3-Player dice game

Exercise 12 (a): 3-player dice game.

Each Player has two choices which we give as 1 (bet one) and 2 (bet two). Hence the space of strategies is the same as in Example 2.5.

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$			
$(1,1,2)$			
$(1,2,1)$			
$(1,2,2)$			
$(2,1,1)$			
$(2,1,2)$			
$(2,2,1)$			
$(2,2,2)$			

There are two possible outcomes to the throw of the dice, 1 and 2. Each of these occurs with probability $1/2$. Hence the expected pay-off for Player 1 when the chosen strategy is $(1,1,1)$ is

$$\underbrace{\frac{1}{2}}_{\text{prob.}} \underbrace{2}_{\text{pay-off}} + \underbrace{\frac{1}{2}}_{\text{prob.}} \underbrace{0}_{\text{pay-off}} = 1.$$

In this manner we can calculate the various expected pay-offs.

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$			
$(1,2,1)$			
$(1,2,2)$			
$(2,1,1)$			
$(2,1,2)$			
$(2,2,1)$			
$(2,2,2)$			

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$			
$(1,2,2)$			
$(2,1,1)$			
$(2,1,2)$			
$(2,2,1)$			
$(2,2,2)$			

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$	2	5	2
$(1,2,2)$	5	2	2
$(2,1,1)$	5	2	2
$(2,1,2)$	2	5	2
$(2,2,1)$	2	2	5
$(2,2,2)$	1	1	1

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$	2	5	2
$(1,2,2)$	5	2	2
$(2,1,1)$	5	2	2
$(2,1,2)$	2	5	2
$(2,2,1)$	2	2	5
$(2,2,2)$	1	1	1

$(1,1,1)$ is not an equilibrium point since Player 3 can improve her pay-off by switching strategies.

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$	2	5	2
$(1,2,2)$	5	2	2
$(2,1,1)$	5	2	2
$(2,1,2)$	2	5	2
$(2,2,1)$	2	2	5
$(2,2,2)$	1	1	1

$(1,1,2)$ is an equilibrium point.

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$	2	5	2
$(1,2,2)$	5	2	2
$(2,1,1)$	5	2	2
$(2,1,2)$	2	5	2
$(2,2,1)$	2	2	5
$(2,2,2)$	1	1	1

$(1,1,2)$ is an equilibrium point.
$(1,2,1)$ is an equilibrium point.

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$	2	5	2
$(1,2,2)$	5	2	2
$(2,1,1)$	5	2	2
$(2,1,2)$	2	5	2
$(2,2,1)$	2	2	5
$(2,2,2)$	1	1	1

$(1,1,2)$ is an equilibrium point.
$(1,2,1)$ is an equilibrium point.
$(1,2,2)$ is an equilibrium point.

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$	2	5	2
$(1,2,2)$	5	2	2
$(2,1,1)$	5	2	2
$(2,1,2)$	2	5	2
$(2,2,1)$	2	2	5
$(2,2,2)$	1	1	1

$(1,1,2)$ is an equilibrium point.
$(1,2,1)$ is an equilibrium point.
$(1,2,2)$ is an equilibrium point.
$(2,1,1)$ is an equilibrium point.

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$	2	5	2
$(1,2,2)$	5	2	2
$(2,1,1)$	5	2	2
$(2,1,2)$	2	5	2
$(2,2,1)$	2	2	5
$(2,2,2)$	1	1	1

$(1,1,2)$ is an equilibrium point.
$(1,2,1)$ is an equilibrium point.
$(1,2,2)$ is an equilibrium point.
$(2,1,1)$ is an equilibrium point.
$(2,1,2)$ is an equilibrium point.

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$	2	5	2
$(1,2,2)$	5	2	2
$(2,1,1)$	5	2	2
$(2,1,2)$	2	5	2
$(2,2,1)$	2	2	5
$(2,2,2)$	1	1	1

$(1,1,2)$ is an equilibrium point.
$(1,2,1)$ is an equilibrium point.
$(1,2,2)$ is an equilibrium point.
$(2,1,1)$ is an equilibrium point.
$(2,1,2)$ is an equilibrium point.
$(2,2,1)$ is an equilibrium point.

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$	2	5	2
$(1,2,2)$	5	2	2
$(2,1,1)$	5	2	2
$(2,1,2)$	2	5	2
$(2,2,1)$	2	2	5
$(2,2,2)$	1	1	1

$(1,1,2)$ is an equilibrium point.
$(1,2,1)$ is an equilibrium point.
$(1,2,2)$ is an equilibrium point.
$(2,1,1)$ is an equilibrium point.
$(2,1,2)$ is an equilibrium point.
$(2,2,1)$ is an equilibrium point.
$(2,2,2)$ is not an equilibrium point.

3-Player dice game

Exercise 12 (a): 3-player dice game.

