate (or an exact multiple or sub-multiple of the signalling rate) to allow synchronisation at the receiver.

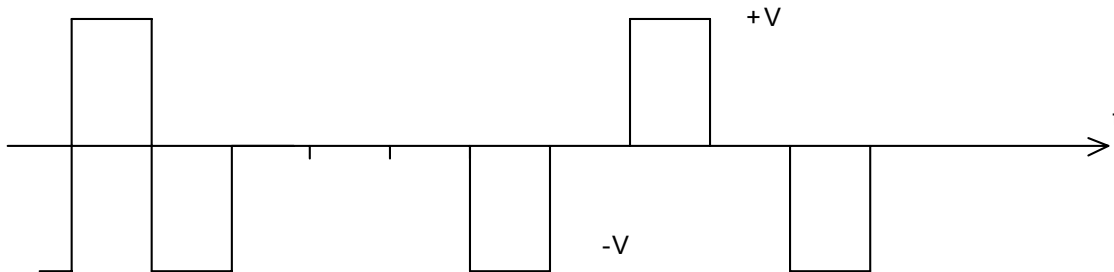
Asynchronous techniques have the advantage of simplicity, but are slow and suitable only for low data rates: e.g. RS232 links between computer peripherals.

Synchronous techniques are considerably more complicated and require a clock to be generated at the receiver exactly synchronised with the data. But the data rate can be much higher and continuous. Suitable for high speed telephone links.

Two possible choices of synchronous techniques (only one needed in answer):

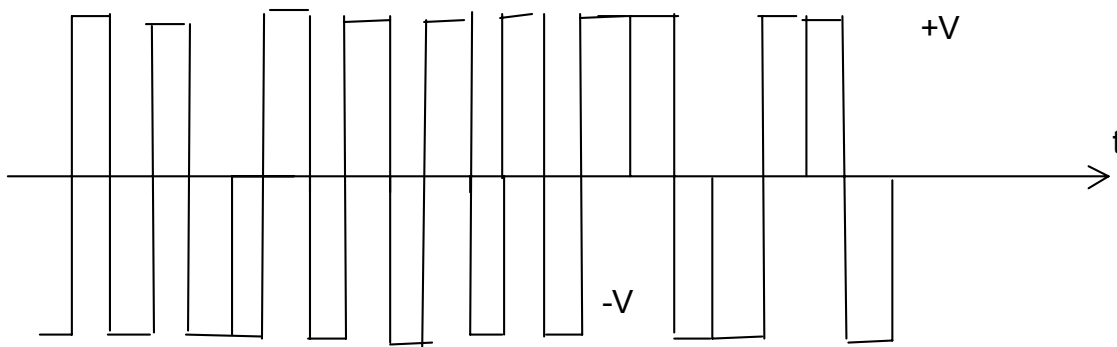
HDB3 coding: (high density bipolar, order 3): This synchronous scheme uses ternary coding to send binary coded data, but with an incorrect "1" in place of any 4th consecutive zero

1 1 0 0 0 0 0 1 0 1



The incorrect "1" is included for clock synchronisation. It is taken to be a data bit of "0" at the receiver.

Bi-phase-L, better known as "Manchester coding", and represents a "one" by a pulse of width T/2 positioned during the first half bit-interval. A zero has a pulse of width T/2 in the 2nd half interval.



Manchester coding has the advantage of absolutely guaranteeing zero dc level and is easy for the receiver clock to synchronise itself to. Its disadvantage in comparison to HDB3 is that the bandwidth requirement is considerably higher.

3(a) (June 2001) If white Gaussian noise (WGN) with 2-sided PSD $N_0/2$ Watts/Hz over $-B$ to B Hz is applied to the filter with frequency response $H(f)$, the power of the output $n_0(t)$ (also Gaussian noise) is:

$$\begin{aligned} \text{Noise power} &= 0.5N_0 \int_{-B}^B |H(f)|^2 df \\ &\approx 0.5N_0 \int_{-\infty}^{\infty} |H(f)|^2 df \end{aligned}$$

when the bandwidth B of the noise is much wider than the bandwidth of the filter $H(f)$.

Let the noise variance be σ^2 before the filter and σ_0^2 after it.

It may be assumed that σ_0^2 is equal to the power of $n_0(t)$.

[2]

$$\therefore \sigma_0^2 \approx 0.5N_0 \int_{-\infty}^{\infty} |H(f)|^2 df$$

If symbol $s(t)$ with FT $S(f)$ is passed through a filter with frequency-response $H(f)$, output waveform produced has FT spectrum $S(f)H(f)$ and by the inverse FT is as follows:

$$a(t) = \int_{-\infty}^{\infty} H(f)S(f)e^{j2\pi ft} df$$

As this is bipolar signalling, we take a threshold γ of zero,
Then the error probability $P_B = Q(z)$ with $z = a(T) / \sigma_0$. [3]

We would like to make $a(T) / \sigma_0$ as large as possible.

Let $F = [a(T)]^2 / \sigma_0^2 =$

$$\frac{\left| \int_{-\infty}^{\infty} H(f) S^*(f) e^{j2\pi ft} df \right|^2}{0.5 N_0 \int_{-\infty}^{\infty} |H(f)|^2 df}$$

Schwartz's inequality for complex valued functions $x(u)$ and $y(u)$ states that:

$$\left| \int x(u)y(u)du \right|^2 \leq \int |x(u)|^2 du \int |y(u)|^2 du$$

Equality applies if $x(u) = k y^*(u)$

Taking u as f , $x(u)$ as $H(f)$ and $y(u)$ as $S(f)e^{2\pi jft}$ this inequality gives:

$$F \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |S(f) e^{j2\pi ft}|^2 df}{0.5 N_0 \int_{-\infty}^{\infty} |H(f)|^2 df}$$

$$\therefore F \leq \frac{1}{0.5 N_0} \int_{-\infty}^{\infty} |S(f) e^{j2\pi ft}|^2 df = \frac{1}{0.5 N_0} \int_{-\infty}^{\infty} |S(f)|^2 df$$

For equality, i.e. to maximise F and hence minimise $Q(a(T) / \sigma_0)$ we must have $H(f) = k S^*(f)e^{-2\pi jfT}$ for some k . We call this the "matched filter".

$H(f)$ is the FT of $h(t)$, the filter's impulse response.

If the FT of $s(t)$ is $S(f)$, then the FT of $s(-t)$ is $S^*(f)$.

Also, if the FT of $s(t)$ is $S(f)$, the Fourier transform of $s(t-T)$ is $S(f)e^{-2\pi jfT}$.

Therefore $h(t)$ is equal to the symbol $s(t)$ modified in the following 3 ways:

- (i) reversed in time,
- (ii) delayed by T seconds, and
- (iii) multiplied by any constant k .

[3]

Substituting $H(f) = k S^*(f)e^{-2\pi jfT}$ in the formula for $a(t)$ with $t=T$, we obtain:

$$a(T) = \int_{-\infty}^{\infty} k S^*(f) S(f) e^{-2\pi jfT} e^{j2\pi fT} df = k \int_{-\infty}^{\infty} |S(f)|^2 df$$

By Parseval's theorem, this also means that when we have a matched filter:

$$a(T) = k \int_{-\infty}^{\infty} (s(t))^2 dt = \text{energy of } s(t)$$

For convenience take $k = 1$.

As this is bipolar signaling with $-s(t)$ for "0" and $s(t)$ for "1", the average energy per bit E_b is equal to the energy of $s(t)$. (Since average energy per bit, E_b , is average of energies of the symbols which are $s(t)$ and $-s(t)$ each having the same energy).

Therefore $a(T) / \sigma_0 = E_b / \sigma_0$ and since $|H(f)|^2 = |S(f)|^2$ for a matched filter, it follows that:

$$\begin{aligned} \text{Therefore } a(T) / \sigma_0 &= E_b / \sqrt{0.5 N_0 E_b} \\ &= \sqrt{2 E_b / N_0} \end{aligned}$$

$$\sigma_0^2 \approx 0.5 N_0 \int_{-\infty}^{\infty} |H(f)|^2 df = 0.5 N_0 \int_{-\infty}^{\infty} |S(f)|^2 df = 0.5 N_0 E_b$$

Therefore, error probability = $Q(a(T) / \sigma_0) = Q(\sqrt{2E_b/N_0})$ as required.

3 (b) Direct approach:

Consider the output from the “integrate and dump” circuit.

The noise has variance: $\sigma_0^2 = \sigma^2 T / (2B) = 4 \times 0.005 / 6000 = 333.3 \times 10^{-8}$ Therefore $\sigma_0 = 1.826 \times 10^{-3}$.

The 2 Volt and -2 Volt pulses become $2T = 0.01$ and $-2T = -0.01$ Volt pulses.

The decision threshold will be 0 Volts.

The error probability is $\text{Prob}("0") \times \text{Prob}(n_0(T) > 0.01) + \text{Prob}("1") \times \text{Prob}(n_0(T) < (0 - 0.01))$

$$\begin{aligned} &= 0.5 \times Q(0.01 / \sigma_0) + 0.5 \times Q(0.01 / \sigma_0) \\ &= Q(5.48) = 0.5 \text{erfc}(5.48 / \sqrt{2}) = 0.5 \text{erfc}(3.88) \approx 2.2 \times 10^{-8} \end{aligned}$$

This may be obtained from the graph of $Q(z)$ against z , tabulations of erfc or the following approximation (Sklar p.88) valid for $z > 3$:

$$\begin{aligned} Q(z) &= 0.5 \text{erfc}(z / \sqrt{2}) = (1/\sqrt{2\pi}) (1/z) \exp(-z^2/2) \\ &\approx (0.4/z) \exp(-z^2/2) \quad \text{when } z > 3 \end{aligned}$$

The error-rate is one bit in about 45,000,000.

Without the integrate & dump circuit, the noise would have variance $\sigma^2 = 4$ and the decision would be on the basis of -2 Volt and 2 Volt pulses.

The threshold would be 0 Volts and the error probability would be the probability of a noise sample (whose standard deviation is 2) exceeding 2 Volts when “0” is transmitted or being less than -2 Volts when a “1” is transmitted.

The error probability without the integrate & dump circuit is therefore

$$P_B = Q(2/2) = Q(1) \approx 1.6 \times 10^{-1}.$$

Approach based on part (a):

With integrate & dump filter which is matched filter for rectangular signalling,

$$N_0 = 4 / 3000 = 1.333 \times 10^{-3}$$

$$E_b = \text{Integral of 4 from 0 to } T = 4T = 0.02.$$

$$\text{Therefore error prob} = Q(\sqrt{0.04 / 0.00133}) = Q(\sqrt{40/1.33}) = Q(\sqrt{30}) = Q(5.48).$$

Hence etc.

4(a) (Jun 2001) This theorem gives us a theoretical maximum bit-rate that can be transmitted with an arbitrarily small bit-error rate (BER), with a given average signal power, over a channel with bandwidth B Hz which is affected by AWGN. By “arbitrarily small BER” this means for any given BER, however small, we can find a coding technique that achieves this BER. The maximum achievable bit-rate (with arbitrary BER) is referred to as the channel capacity C . The Theorem states that:

$$C = B \log_2 \left(1 + \frac{S}{N} \right) \text{ bits / second}$$

where S/N is the mean-square signal to noise ratio (not in dB).

If we wish to transmit average energy/bit of E_b (Joules per bit) and the AWG noise has 2-sided power spectral density $N_0/2$ Watts per Hz, the signal power $S = E_b R$ and the noise power $N = N_0 B$ Watts. Hence, by the S-H theorem:

$$R/B \leq \log_2 \left(1 + \frac{R/B}{E_b/N_0} \right)$$

R/B is called the bandwidth efficiency in units of bit/second/Hz.. We can now write:

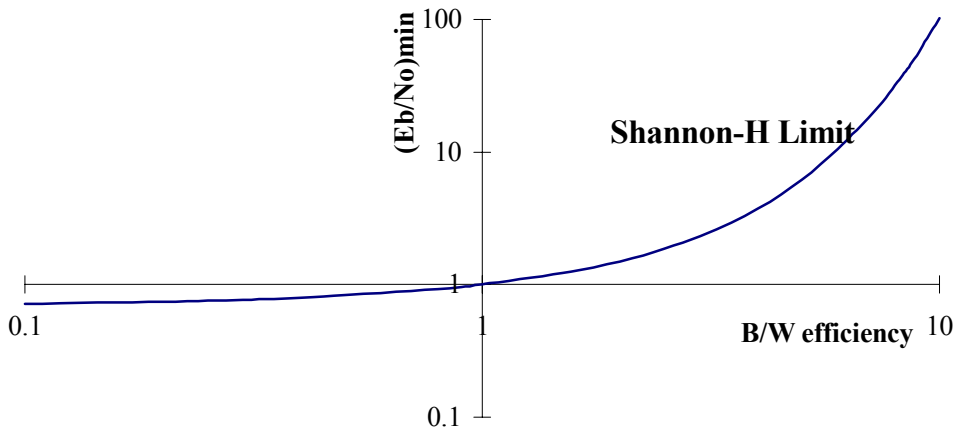
$$2^{R/B} \leq 1 + \frac{R/B}{E_b/N_0}$$

which means that

$$E_b/N_0 \geq \frac{2^{R/B} - 1}{R/B}$$

i.e. $(E_b/N_0)_{\min} \geq \frac{2^{R/B} - 1}{R/B}$

If we draw a graph of $(E_b/N_0)_{\min}$ against R/B we see that $(E_b/N_0)_{\min}$ never goes less than about 0.69 which is about -1.6 dB. Therefore if our normalised energy per bit is less than -1.6dB, we can never satisfy the Shannon-Hartley law.



