CS3222 Exercises 3 (Quantum Computing Basics)

- 1. Check that $\Phi_{0+\pi/2}H\Phi_{\theta}H$ is as stated in the notes.
- 2. By writing down the effect of the circuit on each basis vector $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$, in turn, check that the *SWAP* circuit indeed behaves as advertised.
- 3. What is the matrix representation of the variant of *CNOT* in which the XOR is switched on by a $|0\rangle$ rather than a $|1\rangle$ in the controlling qubit?
- 4. Check that the Toffoli gate implements a permutation of the standard basis of the state space and is therefore unitary.
- 5. Show that provided you are allowed to ignore and/or negate one or both inputs if necessary, any of the 16 functions from two binary bits to one binary bit, can be implemented using a single Toffoli gate.
- 6. Let ε be small and real, and consider the Bell^{ε} basis:

$$|00_{B}\varepsilon\rangle = K^{-1}(e^{\varepsilon}|00\rangle + e^{-\varepsilon}|11\rangle)$$

$$|01_{B}\varepsilon\rangle = K^{-1}(e^{-\varepsilon}|00\rangle - e^{\varepsilon}|11\rangle)$$

$$|10_{B}\varepsilon\rangle = K^{-1}(e^{\varepsilon}|01\rangle + e^{-\varepsilon}|10\rangle)$$

$$|11_{\mathbf{B}}\varepsilon\rangle = K^{-1}(e^{-\varepsilon}|01\rangle - e^{\varepsilon}|10\rangle)$$

where $K = \sqrt{(e^{2\varepsilon} + e^{-2\varepsilon})}$. In superdense coding, if Bob inadvertently used the Bell^{ε} basis to measure the entangled pair instead of the Bell basis, what is the probability that he would: (a) still obtain the correct answer, (b) get an erroneous answer? Assume each bit pattern is transmitted with the same probability.

7. Let ε be small and real, and consider the Hadamard^{ε} transformation:

$$\mathcal{H}^{\varepsilon} = \frac{1}{\sqrt{(e^{2\varepsilon} + e^{-2\varepsilon})}} \begin{bmatrix} e^{\varepsilon} & e^{-\varepsilon} \\ e^{-\varepsilon} & -e^{\varepsilon} \end{bmatrix}$$

In quantum teleportation, if Alice had inadvertently applied H^{ε} instead of H to her first qubit, how is the state that Bob obtains related to the one he would have obtained had Alice used H instead?

- 8. Show that the implementation of C^n -U using Toffolis, ancillas, and a single C-U is correct.
- 9. Show that the implementation of C^2 -U using a V such that $V^2 = U$ given in the notes is correct.
- 10. Show that for any unitary U there is a unitary V such that $V^2 = U$.
- 11. Show that the implementation of Toffoli using V = (1 i)(I + iX)/2 and Q9 is correct.
- 12. Check that the standard quantum implementation of a Boolean function, U_f , is a permutation of the standard basis of the state space and is therefore unitary.