The following table gives the pay-off function for the three players, $P1$, $P2$ and $P3$.

	$P1$	$P2$	$P3$
$(1,1,1)$	1	1	1
$(1,1,2)$	2	2	5
$(1,2,1)$	2	5	2
$(1,2,2)$	5	2	2
$(2,1,1)$	5	2	2
$(2,1,2)$	2	5	2
$(2,2,1)$	2	2	5
$(2,2,2)$	1	1	1

$(1,1,2)$ is an equilibrium point.
$(1,2,1)$ is an equilibrium point.
$(1,2,2)$ is an equilibrium point.
$(2,1,1)$ is an equilibrium point.
$(2,1,2)$ is an equilibrium point.
$(2,2,1)$ is an equilibrium point.

There are plenty of equilibrium points in this game!

Discussion of equilibria

Exercise 13 (a).

The equilibria are

$$\begin{vmatrix} (4,-300) & (10,6) \\ (8,8) & (5,4) \end{vmatrix}$$

Discussion of equilibria

Exercise 13 (a).

The equilibria are $(1, 2)$

$$\begin{vmatrix} (4, -300) & (10, 6) \\ (8, 8) & (5, 4) \end{vmatrix}$$

Discussion of equilibria

Exercise 13 (a).

The equilibria are $(1, 2)$ and $(2, 1)$.

$$\begin{vmatrix} (4, -300) & (10, 6) \\ (8, 8) & (5, 4) \end{vmatrix}$$

Discussion of equilibria

Exercise 13 (a).

The equilibria are $(1,2)$ and $(2,1)$.

Player 1 prefers the former (because it gives him the higher pay-off of 10 over 8)

$$\begin{vmatrix} (4,-300) & (10,6) \\ (8,8) & (5,4) \end{vmatrix}$$

Discussion of equilibria

Exercise 13 (a).

The equilibria are $(1, 2)$ and $(2, 1)$.

Player 1 prefers the former (because it gives him the higher pay-off of 10 over 8) while Player 2 prefers the latter (because it gives her the higher pay-off of 8 over 6).

$$\begin{vmatrix} (4, -300) & (10, 6) \\ (8, 8) & (5, 4) \end{vmatrix}$$

Discussion of equilibria

Exercise 13 (a).

$$\begin{vmatrix} (4,-300) & (10,6) \\ (8,8) & (5,4) \end{vmatrix}$$

The equilibria are $(1,2)$ and $(2,1)$.

Player 1 prefers the former (because it gives him the higher pay-off of 10 over 8) while Player 2 prefers the latter (because it gives her the higher pay-off of 8 over 6).

But if Player 2 chooses her strategy 1 to aim for her preferred equilibrium point then if Player 1 chooses his strategy 1 to achieve his preferred equilibrium point she will get a pay-off of -300.

Discussion of equilibria

Exercise 13 (a).

$$\begin{vmatrix} (4,-300) & (10,6) \\ (8,8) & (5,4) \end{vmatrix}$$

The equilibria are $(1,2)$ and $(2,1)$.

Player 1 prefers the former (because it gives him the higher pay-off of 10 over 8) while Player 2 prefers the latter (because it gives her the higher pay-off of 8 over 6).

But if Player 2 chooses her strategy 1 to aim for her preferred equilibrium point then if Player 1 chooses his strategy 1 to achieve his preferred equilibrium point she will get a pay-off of -300.

It seems therefore much more prudent for her to 'play it safe' by choosing her strategy 2 and settle on $(1,2)$.

Checking equilibria

Exercise 14 (a): Checking equilibria.

Show that the game with the pay-off matrix given below has the mixed strategy equilibrium $((1/2, 0, 0, 1/2), (1/4, 1/4, 1/2))$.

$$\begin{vmatrix} -3 & -3 & 2 \\ -1 & 3 & -2 \\ 3 & -1 & -2 \\ 2 & 2 & -3 \end{vmatrix}$$

Checking equilibria

Exercise 14 (a): Checking equilibria.

$$\left|\begin{array}{rrr} -3 & -3 & 2 \\ -1 & 3 & -2 \\ 3 & -1 & -2 \\ 2 & 2 & -3 \end{array}\right|$$

We calculate the pay-off of playing these two strategies against each other. It is

$$\begin{aligned} &\frac{1}{2}\Big(\frac{1}{4}(-3)+\frac{1}{4}(-3)+\frac{1}{2}2\Big) \\ &+\frac{1}{2}\Big(\frac{1}{4}2 \quad +\frac{1}{4}2 \quad +\frac{1}{2}(-3)\Big) \\ &=-\frac{1}{2}. \end{aligned}$$

Checking equilibria

Exercise 14 (a): Checking equilibria.

$$\begin{vmatrix} -3 & -3 & 2 \\ -1 & 3 & -2 \\ 3 & -1 & -2 \\ 2 & 2 & -3 \end{vmatrix}$$

We calculate the pay-off of playing these two strategies against each other. It is $-1/2$.