To see this mathematically, note that

$$2^{R/B} = e^{(\log_e 2)R/B} = e^{0.693R/B} \approx 1 + 0.693 R/B$$

when R/B is small. Therefore when R/B is small,

$$(E_b/N_0)_{\min} \approx \frac{1 + 0.69 R/B - 1}{R/B} = 0.69 = -1.6dB$$

To avoid calculating logs to the base 2,

$$C = \frac{1}{\log_{10} 2} B \log_{10} \left(1 + \frac{S}{N} \right) \approx 3.32B \log_{10} \left(1 + \frac{S}{N} \right)$$

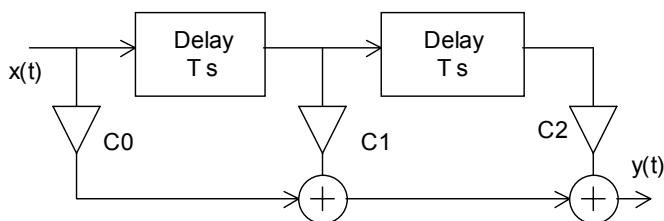
This means that if $S/N \gg 1$, $C \approx 0.332$ times B times the SNR in dB.

Hence if $B=4$ kHz and $SNR = 30$ dB,

$$\text{max bit rate} = 0.332 \times 4000 \times 30 = 39.8 \text{ kb/s}$$

4(b) Because of the channel's finite bandwidth, the response to any symbol will not be time-limited. It will continue ringing forever, though dying away quite rapidly in amplitude. Inter-symbol interference (ISI) can occur due to the ringing of one symbol into the next. [2]

The zero-forcing equaliser is as follows:



When the input is a single signalling pulse of the appropriate shape but distorted by the channel's frequency response, and it is centred on $t=0$ as detected by the timing circuitry, the output must be forced to be zero at $t=0$ and $t=2T$. The output at $t=T$ is arbitrary and may be taken to be 1.

The output at $t=0, T$ and $2T$ is as follows:

$$\begin{aligned} 0 &= y(0) = x(0)C_0 + x(-T)C_1 + x(-2T)C_2 \\ 1 &= y(T) = x(T)C_0 + x(0)C_1 + x(-T)C_2 \\ 0 &= y(2T) = x(2T)C_0 + x(T)C_1 + x(0)C_2 \end{aligned}$$

In matrix form:

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0.1 & -0.1 \\ -0.1 & 1 & 0.1 \\ 0.1 & -0.1 & 1 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix}$$

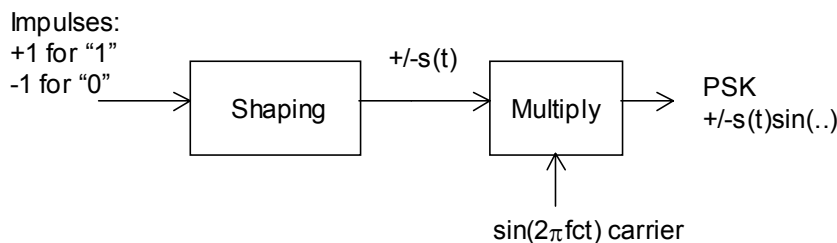
$$\underline{b} = A \underline{c} \quad (\det A = 1.03)$$

Therefore $\underline{c} = A^{-1} \underline{b}$ and it follows that
 $C_0 = -0.0874$; $C_1 = 0.961$; $C_2 = -0.1068$;

$$H(f) = C_0 + C_1 \exp(-2\pi j f T) + C_2 \exp(-4\pi j f T)$$

This will make the channel noise no longer white, introduces correlation in the noise from sampling point to sampling point, increases the noise at the sampling points and thus affects the assumptions used to design the matched filter. This is a disadvantage of this simple equalisation technique.

5.(a) (Jun 2001) (i) Binary PSK modulation (with pulse shaping) & its parameters:-



Transmit a sinusoidal carrier with phase changes determined by the data.

Parameters we need to know are:

- the carrier frequency f_c ,
- the signalling rate $1/T$ baud,
- spectral efficiency (bits/second per Hz)
- channel bandwidth required
- pulse shaping (e.g. 50% raised cosine spectral shaping).
- power of transmission
- time-domain envelope
- other parameters may be mentioned also.

Note inter-dependency of parameters: spectral efficiency depends on required bandwidth which depends on pulse shaping.

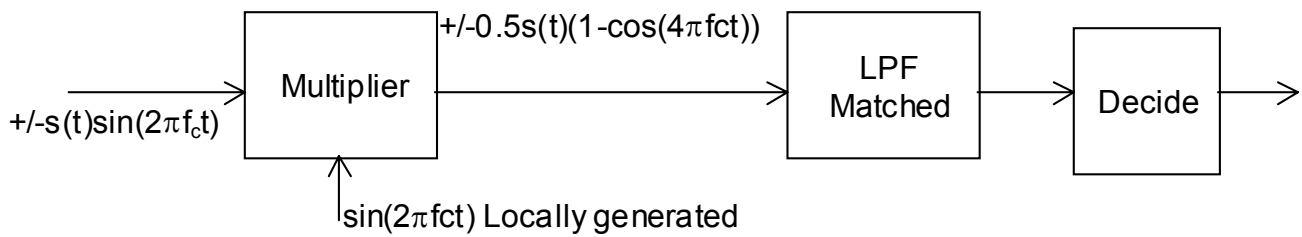
(i) Spectrum of psk is a double side-band suppressed carrier spectrum as for ASK.

The difference between psk & ask is that with psk the modulating signal is anti-podal ($\pm V$) whereas with ask it is unipolar ($+V$ & 0). The difference does not affect spectral occupancy, though it does affect efficiency. Remember that the bandwidth is doubled by the modulation.

The pulse shaping avoids abrupt changes in phase and thus reduces higher frequency components of the spectrum and increases the "sinc"-like decay rate of the time-domain signal that results from the bandwidth limiting. This reduces ISI.

(iii) If signalling rate is $1/T$ and pulse shaping is 100% minimum base-band is $(1/(2T))(1+r)$ Hz. For 50% raised cosine spectral pulse shaping this is $3/(4T)$ Hz at base-band. After modulation this becomes $(1/T)(1+r)$.

(iv) Coherent Detector for binary PSK (with pulse shaping):-



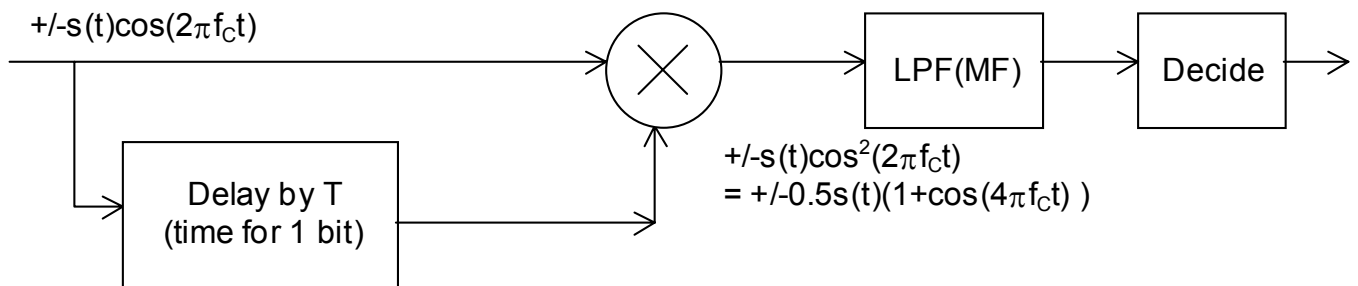
The low-pass filter LPF allows its output to change at the data-rate and eliminates higher frequencies resulting from the $\pm 0.5s(t)\cos(4\pi f_c t)$. If the LPF frequency response is matched to the pulse shape $s(t)$ it will be low-pass as required and optimal for the detection of the polarity of $s(t)$ when the transmission is contaminated by AWGN.

The local carrier must be generated accurately to be of the correct frequency and exactly in phase with the carrier being received. It must do this from the data itself. One method is to square the incoming signal and then divide the frequency of the resulting signal by two.

(i) Differential PSK

Coherently detected PSK has the problem of maintaining long term carrier phase relationship at the receiver. Minor frequency shifts arising from frequency division multiplexing and de-multiplexing appear as continually changing phase shifts whose effect is accumulated over time. Instead, differential PSK (DPSK) may be used where the phase shift of the carrier with respect to the previous bit transmitted indicates the current bit: say 0 degree shift for "1" and 180 degree for "0". (In practice 90 degree and 270 degree phase shifts are often preferred so that phase changes occur at the data rate even when a long succession of "0"s or "1"s are sent).

Differential detection of DPSK: Considering the case where the phase shifts are 0 degrees and 180 degrees to indicate "0" or "1" respectively the following detection techniques may be used. Instead of generating a local carrier, the detector takes the previous symbol as the required carrier segment. Whether this makes the technique classifiable as "coherent" or "non-coherent" is a matter for discussion. There is certainly some penalty to be paid for this form of detection of differentially encoded PSK as compared with a fully coherent technique, but the penalty is not believed to be great. We assume that pulse shaping has been applied to produce shaping $s(t)$.



(If 90° and 270° rather than 0 and 180° phases are used, introduce a 90° phase shift after the delay).

The matched filter "LPF(MF)" acts as a LPF in getting rid of the $\pm 0.5s(t)\cos(4\pi f_c t)$ component and is optimal for the detection of the polarity of $s(t)$ when the transmission is contaminated by AWGN.

Advantages of PSK:

- (i) PSK has superior performance for a given signal-to-noise ratio than FSK or ASK. The same error-rate is achieved by PSK on a channel having a worse signal-to-noise ratio (by up to about 3 dB), than for FSK (except where MSK frequency spacing is used). This advantage applies to differential PSK.
- (ii) PSK (and DPSK) is less susceptible than FSK to the effects of group-delay distortion.
- (iii) Differential PSK has the additional advantage of being insensitive to inversion and relatively insensitive to phase changes introduced by the channel, for example, by carrier mismatches in FDM demodulation

Disadvantage of PSK:

- (i) Not constant envelope when band-limited by channel
- (ii) Receiver quite complicated even at low bit-rates. Receiver normally coherent with the need to derive a local carrier.

Remember that FSK has constant envelope therefore not sensitive to amplitude (gain) variations in the channel. FSK has particular advantages at low data rates in the simplicity of its transmitter and receiver at low data rates. FSK is less bandwidth efficient than ASK or PSK except when MSK frequency spacing is used. Bit error rate performance in AWGN worse than for PSK and requires higher SNR for same BER. We lose the advantage of simplicity for FSK at higher data rates, but there are new advantages to be gained especially with MSK and orthogonal

FSK as used for DAB and digital TV.

5 (c) (Jun 2001) With binary PSK, the best bandwidth efficiency we can hope to achieve is 1 bit/second per Hz.

With QPSK ("4-ary" PSK), say with phases 0° , 90° , 180° , 270° , we can achieve up to 2 bits/second per Hz with the same signalling rate of up to 1 symbol/second per Hz..

With "8-ary" PSK, say with 0° , 45° , 90° , 135° , 180° , 225° , 270° & 305° we could achieve up to 3 bits/second per Hz with the same signalling rate .

In this case we need to achieve only $4/3 = 1.333$ bits/second per Hz .

Therefore let's use QPSK ("4-ary" PSK) with 2000 symbols/second.

At 1 symbol/second per Hz, this would only need 2 kHz, so we can have some nice pulse shaping to reduce ISI.

The use of 100% raised cosine spectral shaping would require a band-width of $2(1+r)$ kHz.

If $2(1+r) = 3$, then $r = 0.5$. Therefore let's use 50% raised cosine pulse shaping to make good use of the available bandwidth.

Solutions to June 2002 exam

1(a). Advantages of digital voice networks in telephony (Choose four):

1. Ease of multiplexing: - TDM (time-division multiplexing) equipment is simpler and less expensive than FDM (frequency-division multiplexing) equipment even when the cost of digitising speech is taken into account.
2. Ease of signalling: - Control information (e.g. on/off hook, dialling, coin deposits, charging) is inherently digital and can therefore be transmitted in exactly the same way as digitised speech.
3. Use of modern technology: - May be applied to switching, multiplexing and signal processing. Digital I/Cs are easier to manufacture than analogue components especially as the latter are usually required to be linear.
4. Integration of transmission and switching: - The multiplexing operations of a transmission system may be easily integrated into the switching equipment.
5. Operability at low signal-to-noise-ratios: - Analogue noise and interference is most noticeable during speech pauses or when the amplitude is low. Noise in digital systems is mostly quantisation noise produced at the A-to-D converter and since companding and adaptive gain control (mirrored at the receiver) may be used to encode low level signals with the same signal to quantisation noise ratio as high level signals (approx.), we do not need an excessively high ratio for high level signals just to ensure that the ratio for low level signals is acceptable. Also, crosstalk in analogue systems is particularly annoying as it tends to be intelligible. Even if crosstalk in digital systems is bad enough to cause bit-errors, the resulting noise will be unintelligible and therefore not as disturbing.
6. Signal regeneration possible: - Distortion in analogue transmission systems cannot be corrected. In a digital system, the probability of transmission errors can be made arbitrarily small by inserting regenerative repeaters, which reconstruct the original pulse shape at intermediate points in the transmission link.
7. Accommodation of other services: - A transmission link can be totally indifferent to the nature of the traffic it carries - it is just a bit-stream. Hence data, video, music etc., can be accommodated in a totally integrated system.
8. Performance monitoring: - Possible, for example by recording parity errors.
9. Ease of encryption: - Scrambling and unscrambling of the transmitted bit stream for security is much easier.

Disadvantages (choose 1 considered most important)

1. Higher bandwidth requirements: - PCM requires a number of pulses (typically 8) to be transmitted for each sample. With binary signalling, this would mean that the bandwidth required would be multiplied by at least this number, as compared with analogue, since the sampling rate, f_s must be greater than twice the signal bandwidth and a transmission bandwidth of at least $1/(2T) = f_s/2$ Hz is needed for normal pulse transmission. The bandwidth requirement may be improved by bit-rate compression and multi-level signalling but at the expense of greater complexity. Currently much higher bandwidths than are required for analogue transmission are the norm.

2. Cost of analogue/digital conversion needed: - Although the cost of this conversion is usually offset by savings in other parts of the system.
3. Need for time synchronisation: - Whenever digital information is transmitted, a timing reference or 'clock' is needed for deciding when to sample the waveform, to decide what bit is being transmitted. This increases cost & complexity.
4. Incompatibility with existing analogue facilities: - Until the telephone system becomes totally digital, many awkward and expensive interfaces will be needed.

Digital wired domestic links still not widely used:-

The technology required for digital voice transmission is more efficiently used when multiplexing is possible as is the case with exchange to exchange transmission and mobile telephony. Wired links into the home are not generally multiplexed, and are still rather expensive to digitise (though this is happening now in various ways). Further, the bit-rate provided using current modems over domestic analogue links (of the order of 50 kHz) is barely high enough for acceptable "toll" quality telephone speech except where compression is used at the expense of complexity. Again this is changing with reducing prices of ISDN & the introduction of ASDL. Other reasons: historical etc.

1 (b) Some of the characteristics of speech which may be exploited to reduce the bit-rate:-

- (i) Non-uniform amplitude distributions: e.g. lower amplitude sample values are more common than higher ones.
- (ii) Sample-to-sample correlation: e.g. adjacent samples are usually fairly close in value.
- (iii) Short term periodicity due to vocal tract resonances.
- (iv) Long term (pitch-interval to pitch-interval) quasi-periodicity in voiced speech.: characteristic waveform repeated after each pitch-interval whose duration is determined by the rate of vibration of the vocal cords (for voiced speech) which determines the pitch of the voice. Unvoiced speech (consonants) does not have this correlation, but tends to be at lower levels than voiced speech.
- (v) Inactivity factors: pauses in speech (60 % duration per speakers)
- (vi) Perceptual effects: e.g. ability to band-limit speech to 4kHz without loss of intelligibility and non-sensitivity of hearing to phase,

G711 (64 kb/s pcm) band-limits to 4 kHz. With its a-law companding it quantises the lower level samples more accurately than the higher ones. This tends to lower the quantisation noise when the signal is quiet, and allows it to increase in amplitude when the speech gets louder. Hence it exploits perception. It is a "waveform coding" approach. RPE-LTP is a "parametric" encoding technique which exploits all the factors apart possibly from (v). It exploits (ii) and (iii) particularly by parametrising the vocal tract resonances and measuring the fundamental frequency of voiced speech portions at suitable intervals of time.

1 (c) Various answers possible: a nice design exercise. Must be multi-level and allow some bandwidth for pulse shaping. e.g. 16-level QAM (gives 4 bits per symbol) and a Baud rate of $13/4 = 3.25$ kBaud.

This allows pulse shaping with 100% RC spectra such that $3.25(1+r) = 6$.

This gives $r = 0.85$, i.e. we can use 85 % RC pulse shaping.

2. (a) (Jun 2002) "Power" or "finite power" signal has finite average power and therefore infinite energy.

Auto-correlation function:

$$R_x(\tau) = \lim_{D \rightarrow \infty} \frac{1}{D} \int_{-D/2}^{D/2} x(t)x(t+\tau)dt$$

Two sided power spectral density:

$$PSD(f) = \lim_{D \rightarrow \infty} \frac{1}{D} |X_D(f)|^2$$

If $x(t)$ is passed through an ideal band-pass filter with cut-off frequencies f_L and f_U Hz, the power of the resulting signal is obtained by integrating $PSD(f)$ between $-f_U$ to $-f_L$ and f_L to f_U .

The units of $PSD(f)$ are Watts (relative to 1 Ohm) per Hz.

For a "finite power" signal, $PSD(f)$ is the Fourier transform of $R_x(t)$

$$\text{Let } D = 2mT \text{ and } x(t) = \sum_{k=-\infty}^{\infty} r_k(t) \text{ with } r_k(t) = \begin{cases} \pm 1 : (k-0.5)T \leq t \leq (k+0.5)T \\ 0 : \text{otherwise} \end{cases}$$





This expresses $x(t)$ as the sum of individual pulses whose values are randomly distributed between ± 1 . We have $2m$ such pulses in the range $-mT \leq t \leq mT$.

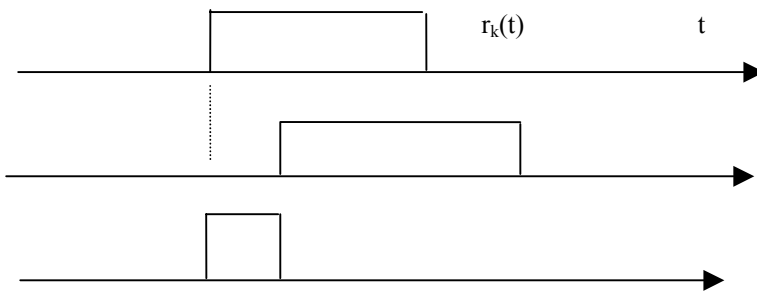
$$ACF(\tau) = \lim_{m \rightarrow \infty} \frac{1}{2mT} \int_{-mT}^{mT} \sum_{k=-\infty}^{\infty} r_k(t) \sum_{\ell=-\infty}^{\infty} r_\ell(t + \tau) dt$$

$$= \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=-m}^m \sum_{\ell=-m}^m \int_{-\infty}^{\infty} r_k(t) r_\ell(t + \tau) dt$$

When $k \neq \ell$, $r_k(t)$ and $r_\ell(t)$ will either have no overlap, (therefore $r_k(t)r_\ell(t) = 0$ for all t) or any overlap will be randomly $+1$ or -1 producing an average of $r_k(t)r_\ell(t)$ close to zero, so that in the limit, the contribution of $r_k(t)r_\ell(t)$ is zero.

Therefore $ACF(\tau) = \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=-m}^m \int_{-\infty}^{\infty} r_k(t) r_k(t + \tau) dt$

Now $\int_{-\infty}^{\infty} r_k(t) r_k(t + \tau) dt = \begin{cases} T - |\tau| & : |\tau| \leq T \\ 0 & : |\tau| > T \end{cases}$



Therefore $ACF(\tau) = \lim_{m \rightarrow \infty} \frac{1}{2mT} \sum_{k=-m}^m (T - |\tau|) = \lim_{m \rightarrow \infty} \frac{1}{2mT} (2m)(T - |\tau|)$ when $|\tau| \leq T$

$$= \frac{T - |\tau|}{T} \lim_{m \rightarrow \infty} \frac{1}{2m} (2m) = \frac{T - |\tau|}{T}$$
 when $|\tau| \leq T$

This means that $ACF(\tau) = \begin{cases} 1 - |\tau|/T & : |\tau| \leq T \\ 0 & : |\tau| > T \end{cases}$ as required.

Let $x(t) = (\delta(t+T) - 2\delta(t) + \delta(t-T))/T$.

Then if $y(t) = \int_{-\infty}^t x(t) dt$ and $z(t) = \int_{-\infty}^t y(t) dt$, the required $ACF(\tau)$ is $z(\tau)$.

Also, if the FT of $x(t)$ is $X(f)$ then the FT of $y(t) = \int_{-\infty}^t x(t) dt$ is $Y(f) = (1/(2\pi j f)) X(f)$.

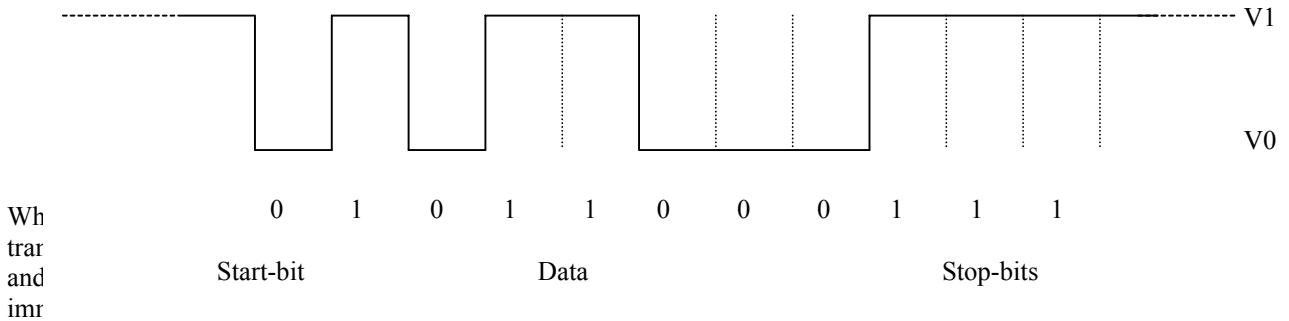
Further, the FT of $z(t) = \int_{-\infty}^t y(t) dt$ is $(1/(2\pi j f)) Y(f) = (1/(2\pi j f))^2 X(f) = -(1/(4\pi^2 f^2)) X(f)$

Now the FT of $x(t) = (\delta(t+T) - 2\delta(t) + \delta(t-T))/T$ is $X(f) = (e^{2j\pi f T} - 2 + e^{-2j\pi f T})/T = -2(1 - \cos(2\pi f T))/T$
 Therefore FT of $z(t)$ is $(1/(2\pi^2 f^2))(1 - \cos(2\pi f T)) = (1/(2\pi^2 f^2))(2\sin^2(\pi f T))$ by rule: $\cos(2\theta) = 1 - 2\sin^2(\theta)$.
 $= T \sin^2(\pi f T) / (\pi^2 f^2 T^2) = T \text{sinc}^2(\pi f T)$

Therefore $PDS(f) = \text{FT of } z(\tau)$ is $T \text{sinc}^2(\pi f T)$

2 (b) (Jun 2002)

Asynchronous transmission (low data rates): Transmitter and receiver clocks are only approximately matched. Data is sent in short words, say 8 bits long, with synchronising start and stop bits. Clocks resynchronise at each start-bit. Consider the transmission of 8-bit ASCII characters according to the well known RS232 protocol:



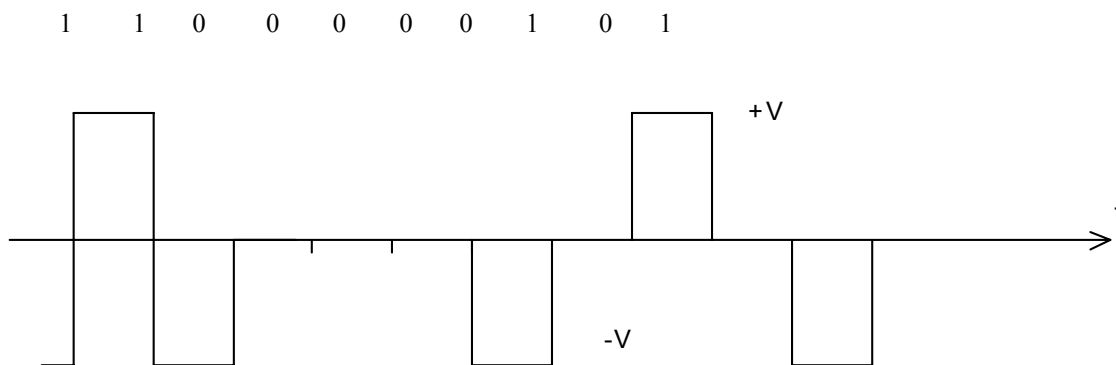
- Synchronous transmission: Continuous, at high data rate. Synchronising code (say 10101010) sent at start of transmission, and thereafter, the receiver clock must be kept synchronised from the transmission itself. Note that:
- (2) The DC component of a signal is normally lost over wire lines, because of AC coupling, the use of transformers, etc. We would therefore like to keep the average voltage level zero for data transmission.
 - (3) For synchronous transmission, we need to ensure that the signal always has a frequency component at the signalling rate (or an exact multiple or sub-multiple of the signalling rate) to allow synchronisation at the receiver.

Asynchronous techniques have the advantage of simplicity, but are slow and suitable only for low data rates: e.g. RS232 links between computer peripherals.

Synchronous techniques are considerably more complicated and require a clock to be generated at the receiver exactly synchronised with the data. But the data rate can be much higher and continuous. Suitable for high speed telephone links.

Two possible choices of synchronous techniques (only one needed in answer):

HDB3 coding: (high density bipolar, order 3): This synchronous scheme uses ternary coding to send binary coded data, but with an incorrect "1" in place of any 4th consecutive zero



2(b) continued

The incorrect "1" is included for clock synchronisation. It is taken to be a data bit of "0" at the receiver.

Bi-phase-L, better known as "Manchester coding", and represents a "one" by a pulse of width T/2 positioned during the first half bit-interval. A zero has a pulse of width T/2 in the 2nd half interval.

Taking u as f , $x(u)$ as $H(f)$ and $y(u)$ as $(S_1(f) - S_2(f))e^{2\pi jft}$ this inequality gives:

$$F \leq \frac{\int_{-\infty}^{\infty} |H(f)|^2 df \int_{-\infty}^{\infty} |(S_1(f) - S_2(f))e^{2\pi jft}|^2 df}{0.5 N_0 \int_{-\infty}^{\infty} |H(f)|^2 df}$$

$$\therefore F \leq \frac{1}{0.5 N_0} \int_{-\infty}^{\infty} |(S_1(f) - S_2(f))e^{2\pi jft}|^2 df = \frac{1}{0.5 N_0} \int_{-\infty}^{\infty} |S_1(f) - S_2(f)|^2 df$$

3(a) continued

For equality, i.e. to maximise F and hence minimise $Q(|a_1(T) - a_2(T)| / (2\sigma_0))$ we must have $H(f) = k(S_1^*(f) - S_2^*(f))e^{-2\pi jfT}$ for some k .

We call this the “matched filter”.

$H(f)$ is the FT of $h(t)$, the filter’s impulse response.

If the FT of $s(t)$ is $S(f)$, then the FT of $s(-t)$ is $S^*(f)$.

Also, if the FT of $s(t)$ is $S(f)$, the Fourier transform of $s(t-T)$ is $S(f)e^{-2\pi jfT}$.

Therefore $h(t)$ is equal to $s_1(t) - s_2(t)$ modified in the following 3 ways:

- (iv) reversed in time,
- (v) delayed by T seconds, and
- (vi) multiplied by any constant k .

[3]

Substituting $H(f) = k[S_1^*(f) - S_2^*(f)]e^{-2\pi jfT}$ in the formulae for $a_1(t)$ & $a_2(T)$ with $t=T$, we obtain:

$$|a_1(T) - a_2(T)| = k \int_{-\infty}^{\infty} |S_1(f) - S_2(f)|^2 df$$

By Parseval’s theorem, this also means that when we have a matched filter:

$$|a_1(T) - a_2(T)| = k \int_{-\infty}^{\infty} (s_1(t) - s_2(t))^2 dt = kE_d = \text{scaled energy of } (s_1(t) - s_2(t))$$

For convenience take $k = 1$.

Therefore $|a_1(T) - a_2(T)| / (2\sigma_0) = E_d / (2\sigma_0)$ and since $|H(f)|^2 = |S_1(f) - S_2(f)|^2$ for a matched filter, it follows that:

$$\text{Therefore } |a_1(T) - a_2(T)| / (2\sigma_0) = E_d / 2\sqrt{0.5N_0E_d}$$

Therefore, error probability = $Q(|a_1(T) - a_2(T)| / (2\sigma_0)) = Q(\sqrt{E_d / (2N_0)})$ as required.

$$\sigma_0^2 \approx 0.5N_0 \int_{-\infty}^{\infty} |H(f)|^2 df = 0.5N_0E_d$$

(b) If $s_1(t)$ is 2 volts for T seconds and $s_2(t)$ is -2 volts for T seconds, the energy of $|s_1(t) - s_2(t)|$ is $4^2 T = 16 T$.

Given that $N_0 = 2 \times 10^{-3}$, $Q(\sqrt{E_d / (2N_0)}) = Q(\sqrt{16T / (4 \times 10^{-3})}) = Q(\sqrt{4 \times 10^3 T})$.
if $Q(z) < 0.001$, from the graph $z > 3.1$.

Therefore $4 \times 10^3 T > 9.61$

$$1/T < (4/9.61) \times 10^3 \text{ Hz} \\ = 0.416 \text{ kHz}$$

This is the minimum symbol rate required.

Question 4 (Jun 2002) Solution:

Rectangular symbols are not suitable for transmitting data over band-limited channels. Any shaped pulse which is time-limited will require infinite bandwidth. A symbol with finite bandwidth must have infinite time-duration.

In practice, a pulse must be non-zero for more than T seconds when the signaling rate is $1/T$ Baud to achieve approximate bandlimiting. Each symbol will run into the previous and next symbol.

“Pulse shaping” means that we carefully choose the time-domain shapes of the symbols such that zero-crossings occur at $t = \pm T, \pm 2T, \dots$ when they are detected at the decision block within the receiver. So at the centre of the next pulse, the current one will be zero and not affect the measurement at the decision block.

A convenient way of generating symbols with the time-domain shape we require is to generate an impulse of the appropriate strength for each symbol and then to shape this impulse by passing it through a “transmitting” filter. The impulse-response of the transmitting filter is the symbol shape we wish to launch into the channel.

To achieve zero ISI the frequency response of the pulse shaping filter must be such that the combined effect of all filtering, and the channel (with equalisation) has the frequency response on a “Nyquist” filter. A “100%” raised cosine frequency response is commonly used to achieve this Nyquist frequency response (giving the correctly positioned zero-crossings). In order to make the overall frequency response, (from impulses at the transmitter to the detector within the receiver) into a 100% RC response the transmitter shaping filter must be “100% root-RC (RRC)” frequency response.

- (b) Because of the channel’s finite bandwidth, the response to any symbol will not be time-limited. It will continue ringing forever, though dying away quite rapidly in amplitude. Inter-symbol interference (ISI) can occur due to the ringing of one symbol into the next.

Three main requirements for the pulse shaping filter are:

1. The shaping filter must produce pulses whose spectra fall within the band width of the channel.
2. To avoid ISI occurring at the receiver’s threshold detector by ensuring that the transmitting filter shapes the symbols so that zero-crossings at the output of the receiving filter (i.e. at the threshold detector) occur T seconds, $2T$ seconds, and so on after (and before) the centre of the symbol. So when we sample at $t=0, T, 2T, \dots$ we only see the centre of one symbol, all the other symbols being zero at those instants.

3. To shape the pulses such that the ringing dies away in amplitude as quickly as possible, so that timing “jitter” or timing error at the receiver, causing samples to be taken not exactly at the zero crossing times, does not cause very serious error.

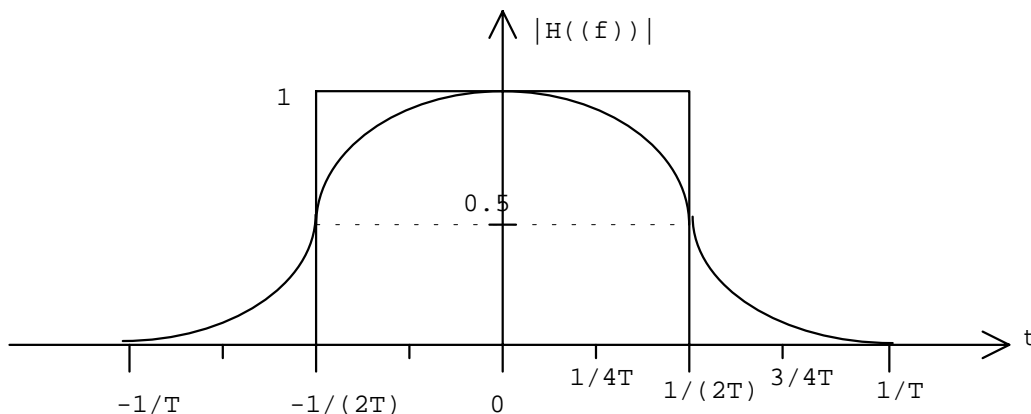
If we combine the transmitting filter, the channel and the receiving filter into a single frequency response $H(f)$ say, then requirement is to make $H(f)$:

- (iii) a “Nyquist frequency response” i.e. a frequency response with the required time-domain zero-crossings
- (iv) a frequency response whose inverse FT (i.e. the corresponding impulse response) dies away as quickly as possible.

- (b) If a time-domain waveform $s(t)$ say has the required zero crossings, sampling it at $t = 0, \pm T, \pm 2T, \pm 3T, \dots$ gives a discrete time signal $\{s[n]\}$ which is equal to $s(0)$ at $n=0$ and zero for $n \neq 0$.

Its DTFT is $S(e^{j\Omega}) = s(0)$ for all Ω and by the sampling theorem, this is equal to the sum $\sum_{k=-\infty}^{\infty} S(f - k/T)$ with $f =$

$\Omega T/(2\pi)$. If $S(f)$ is band-limited between $f = \pm 1/T$ Hz, only $S(f)$ and $S(f-1/T)$ contribute to $S(e^{j\Omega})$ in the range $0 \leq f \leq T$. Hence $S(f) + S(f-1/T) = s(0)$.



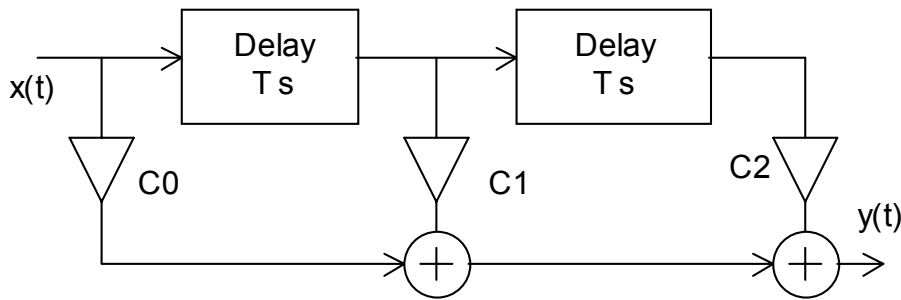
We need to make the overall frequency response $H(f)$ satisfy Nyquist’s first criterion by being a raised cosine spectrum. With a bandlimiting pulse shaping filter at the transmitter and its matched filter equivalent at the receiver,

assuming the frequency response of the channel is cancelled out by an equaliser, $H(f)$ will have magnitude spectrum equal to the square of that of the pulse-shaping filter. Hence we make the magnitude spectrum of the pulse shaping filter RRC.

4(b) Because of the channel's finite bandwidth, the response to any symbol will not be time-limited. It will continue ringing forever, though dying away quite rapidly in amplitude. Inter-symbol interference (ISI) can occur due to the ringing of one symbol into the next. [2]

(iii) The channel equaliser aims to cancel out the gain and phase distortion introduced by the filtering effect of the channel which will have changed the shape of the signal launched into it. It tries to correct the "100 r % RC" shapes as seen at the detector. A zero-crossing equaliser is a simple way of doing this. More sophisticated adaptive FIR digital filtering schemes are much more successful for radio communications.

The zero-forcing equaliser is as follows:



When the input is a single signalling pulse of the appropriate shape but distorted by the channel's frequency response, and it is centred on $t=0$ as detected by the timing circuitry, the output must be forced to be zero at $t=0$ and $t=2T$. The output at $t=T$ is arbitrary and may be taken to be 1.

The output at $t=0, T$ and $2T$ is as follows:

$$\begin{aligned} 0 &= y(0) = x(0)C_0 + x(-T)C_1 + x(-2T)C_2 \\ 1 &= y(T) = x(T)C_0 + x(0)C_1 + x(-T)C_2 \\ 0 &= y(2T) = x(2T)C_0 + x(T)C_1 + x(0)C_2 \end{aligned}$$

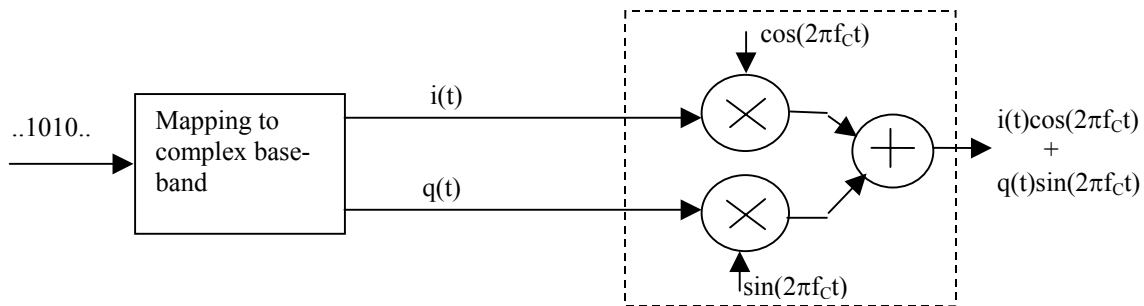
Given $x(-2T), x(T), x(0), x(T)$ & $x(2T)$, values of the three coeffs may be found.

$$H(f) = C_0 + C_1 \exp(-2\pi j f T) + C_2 \exp(-4\pi j f T)$$

This will make the channel noise no longer white, introduces correlation in the noise from sampling point to sampling point, increases the noise at the sampling points and thus affects the assumptions used to design the matched filter. This is a disadvantage of this simple equalisation technique.

Question 5 (Jun 2002) Solution:

(a) An "analogue vector-modulation" is as follows:



The signal $i(t)$ is the "in-phase" component of a "complex base-band signal" and $q(t)$ is the "quadrature" component. The complex base-band signal is said to be complex valued and equal to $i(t) + jq(t)$.

The block shown with a dashed edge is a "vector modulator" multiplying $i(t)$ by the "in-phase carrier component" $\cos(2\pi f_c t)$, $q(t)$ by the "quadrature carrier component" $\sin(2\pi f_c t)$, and adding the two resulting signals together.

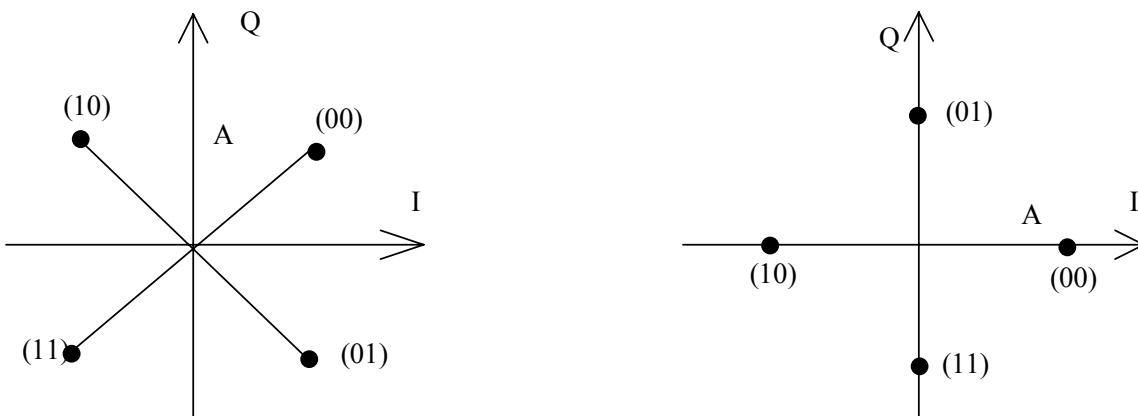
QPSK is achieved by taking alternate bits to produce $\pm A$ amplitude bipolar pulses on $i(t)$ and $q(t)$ alternately. A possible approach is summarised by the table below:

Pair	$i(t)$	$q(t)$
0 0	-A	-A
0 1	-A	A
1 0	A	-A
1 1	A	A

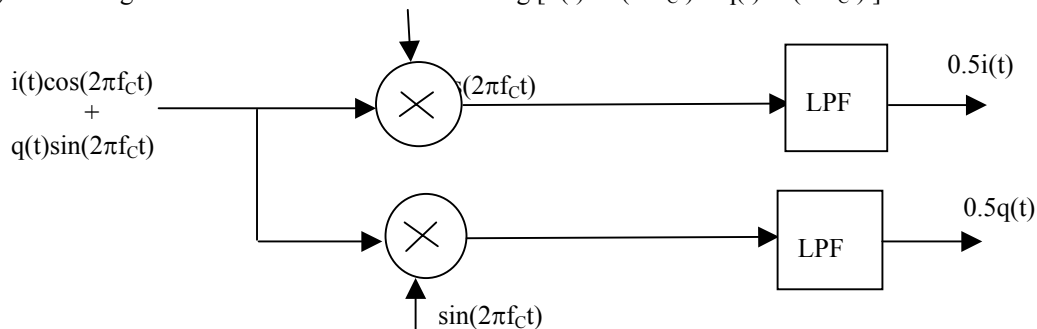
This may be summarised as a constellation diagram. In practice, suitably shaped pulse of amplitude A or $-A$ would be used.

An alternative of $(A, 0), (0, A), (-A, 0), (0, -A)$ is acceptable.

Constellations for QPSK (either will do):-



5 (b) An "analogue vector-demodulator" for receiving $[i(t)\cos(2\pi f_c t) + q(t)\sin(2\pi f_c t)]$ is shown below:



The blocks labelled "LPF" remove the unwanted high frequency components at $2f_c$ Hz..

Since

$$[i(t)\cos(2\pi f_c t) + q(t)\sin(2\pi f_c t)] \cos(2\pi f_c t) = 0.5 [i(t) + i(t)\cos(4\pi f_c t) + q(t)\sin(4\pi f_c t)]$$

and

$$[i(t)\cos(2\pi f_c t) + q(t)\sin(2\pi f_c t)] \sin(2\pi f_c t) = 0.5 [i(t)\sin(4\pi f_c t) + q(t) - q(t)\sin(4\pi f_c t)]$$

it can be seen how this works.

The vector demodulator is capable of receiving "two signals for the price of one".

It is therefore capable of receiving the $i(t)$ and $q(t)$ channels transmitted by QPSK. Looking up the received amplitudes in the table above gets us back to the bit pairs.

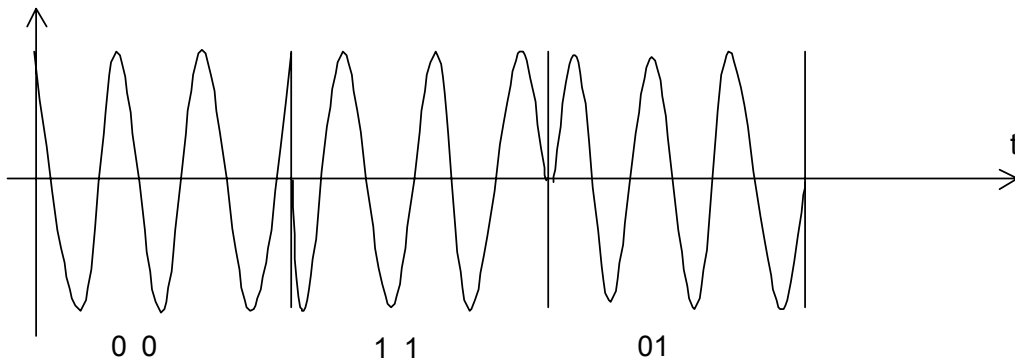
5 (c) Over ASK, constant envelope, reduction in BER in same SNR conditions.

Over FSK, higher bandwidth efficiency generally except when comparing with MSK.

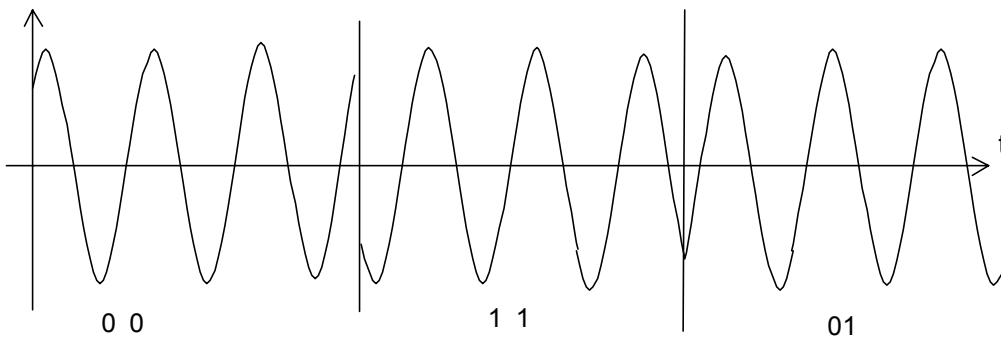
5 (d) Max bit-rate 2 b/s per Hz with 0% pulse shaping.

Difficult to achieve because of sinc time-domain shaped pulse which dies away slowly, needs high order pulse shaping filter to approximate it and because of this, high susceptibility to symbol timing errors.

For second constellation:



For first constellation:-



5 (e) If BER is 1 in 10^{10} , $P_B = 10^{-9}$

If $Q(a(T) / (2\sigma_0)) = 10^{-9}$, then $a(T) / 2\sigma_0 = 6.0$ (from $Q(z)$ graph)

If we decrease $a(T)$ to $a_1(T)$ say, such that $Q(a_1(T) / 2\sigma_0) = 10^{-5}$, then

$a_1(T) / 2\sigma_0 = 4.25$ (from graph)

Therefore $a(T) / a_1(T) = 6 / 4.25 = 1.41$.

In dB the increase is $20 \log_{10}(1.41) = 3$ dB.

Therefore decreasing the power by 3 dB will increase the BER from 1 in 10^{10} to 1 in 10^5 .

Old question not used:-

5(a) fsk: Transmit constant amplitude sine wave segments of different frequencies – say f_0 Hz for logic '0' and f_1 Hz for logic '1'.

psk: transmit a sinusoidal carrier of fixed frequency with phase changes

determined by the binary data. Differential psk transmits one phase change

(e.g. +90 degrees) for a change of binary digit (i.e. from 0 to 1, or 1 to 0), and a different phase change (e.g. –90 degrees) for no change (0 following 0, or 1 following 1).

Advantage of fsk:

Constant envelope.

Simplicity of transmitter and receiver at low data rates.

Disadvantages of fsk:

(i) More susceptible to the effects of group-delay characteristics of telephone lines than psk.

(ii) Requires higher SNR for same BER.

(iii) A zero-crossing detector (or a detector of similar complexity) must be used for fsk at higher data rates (say 1200 bits/s and above) since only one or two complete cycles can be transmitted per bit, and this is not enough to allow simple detectors to operate. This is probably as complicated as (or more complicated

than) a psk detector operating at the same data rate. Hence we lose the advantage of simplicity for fsk at higher data rates.

Advantages of dpsk:

(i) psk has superior performance for a given signal-to-noise ratio. The same error rate is achieved by psk on a channel having a worse signal-to-noise ratio (by up to about 3 dB), than for fsk. This advantage applies to differential dpsk.

(ii) psk (and dpsk) is less susceptible than fsk to the effects of group delay distortion.

(ii) Differential dpsk has the additional advantage of being insensitive to inversion and relatively insensitive to phase changes introduced by the channel, for example, by carrier mismatches in FDM demodulation

Disadvantage of psk:

Not constant envelope when bandlimited by channel

Receiver quite complicated even at low bit-rates. Receiver normally coherent with the need to derive a local carrier.

For fsk, any of 4 methods may be chosen from lecture notes: discriminator type, phase-locked loop, zero-crossing detector and a coherent detector.

A discriminator type:-

For each of the two frequencies, f_0 and f_1 say, there is a bandpass filter and an envelope detector (diode & smoothing capacitor say followed by low-pass filter). At the appropriate sampling points, the two outputs are compared, the larger indicating the frequency of the current symbol being transmitted. This is a non-coherent detector.

Differential detection of DPSK:-

Assume an increase of 90 degrees signifies "1" and -90 degrees signifies "0".

The 90 deg phase increase in the diagram means that the symbols will be in phase producing $A^2 \cos^2(\omega_c t)$ or $A^2 \sin^2(\omega_c t)$ when a 0 is received.

They will be 180 degrees out of phase producing $-A^2 \cos^2(\omega_c t)$ or $-A^2 \sin^2(\omega_c t)$ when a 1 is transmitted.

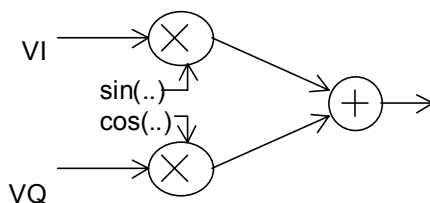
Since $\cos^2(\omega_c t) = 0.5(1 + \cos(2\omega_c t))$ and $\sin^2(\omega_c t) = 0.5(1 - \cos(2\omega_c t))$, the low-pass filter can remove the component at $2\omega_c$. We are left with A^2 when 0 is received and $-A^2$ when 1 is received. Hence we can decide according to the sign of the LPF output. This is non coherent

2(b) contin Coherent detectors operate by mixing the incoming data stream with a locally generated carrier signal. This carrier signal must be of exactly the same frequency and in phase with the carrier of the received signal. Non-coherent detectors do not require such a phase matched carrier and operate by determining the amplitude of the envelope carried by the carrier at appropriate sampling points. The phase relationship between the symbol stream and the carrier becomes unimportant with non-coherent detection. A given symbol, or component of noise will affect the detector output in the same way, regardless of its phase relationship with the carrier. A better performance is obtained with coherent detectors when noise is present, because only half the noise power will be detected by the "in-phase" detector, the other half being seen by a quadrature detector. Hence for the same BER a 3dB worse SNR can be tolerated by a coherent detector than by a non-coherent detector.

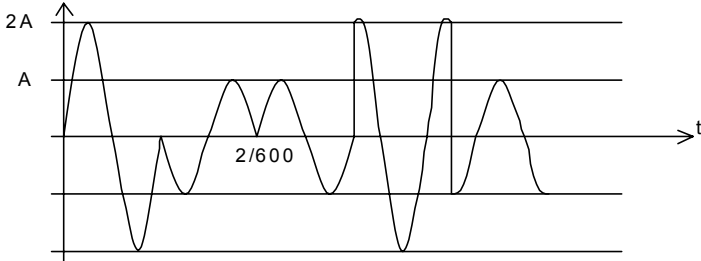
2(c) Assume carrier phase is zero at $t=0$ (start of first symbol). There are 3 bits per symbol, therefore 600 symbols/second. One symbol per cycle:

Symbol allocation table:-

Symbol:	000	001	010	011	100	101	110	111
VI:	+A	0	-A	0	2A	0	-2A	0
VQ:	0	A	0	-A	0	2A	0	-2A



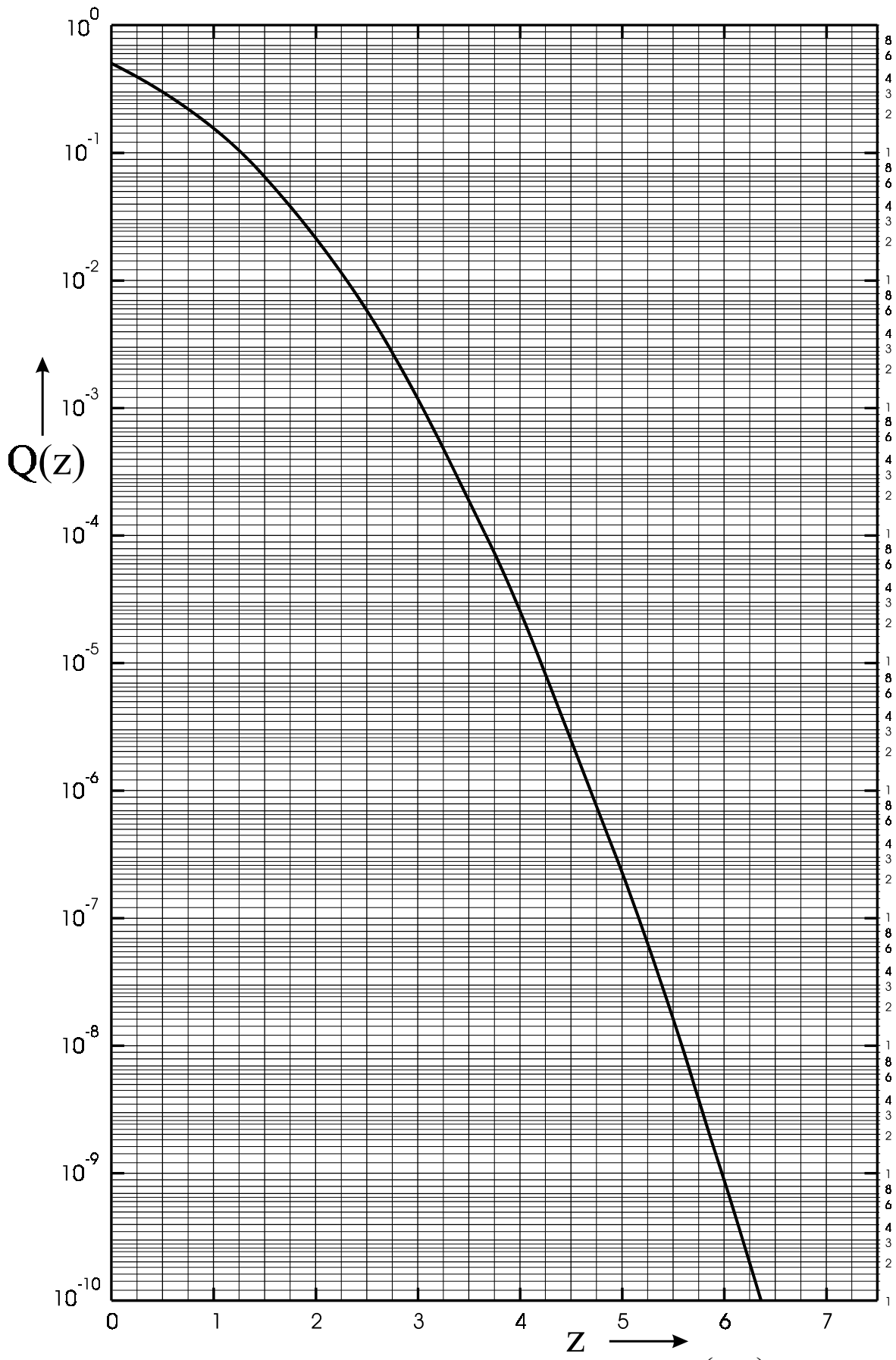
NB VI multiplies "sin" here as in some textbooks.



EE3262 Appendix C: Properties of the Fourier Transform

2/3/00 BMGC

<u>Property</u>	<u>Signal</u>	<u>Fourier Transform</u>
Transform:	$x(t)$	$X(f) = \int_{-\infty}^{\infty} x(t)e^{-2\pi jft} dt$
Inverse transform:	$\int_{-\infty}^{\infty} X(f)e^{2\pi jft} df$	$x(t)$
Complex conjugate:	$x^*(t)$	$X^*(-f)$
Time-reversal:	$x(-t)$	$X(-f)$
For real signal:	$x^*(t) = x(t)$	$X^*(-f) = X(f)$
Real signal time-reversed:	$x^*(-t) = x(-t)$	$X^*(f) = X(-f)$
Symmetry	$x(t)$ real & even $x(t)$ real & odd $x(t)$ real	$X(f)$ real & even $X(f)$ imaginary & odd $ X(f) $ is even, $\phi(f)$ is odd
Interchange	$X(t)$ $\text{Rect}(t/A)$ $\text{Sinc}(\pi t/A)$	$x(-f)$ $A \text{sinc}(\pi Af)$ $A \text{rect}(Af)$
Amplitude scaling	$Ax(t)$	$AX(f)$
Superposition	$Ax_1(t) + Bx_2(t)$	$AX_1(f) + BX_2(f)$
Delay	$x(t-\tau)$	$e^{-2\pi jf\tau} X(f)$
Level shift	$A+x(t)$	$A\delta(f) + X(f)$
Frequency shift	$e^{2\pi jFt} x(t)$	$X(f-F)$
Time-scaling	$x(At)$	$(1/ A) X(f/A)$
Differentiation	$\frac{d^m}{dt^m} \{x(t)\}$	$(2\pi jf)^m X(f)$
Time-domain product	$x_1(t)x_2(t)$	$X_1(f) \otimes X_2(f)$
Time-domain convolutn	$x_1(t) \otimes x_2(t)$	$X_1(f) X_2(f)$
Correlation	$x_1(t)$ with $x_2(t)$	$X_1(f) X_2^*(f)$



Graph of Complementary Error function, $Q(z) = \int_z^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt$

EE3262: Trigonometric formulae

$$\begin{aligned}\sin(A \pm B) &= \sin(A) \cos(B) \pm \cos(A) \sin(B) \\ \cos(A \pm B) &= \cos(A) \cos(B) \mp \sin(A) \sin(B) \\ \tan(A \pm B) &= (\tan(A) \pm \tan(B)) / (1 \mp \tan(A) \tan(B))\end{aligned}$$

$$\begin{aligned}\sin(2A) &= 2 \sin(A) \cos(A) \\ \cos(2A) &= 2 \cos^2(A) - 1 = 1 - 2 \sin^2(A) \\ \tan(2A) &= 2 \tan(A) / (1 - \tan^2(A))\end{aligned}$$

$$\begin{aligned}2 \cos(A) \cos(B) &= \cos(A + B) + \cos(A - B) \\ 2 \sin(A) \cos(B) &= \sin(A + B) + \sin(A - B) \\ 2 \sin(A) \sin(B) &= \cos(A - B) - \cos(A + B)\end{aligned}$$

$$\begin{aligned}\cos(A) + \cos(B) &= 2 \cos((A + B)/2) \cos((A - B)/2) \\ \sin(A) + \cos(B) &= \sin(A) + \sin(B + \pi/2) \\ \sin(A) + \sin(B) &= 2 \sin((A + B)/2) \cos((A - B)/2)\end{aligned}$$

$$\begin{aligned}\cos(\theta) &= (e^{j\theta} + e^{-j\theta}) / 2 \\ \sin(\theta) &= (e^{j\theta} - e^{-j\theta}) / (2j)\end{aligned}$$

Some further revision questions

1. With respect to the “TCP/IP” reference model for network communications, what is the difference between its transmission control (TCP) and user datagram (UDP) protocols? Show how the layers of TCP/IP are related to those of the OSI model.

2. As a means of providing computing power and communications within a large organisation such as a university, what do you consider to be the two main advantages and the two main disadvantages of a local area computer network (LAN) rather than a large central time-shared computer with terminals for all users.

Advantages: reliability and robustness since if one computer goes down the rest of the service can continue; cost since large mainframe computers are very expensive; ease of cabling; possibility of multicasting (sending same message to everyone without duplicating it), etc.

Disadvantages: Less secure since every user has physical access to messages intended to other users, though they cannot usually decode a message not intended for them; perhaps a speed penalty especially during busy times; possible misuse affecting the link for others, centralised control of activity much harder, etc.

3 What is the principal difference between connectionless and connection oriented communication?

4 Which of the OSI reference model layers handles each of the following:

- (i) Breaking the transmitted bit-stream into frames.
- (ii) Determining which route to use to send the data.

5 A cellular mobile radio communication system has equal sized hexagonal cells covering a given environment. If a total of 840 frequencies are available for frequency division multiplexed speech channels, how many channels are available in a given cell if cells using the same frequency must be separated by at least the distance of one other cell. What if the separation must be the distance of at least two other cells?

6. Why is “2 to 4” and “4 to 2” line conversion necessary in the POTs?

Is it true that telephone quality speech is restricted in bandwidth to the range 300 –3400 Hz because below 300 Hz and above 3400 Hz cannot be heard by humans?

What is a regenerative repeater? Is it used for (a) analogue transmission, (b) digital transmission or (c) both? .

7(a) What do you believe to be the most important two advantages of digital voiced networks in telephony? Why is digital voice transmission used for mobile telephony, exchange to exchange transmission, but not yet widely for links into the home or office?

7(b) Consider a 64 kb/s PCM speech channel when the speech signal is quantised and digitised at 8 bits/sample.

Assume speech waveforms are approximately sinusoidal.

Draw a graph of SQNR against the power of the input signal, both in dB when uniform quantisation is used.

Now estimate as best you can without doing lots of calculations a similar graph of SQNR against input power when A-law quantisation is used with $A = 87.6$.

Hint: The linear region for $|x| < 1/A$ is straightforward. Then the effect of logarithmic quantisation takes over.

You are given the following formula for A-law compression applied to $x(t)$ in the range $-V$ to V , with $K=1+\log_e(A)$,

$$y(t) = \begin{cases} \frac{A}{K} \left(\frac{x(t)}{V} \right) & : |x(t)| \leq V/A \\ 1 + \frac{1}{K} \log_e \left(\frac{x(t)}{V} \right) & : V/A \leq x(t) \\ - \left[1 + \frac{1}{K} \log_e \left(\frac{x(t)}{V} \right) \right] & : -V/A \geq x(t) \end{cases}$$

$y(t)$ will lie between -1 and $+1$.

7(c): Let the dynamic range of a digital telephone link be defined as:

$$D_y = 10 \log_{10} \left(\frac{\text{Maximum signal power such that there is no overload}}{\text{Minimum signal power giving at least 30dB SQNR}} \right)$$

From the graphs in part (b) estimate D_y .

7(d) Sketch the shape of $y(t)$ when $x(t)$ is a sinusoid of amplitude A .

7(e) Give a block diagram to show how speech is transmitted at 64kb/s by A-law PCM. Give a formula for the “expander” circuit.

8(a) Assuming the Schwartz (or Cauchy-Schwartz) inequality:

$$\left| \int_a^b x(u)y(u)du \right|^2 \leq \int_a^b |x(u)|^2 du \int_a^b |y(u)|^2 du$$

show that if binary signalling at $1/T$ Baud with equally likely symbols $s(t)$ and 0 is received from a channel affected by additive white Gaussian noise, a matched filter with impulse-response proportional to $s(T-t)$ will minimise the bit-error rate produced by a threshold detector. If the one-sided power spectral density of the noise is N_0 Watts/Hz, and the average energy of the signalling component of the received signal is E_b joules per bit, show that the minimised error probability is $P_b = Q(\sqrt{E_b / N_0})$.

8(b). A receiver receives equally likely binary symbols S_0 and S_1 of energy 0 and 9.8 joules respectively at 100 Baud. The transmission is distorted by zero mean AWGN bandlimited from -1 kHz to $+1$ kHz with one sided PSD $N_0 = 0.1$ Watts/Hz. An matched filter is employed. Estimate the error probability assuming an equal occurrence of 1 's and zeros and an appropriate threshold. Compare with what would have been obtained without the matched filter when the pulse shape of s_1 may be assumed rectangular.

9(a) A channel has bandwidth 3 kHz. What is the maximum achievable bit-rate using (i) binary FSK non-coherently detected (ii) binary FSK coherently detected (iii) binary PSK (iv) QPSK (v) 8-ary MSK. How would the use of a 50% raised cosine spectrally shaped data pulse affect each of these schemes?

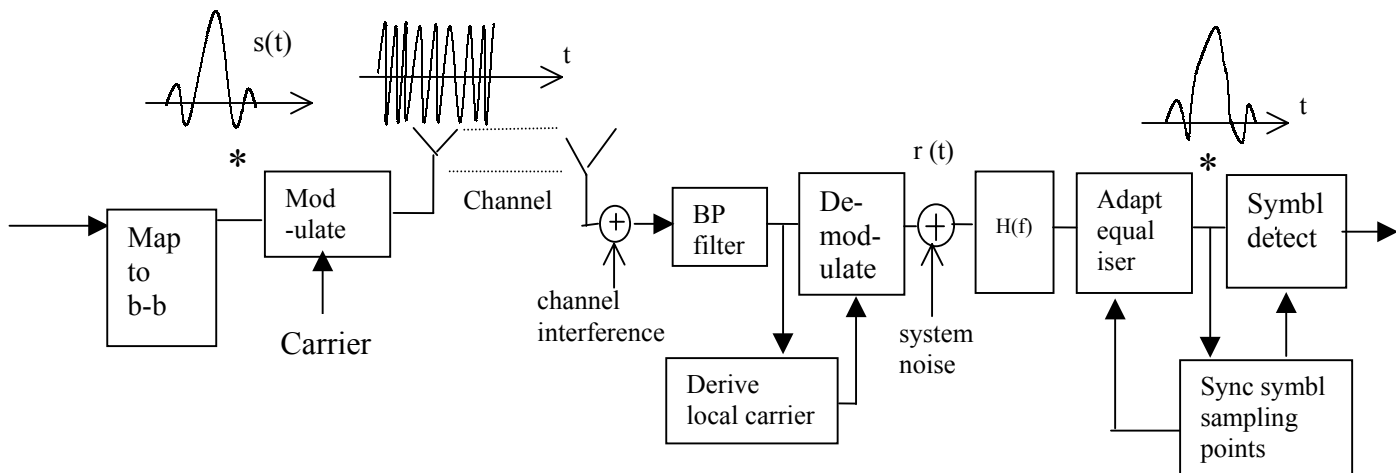
(b) Show that for any value of the "phase offset" ϕ , $\cos(2\pi f_0 t)$ & $\cos(2\pi f_1 t + \phi)$ are orthogonal over 0 to T seconds when $|f_1 - f_0| = 1/T$ Hz.

Show that when $\phi = 0$, i.e. when the two sinusoids are "coherent" or "in-phase", $\cos(2\pi f_0 t)$ & $\cos(2\pi f_1 t + \phi)$ are orthogonal over 0 to T seconds when $|f_1 - f_0| = 1/(2T)$ Hz.

Hence what is the minimum frequency spacing for coherently and non-coherently detected FSK with signalling rate $1/T$ symbols/second. What is meant by "binary MSK with Gaussian shaping" as used in GSM mobile phones. What is the bandwidth efficiency of binary MSK and why is the Gaussian shaping applied?

The answer to the numerical part of this question is in Sklar pp152-154.

Question 1(a): Physical layer controls the method by which "raw" binary digits are transmitted over the real channel and received. Must ensure that when a stream of bits are sent we receive the correct bits and also the correct number of bits. The received bit-stream will normally contain errors. [1]



[3]

(BP filter: removes out of band interference.)

Channel distorts waveform

- (i) by having a gain-response and group-delay which varies with frequency and, especially with mobile, with time.
- (ii) Doppler-shift with mobile.
- (iii) interference from other users and background.

Thermal noise, modelled as AWGN (system noise) added at the point shown in the block-diagram, is generated by electronic components at various stages in the receiver system.

(Of course the transmitter circuitry adds thermal noise as well so we transmit the intended signal with AWGN)

Map to base-band: converts a bit stream to a sequence of suitably shaped symbols or pulses of voltage. This process is achieved by impulsively exciting a pulse-shaping filter whose impulse-response combined with that of the matched filter is that of a Nyquist frequency-response.

Matched filter H(f): minimises the effect of AGWN on the detection process.

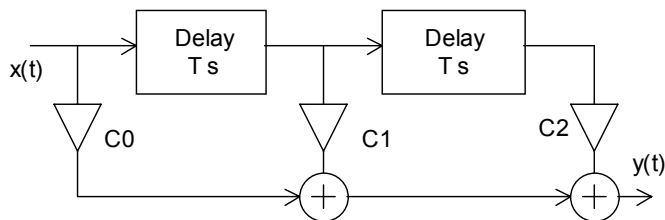
Adaptive equaliser: An adaptive filter which cancels out the possibly time-varying gain and phase distortion of the channel.

Nyquist frequency response: between points marked "*" in block-diagram.

Ensures that centre of a given symbol coincide with zero-crossings of previous and subsequent symbols.

(Achieved by making the product of the pulse shaping filter's frequency response and the matched filter's frequency response a Nyquist frequency response. Can use root-raise cosine pulse shaping or root-Gaussian.)

1(b) The zero-forcing equaliser is as follows:



When the input is a single signalling pulse of the appropriate shape but distorted by the channel's frequency response, and it is centred on $t=0$ as detected by the timing circuitry, the output must be forced to be zero at $t=0$ and $t=2T$. The output at $t=T$ is arbitrary and may be taken to be 1.

The output at $t=0, T$ and $2T$ is as follows:

$$\begin{aligned} 0 &= y(0) = x(0)C_0 + x(-T)C_1 + x(-2T)C_2 \\ 1 &= y(T) = x(T)C_0 + x(0)C_1 + x(-T)C_2 \\ 0 &= y(2T) = x(2T)C_0 + x(T)C_1 + x(0)C_2 \end{aligned}$$

In matrix form:

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0.2 & -0.1 \\ -0.3 & 1 & 0.2 \\ 0.2 & -0.3 & 1 \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \end{bmatrix}$$

$$\underline{b} = A \underline{c}$$

Therefore $\underline{c} = A^{-1} \underline{b}$ with

$$A^{-1} = \begin{bmatrix} 0.93 & -0.15 & 0.12 \\ 0.30 & 0.90 & -0.15 \\ -0.10 & 0.30 & 0.93 \end{bmatrix}$$

It follows that:

$$C_0 = -0.15 ; \quad C_1 = 0.9 ; \quad C_2 = -0.3;$$

$$H(f) = C_0 + C_1 \exp(-2\pi j f T) + C_2 \exp(-4\pi j f T)$$

This will make the channel noise no longer white, introduces correlation in the noise from sampling point to sampling point, increases the noise at the sampling points and thus affects the assumptions used to design the matched filter. This is a disadvantage of this simple equalisation technique.

Question 2: Parseval's thm for finite energy signal $s(t)$:

$$\begin{aligned} \int_{-\infty}^{\infty} |s(t)|^2 dt &= \int_{-\infty}^{\infty} |S(f)|^2 df \\ \int_{-\infty}^{\infty} s^*(4T-t)e^{-j2\pi ft} dt &= \int_{-\infty}^{\infty} -s^*(\tau)e^{-j2\pi(4T-\tau)} d\tau \\ &= \int_{-\infty}^{\infty} s^*(\tau)e^{-2j\pi 4T} e^{j2\pi\tau} d\tau \\ &= e^{-2j\pi 4T} \int_{-\infty}^{\infty} s^*(\tau)e^{j2\pi\tau} d\tau \\ &= e^{-2j\pi 4T} \int_{-\infty}^{\infty} \overline{s(\tau)e^{-j2\pi\tau}} d\tau \\ &= e^{-2j\pi 4T} S^*(f) \quad \text{where '*' and the bar denote complex conj} \end{aligned}$$

'2-sided' energy spectral density of $s(t)$: $ESD(f) = |S(f)|^2$

If $s(t)$ passed thro' filter, output spectrum is: $S(f)H(f)$ and '2-sided' ESD is therefore $|S(f)H(f)|$

Therefore energy is:

$$\int_{-\infty}^{\infty} |S(f)H(f)|^2 df$$

Two-sided PSD of $n(t)$ is:

$$PSD(f) = \lim_{D \rightarrow \infty} \frac{1}{D} |N_D(f)|^2$$

where $N_D(f) = \text{Fourier Transform of } n_D(t)$

$$\text{and } n_D(t) = \begin{cases} n(t) & : -D/2 \leq t \leq D/2 \\ 0 & : \text{otherwise} \end{cases}$$

Result of passing $n(t)$ thro' $H(f)$:

Result of passing the finite energy signal $n_D(t)$ thro' $H(f)$ is a signal whose 2-sided ESD is:

$|N_D(f)H(f)|^2$ and therefore whose energy is:

$$\int_{-\infty}^{\infty} |N_D(f)H(f)|^2 df$$

Power is average energy per unit time, therefore an estimate of the power of $n(t)$ is:

$$\text{Power spectral estimate: } (1/D) \int_{-\infty}^{\infty} |N_D(f)H(f)|^2 df$$

As D gets bigger, this estimate becomes more accurate, and therefore the power of $n(t)$ is:

$$\begin{aligned} \lim_{D \rightarrow \infty} (1/D) \int_{-\infty}^{\infty} |N_D(f)H(f)|^2 df \\ &= \int_{-\infty}^{\infty} \lim_{D \rightarrow \infty} (1/D) |N_D(f)|^2 |H(f)|^2 df \\ &= \int_{-\infty}^{\infty} PSD(f) |H(f)|^2 df \end{aligned}$$

Question 2(b): By time-domain convolution, response of $H(f)$ to $s(t)$ is:

$$s_0(t) = \int_{-\infty}^{\infty} h(\tau)s(t-\tau)d\tau = \int_{-\infty}^{\infty} s(4T-\tau)s(t-\tau)d\tau$$

At the sampling point $t=4T$,

$$\begin{aligned} s_0(4T) &= \int_{-\infty}^{\infty} s(4T-\tau)s(4T-\tau)d\tau = \int_{-\infty}^{\infty} |s(4T-\tau)|^2 d\tau \\ &= \int_{-\infty}^{\infty} |s(t)|^2 dt = E \text{ as defined} \end{aligned}$$

Response of $H(f)$ to 0 is clearly 0 for all t .

What about the noise? Can't predict its value at $t=4T$, but can estimate its power.

Its '2-sided' PSD is $N_0/2$ therefore the noise power is:

$$\begin{aligned} \int_{-\infty}^{\infty} (N_0/2) |H(f)|^2 df &= (N_0/2) \int_{-\infty}^{\infty} |H(f)|^2 df \\ &= (N_0/2) \int_{-\infty}^{\infty} |h(t)|^2 dt = (N_0/2) \int_{-\infty}^{\infty} |s(4T-t)|^2 dt \\ &= (N_0/2) \int_{-\infty}^{\infty} |s(t)|^2 dt = (N_0/2)E \end{aligned}$$

Under the assumption that the noise is a signal whose samples may be considered samples of a zero mean random variable, the variance of this random variable, n_0 say, is equal to the power $(N_0/2)E$. Therefore when the input is $s(t)$ or zero plus AWGN of '2-sided' PSD $N_0/2$ the filter, the output at $t=4T$ is:

$$a(4T) = \begin{cases} E + n_0 & \text{for logic "1"} \\ n_0 & \text{for logic "0"} \end{cases} \quad \text{where } E = \int_{-\infty}^{\infty} |s(t)|^2 dt$$

Taking a threshold at $E/2$, the bit-error probability $P_B = Q((E/2)/\sigma)$ where $\sigma =$ standard deviation of noise which is square root of variance. Therefore $\sigma = \sqrt{(EN_0/2)}$ and it follows that:

$$P_B = Q((E/2)/\sqrt{(EN_0/2)}) = Q(\sqrt{(E/(2N_0))})$$

4(c) The filter $H(f)$ must be causal, therefore its impulse-response $h(t)$ must satisfy: $h(t)=0$ for $t<0$.

Its impulse response is $s(4T-t)$, therefore $s(4T-t)=0$ for $t<0$.

It follows that $s(t)=0$ for $t>4T$.

If a stream of bits shaped like $s(t)$ but delayed by $\pm T, \pm 2T$, etc. are not to interfere with the detection of $s(t)$ at $t=4T$, the response of $H(f)$ to each delayed pulse must be equal to zero at $t=4T$. This means that $a(t)$, the filter's response to $s(t)$, must have zero-crossings at $t=\pm T, \pm 2T$, etc.

It is advantageous to design $H(f)$ with impulse-response $s(4T-t)$ and detect $s(t)$ at $t=4T$ since it allows us to use a pulse shape $s(t)$ (which in theory should extend from $t = -\infty$ to ∞) with a finite duration pulse remaining non-zero up to $4T$ seconds after the central point. The faster the sinc-like ripples decay, the better will be the approximation to the infinite duration pulse-shape when it is truncated at $t=4T$.

Question 3 (a)

$$D_Y = 10 \log_{10} \left(\frac{\text{Max signal power avoiding overload}}{\text{Min signal power with acceptable SQNR}} \right) \text{ dB.} \quad [1]$$

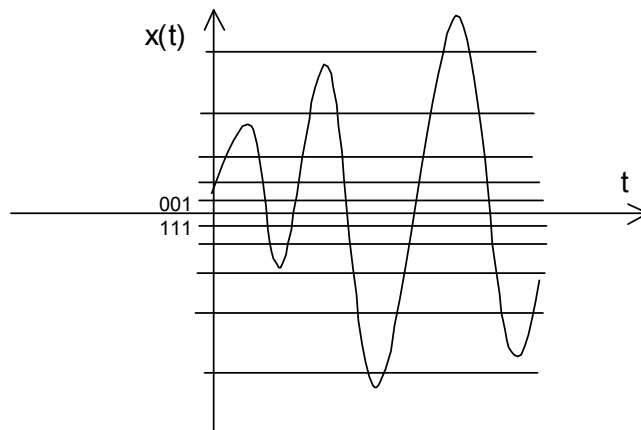
With uniform quantisation (with step-size Δ), the power of the noise created by quantisation-error remains constant (at $\Delta^2/12$) as signal power varies. [1]

To make signal-to-quantisation-noise ratio (SQNR) acceptable for the lowest signal level within a specified dynamic range, Δ must be chosen such that the SQNR is unnecessarily high for higher signal levels. [1]

The ideal would be to have the SQNR independent of signal power. [1]

To achieve this the step-size Δ would have to vary with signal level.

The smaller the voltage the smaller the value of Δ as illustrated below: [1]



Implemented by passing the samples of the input signal $x(t)$ through a logarithmic “compressor” circuit to produce samples of a signal $y(t)$ which are then quantised uniformly and transmitted. At the receiver, the quantised samples of $y(t)$ are passed through an “expander” which reverses the effect of the compressor to produce output samples which are close to the original samples of $x(t)$.

[1]

An ideal logarithmic compressor would require an infinitesimally small value of Δ and hence an infinite number of quantisation levels to span the dynamic range. A compromise is to use logarithmic spacing between quantisation levels for voltages higher than a certain threshold and uniform (constant) spacing for voltages below this threshold.

[1]

3(b) Sampling rate = 8kHz. Let number of bits/sample be b .

Then bit-rate = 8000 b bits/seconds.

Symbol time $T = 1 / (8000 b)$ seconds

[1]

Channel bandwidth = 42 kHz and QPSK used with pulse shaping is 50% RRC

Bandwidth of each symbol at base-band would be:

$$(1 + 50/100)/(2T) = 4000 b (1.5) = 6000b \text{ Hz}$$

[1]

This bandwidth is doubled by dsb modulation (multiplication by carrier) but with QPSK, we have in-phase and quadrature components each carrying data.

[1]

Therefore QPSK bandwidth = 6000 b Hz.

[1]

(Bandwidth doubles but bit-rate doubles as well, so we get same bandwidth efficiency as at baseband).

Bandwidth available = 42 kHz, therefore $b = 42k / 6k = 7$ bits/sample

[1]

Therefore max SQNR = $6 b + 1.8 \text{ dB} = 43.8 \text{ dB}$.

[1]

Bandwidth efficiency = $42/56 \text{ b/s} / \text{Hz} = 0.75 \text{ b/s per Hz}$

[1]

3(c) Channel capacity $C = B \log_2(1 + \text{SNR})$ b/s where B Hz is bandwidth.

[1]

$$\approx 0.33 B \text{ SNR (dB)}$$

[1]

$$= 0.33 \cdot 42k \cdot 10 = 140 \text{ kb/s}$$

[1]

At 8 kb/s sampling rate this gives $140 / 8 = 17$ bits/sample

[1]

$$\text{Max SQNR} = 6 \times 17 + 1.8 = 103.8 \text{ dB}$$

[1]

$$\text{Dynamic range} : 103.8 - 30 \text{ dB} = 73.8 \text{ dB}$$

[1]

Question 4(a) Solution:

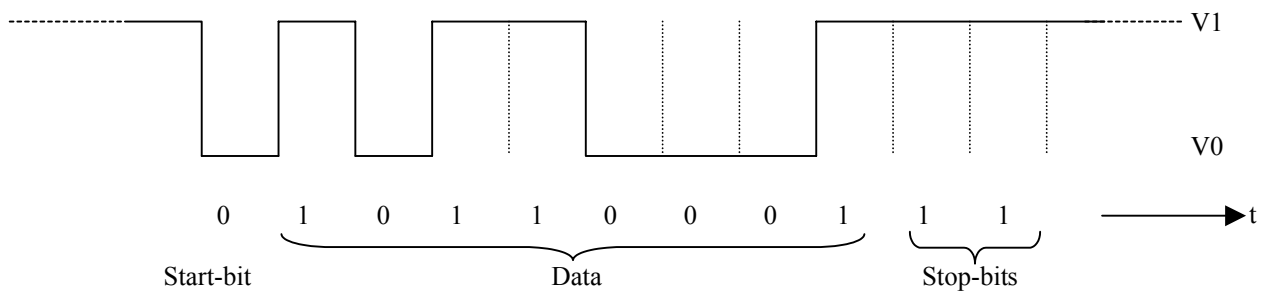
Asynchronous transmission: A digital transmitter must apply suitably shaped symbols to the channel at times specified by a timing reference or 'clock'. For short block-length transmissions as used for transmitting 8-bit binary numbers between computers and peripherals over short distances the transmitter and receiver clocks need only be approximately matched and they may resynchronise at the beginning of each short block. This is 'asynchronous' transmission and is the basis of the well known RS232 standard..

Data is sent in short words, say 8 bits long, with synchronising start and stop bits. The receiver clock resynchronises itself at each start-bit.

Transmission of 8-bit ASCII characters according to the RS232 protocol:

When idle, the line remains high at voltage V_1 . A “start-bit” signifies the start of a transmission. This bit is always “0”. The eight bits of data are then transmitted using “non-return to zero” (NRZ) pulses and finally a number of “1” stop-bits (in this case two) are transmitted to ensure that the next character is not sent immediately.

The receiver waits for a transition from the 'idle state' "1" to "0" indicating a 'start bit'. It delays for half a symbol period according to its own free running clock having approximately the same frequency as that of the transmitter, and then samples the channel eleven times at intervals of T seconds. The samples will hopefully lie in or close to the centre of each symbol, but the timing will drift over the eleven samples. The drift is acceptable because of the frequent resynchronisation.



Advantage: simplicity of transmitter and especially the receiver.

Disadvantage: inefficient in its utilisation of the channel capacity.

4(b) Synchronous techniques are used for the efficient transmission of continuous data for long periods of time, often at data rates close to the maximum possible over a channel of specified bandwidth. A synchronising code (say 10101010) is sent at start of transmission, and thereafter, the receiver clock must be kept synchronised in frequency and symbol-timing from the transmission itself.

When considering how to synchronously transmit digital information over wires, two factors must be borne in mind:-

- (i) We would like to keep the average voltage level as close as possible to zero since any voltage offset carries no data and just wastes power. In many cases, the DC component of a signal is lost over wire lines sometimes because of AC coupling, the use of transformers and/or because the line is used to carry power as well as the data. This is certainly the case with telephone lines.
- (ii) For synchronous transmission, we need to ensure that the signal always has a frequency component at the signalling rate (or an exact multiple or sub-multiple of the signalling rate) to allow a timing waveform to be extracted at the receiver for synchronising the detection process.

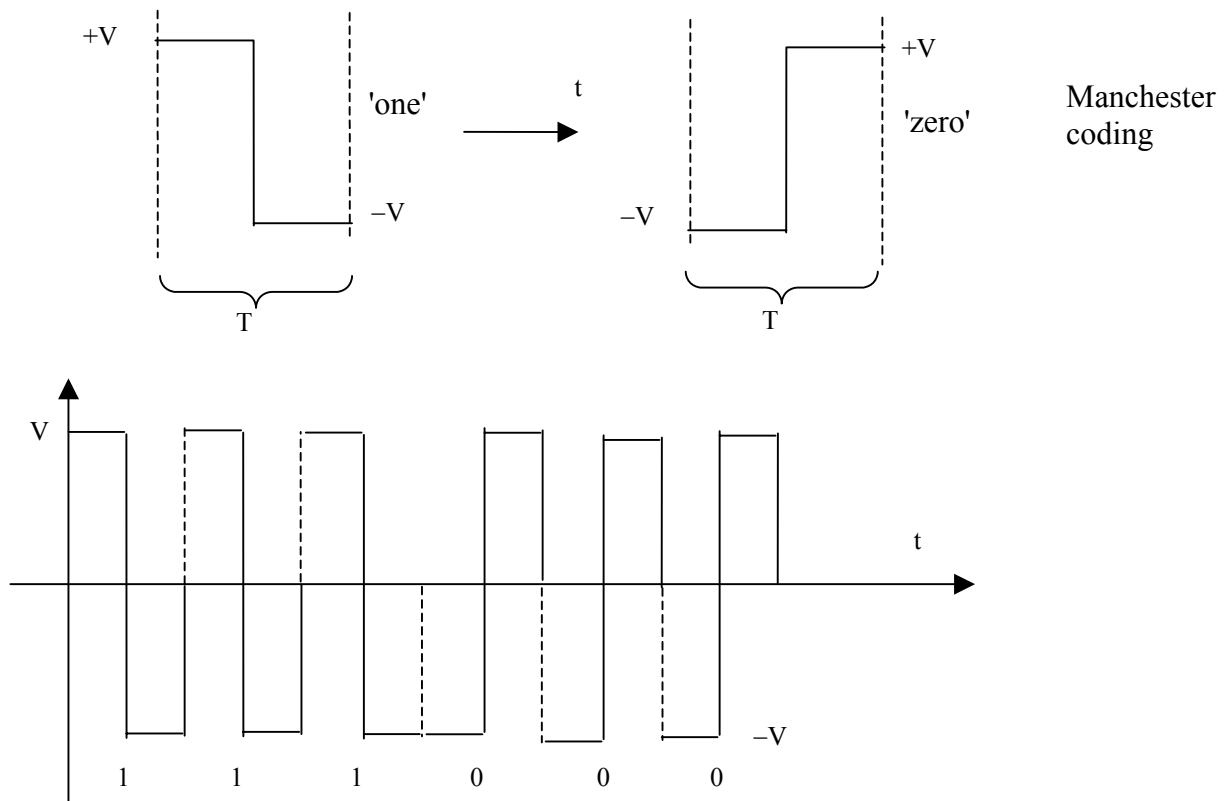
HDB3 coding: (high density bipolar, order 3): uses ternary coding to send binary coded data, as described above for AMI, but places an incorrectly signed pulse in place of any 4th consecutive zero.

E.g. for ' 1 1 1 0 0 0 0 0 0 0 1 ... ' we send:

' +V -V +V 0 0 0 +V 0 0 0 +V -V ... '

The incorrectly phased pulses are included only for clock synchronisation. They are taken to be a “0” at the receiver. The average voltage is no longer zero in the short block above, but over a longer time-span the average will still remain zero since incorrect +V pulses and -V pulses will occur equally often.

Bi-phase-L is better known as “Manchester coding”, and represents a “one” by a pulse of width $T/2$ positioned during the first half bit-interval. A zero has a pulse of width $T/2$ in the second half interval.



Manchester coding has the advantage of absolutely guaranteeing zero dc level and is easy for the receiver clock to synchronise itself to.

Its disadvantage in comparison to NRZ-HDB3 is that the bandwidth requirement is considerably higher.

Question 4(c) Solution: Each regenerative repeater has within it a receiver and a re-transmitter. At each sampling point, the receiver receives a $+V$ or a $-V$ corrupted by noise and possibly other distortion. It must decide whether a “0” or a “1” was intended and then the re-transmitter reconstructs a perfectly shaped symbol of appropriate amplitude and sends it on to the next repeater.

Assume that, at a receiver, the symbols are expected to be $+V$ and $-V$ at the sampling points to signal “1” and 0 respectively. We could take the threshold to be 0 volts, and decide a $+V$ symbol was intended if the voltage is greater than 0 and a $-V$ symbol was intended if the received voltage is negative at the sampling point. To produce an error, the noise must exceed $+V$ when $-V$ (logic “0”) is sent, or be less than $-V$ when $+V$ (logic “1”) is sent. Hence the probability of an error is:

$$(\text{Prob of a “0”}) \times (\text{Prob of noise sample being greater than } +V) + (\text{Prob of “1”}) \times (\text{Prob of noise sample being less than } -V)$$

We assume an equal probability of “1”s and “0”s. Therefore probability of an error is:

$$0.5 \times Q(V/\sigma) + 0.5 \times Q(V/\sigma)$$

where σ is the standard deviation ($\sigma^2 = \text{variance}$) of the noise. The probability of a noise sample being less than $-V$ is same as the probability of a noise sample being greater than V .

$Q(z)$ as plotted in these notes is the probability of a noise sample being greater than z for Gaussian noise of zero mean and standard deviation equal to one. Therefore $Q(z/\sigma)$ is the probability of a noise sample being greater than z/σ for Gaussian noise of zero mean and standard deviation equal to

one. But, most importantly, $Q(z/\sigma)$ is also the probability of a Gaussian noise sample being greater than z when the noise has zero mean and standard deviation equal to σ . It is also the probability of a Gaussian noise sample being less than $-z$ when the noise has zero mean and standard deviation equal to σ .

If $Q(V/\sigma) = 10^{-3}$, from the graph we find that $(V/\sigma) = 3$

Therefore $\sigma = V/3$. This is the standard deviation of the zero mean Gaussian noise that is causing the errors.

With the same noise level, to produce an error probability of 10^{-9} rather than 10^{-3} , we need the regenerative repeaters to receive higher voltage levels for the symbols.

Assume they are raised from $\pm V$ to amplitude $\pm U$ at the sampling points.

Then $Q(U/\sigma) = 10^{-9}$ and from the graph, we find that $U/\sigma = 6$.

This means that $U = 6\sigma = (6/3)V = 2V$.

$20\log_{10}(2) = 6 \text{ dB}$.

Therefore we need to arrange that the voltages as received at the receiver are raised by 6 dB.

If we cannot raise the voltages transmitted by the regenerative repeaters because of cross-talk, we can only reduce the distance between the repeaters so that less attenuation occurs between one regenerative repeater and the next.

The attenuation over 2 km must be reduced from 40 dB to 34 dB.

$20 \times \text{distance} = 34$

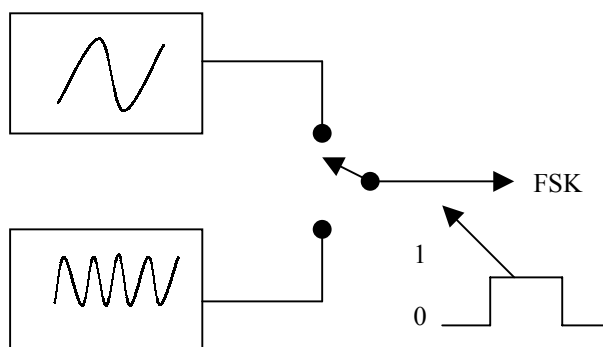
Distance between repeaters must be reduced to $34/20 = 1.7 \text{ km}$ instead of 2 km.

Originally, need $34/2 = 17$ repeaters therefore error rate (approx adds): 17 in 1000, i.e, 1 in 143.

With reduced spacing, need $34/1.7 = 20$ repeaters, therefore error-rate is 20 in 10^9 i.e. 1 in 5×10^7 .

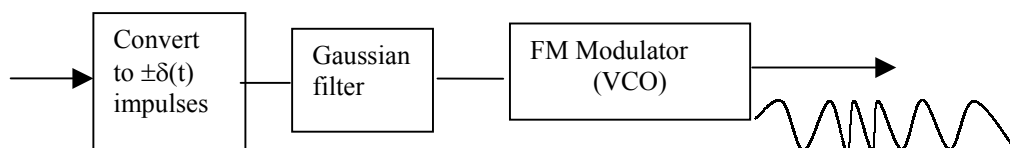
Question 5 (a) solution:

Binary frequency Shift Keying (BFSK) can be a very straightforward form of digital modulation, simple to generate and detect and, being of constant amplitude throughout a transmission, insensitive to fluctuations of the channel attenuation. It is effectively frequency modulation, but uses a set of two distinct frequencies to represent the required symbols. The principle is to transmit a constant amplitude sine wave whose frequency varies between the frequencies assigned to each symbol. For binary signalling there would be two frequencies, f_0 and f_1 say. The simple generation methods may be referred to illustrate the answer.



Minimum shift keying (MSK) is a form of FSK where the difference between f_0 and f_1 is not $1/T$ but $1/(2T)$ Hz. This narrow spacing makes MSK very efficient in its spectral utilisation, but the price to be paid is increased complexity in the generation and detection process.

The spectrum of FSK can be reduced by a pulse-shaping filter which may be a 100% root raised cosine frequency response filter. This is placed just before the FSK modulator and therefore controls how the frequency changes from f_0 to f_1 and vice-versa. In GSM based digital cellular mobile phone systems the shaping is not 100% root raised cosine but something similar called a root-Gaussian filter. The latter has a root-Gaussian shaped gain response. GSM phones use Gaussian shaping in conjunction with MSK. i.e. “*Gaussian MSK*”.



Advantages of GMSK:

1. Constant envelope hence not too sensitive to varying attenuation on the channel and non-linearity of RF amplifiers.
2. Detection based on frequency changes, therefore not very sensitive to frequency shifts of channel, Doppler shifts in mobile systems, etc.

Disadvantages of FSK:

1. Less bandwidth efficient than QPSK
2. Bit error rate performance in AWGN a bit worse than QPSK.

Question 5(b): Gray coding makes the bit error rate equal to the symbol error rate except when the noise is high enough to cause jumps to non-adjacent symbols .

With Gray coding there is only one bit difference between adjacent M-ary levels.

'8-APK': Average power = $(1/8)(4A^2/2 + 4(4A^2)/2) = 5A^2/4$

Symbol rate = 1 k baud therefore bit rate is 3 kb/s.

$$E_B = (5A^2/4) / 3k = 5A^2/12 \times 10^{-3} \text{ Joule}$$

'8-PSK': Average power = $R^2/2$

Bit rate as before.

$$E_B = R^2/6 \times 10^{-3} \text{ Joule}$$

$$\text{If } E_B(8\text{-APK}) = E_B(8\text{-PSK}) \quad 5A^2/12 = R^2/6$$

$$\text{Therefore } R^2 = 2.5 A^2 \quad \text{and } R \approx 1.58 A$$

For 8-APK, min distance = A

$$\text{For 8-PSK min distance} \approx 2\pi R/8 \approx (2\pi R/8) \times 1.58 A = 1.24 A$$

Therefore 8-PSK is better for equal E_B/N_0 .

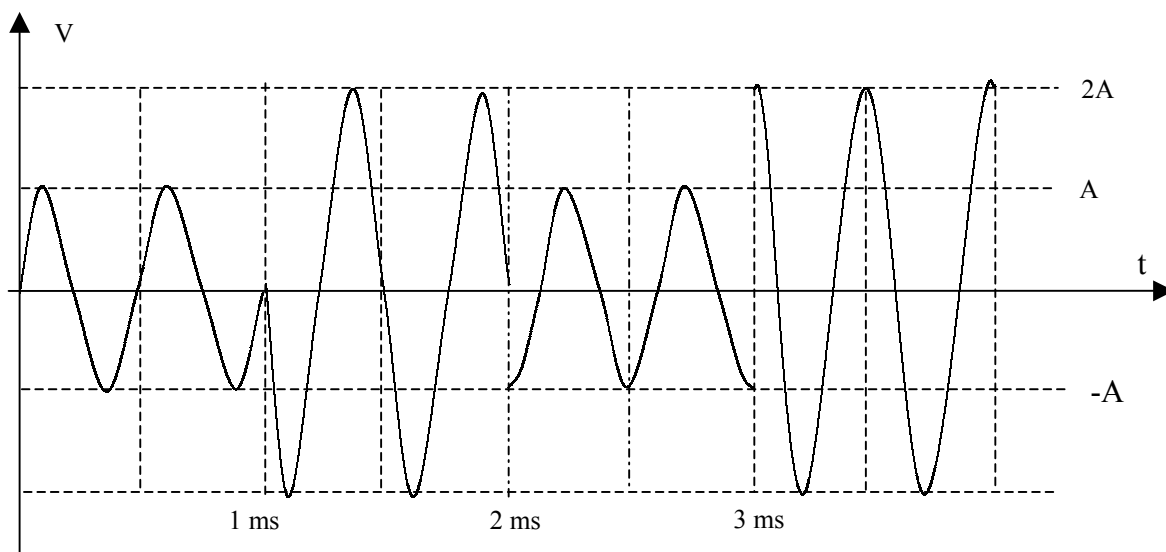
If amplitude limited, $2A = R$.

$$\text{For 8-APK min dist} = A \quad \text{and for 8-PSK min dist} = (2\pi R/8) = \pi A/2 \approx 1.6A.$$

So 8-PSK is still better of course.

Question 5(b) concluded:

Waveform for given '8-APK' scheme coding 000 110 011 101



Waveform for given '8-PSK' scheme coding 000 110 011 101 :-