If Player 1 chooses his mixed strategy $(1/2, 0, 0, 1/2)$ his pay-offs against Player 2's pure strategies are as follows.

	1	2	3
$(1/2, 0, 0, 1/2)$	$-1/2$	$-1/2$	$-1/2$

Since no number appearing in this table is smaller than $-1/2$, changing from the equilibrium point to a pure strategy won't leave Player 2 better off. So this part checks out.

Checking equilibria

Exercise 14 (a): Checking equilibria.

$$\begin{vmatrix} -3 & -3 & 2 \\ -1 & 3 & -2 \\ 3 & -1 & -2 \\ 2 & 2 & -3 \end{vmatrix}$$

We calculate the pay-off of playing these two strategies against each other. It is $-1/2$.

When Player 2 matches her mixed strategy $(1/4, 1/4, 1/2)$ against Player 1's pure strategies the pay-offs are as follows.

	1	2	3	4
$(1/4, 1/4, 1/2)$	$-1/2$	$-1/2$	$-1/2$	$-1/2$

Since no number appearing in this table is larger than $-1/2$, changing from the equilibrium point to a pure strategy won't leave Player 1 better off. So this part checks out too.

Checking equilibria

Exercise 14 (a): Checking equilibria.

$$\begin{vmatrix} -3 & -3 & 2 \\ -1 & 3 & -2 \\ 3 & -1 & -2 \\ 2 & 2 & -3 \end{vmatrix}$$

We calculate the pay-off of playing these two strategies against each other. It is $-1/2$.

By Proposition 3.9 this is indeed an equilibrium point. Note that in the long run that means that Player 1 will lose $1/2$ unit per game!

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 2 & 4 & 0 & -2 \\ 4 & 8 & 2 & 6 \\ -2 & 0 & 4 & 2 \\ -4 & -2 & -2 & 0 \end{vmatrix}$$

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 2 & 4 & 0 & -2 \\ 4 & 8 & 2 & 6 \\ -2 & 0 & 4 & 2 \\ -4 & -2 & -2 & 0 \end{vmatrix}$$

Strategy 1 for Player 2 dominates her strategy 2

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 2 & 4 & 0 & -2 \\ 4 & 8 & 2 & 6 \\ -2 & 0 & 4 & 2 \\ -4 & -2 & -2 & 0 \end{vmatrix}$$

Strategy 1 for Player 2 dominates her strategy 2 and strategy 2 for Player 1 dominates his strategies 1 and 4.

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 & 6 \\ -2 & 4 & 2 \end{vmatrix}$$

Strategy 1 for Player 2 dominates her strategy 2 and strategy 2 for Player 1 dominates his strategies 1 and 4.

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 & 6 \\ -2 & 4 & 2 \end{vmatrix}$$

The first strategy for Player 2 dominates the third one.

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 \\ -2 & 4 \end{vmatrix}$$

The first strategy for Player 2 dominates the third one.

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 \\ -2 & 4 \end{vmatrix}$$

This matrix can be solved with the methods for (2×2)-matrices lecture notes.

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 \\ -2 & 4 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 \\ -2 & 4 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = -6x + 4 \qquad \text{and} \qquad y = 2x + 2.$$

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 \\ -2 & 4 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = -6x + 4 \qquad \text{and} \qquad y = 2x + 2.$$

We set

$$-6x + 4 = 2x + 2,$$

which is equivalent to

$$8x = 2 \qquad \text{or} \qquad x = \frac{1}{4}.$$

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 \\ -2 & 4 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = -6x + 4 \qquad \text{and} \qquad y = 2x + 2.$$

We get $x = 1/4$. The value of the game is the corresponding y-value,

$$-6 \times \frac{1}{4} + 4 = 5/2.$$

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 \\ -2 & 4 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = -6x + 4 \qquad \text{and} \qquad y = 2x + 2.$$

We get $x = 1/4$. The value of the game is the corresponding y-value, $5/2$. For Player 2 we have to intersect the two lines given by

$$y = -2x + 4 \qquad \text{and} \qquad y = 6x - 2.$$

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 \\ -2 & 4 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = -6x + 4 \qquad \text{and} \qquad y = 2x + 2.$$

We get $x = 1/4$. The value of the game is the corresponding y-value, $5/2$. For Player 2 the intersection is at $x = 3/4$.

Dominance

Exercise 15 (a): Dominance.

$$\begin{vmatrix} 4 & 2 \\ -2 & 4 \end{vmatrix}$$

$$\begin{vmatrix} 2 & 4 & 0 & -2 \\ 4 & 8 & 2 & 6 \\ -2 & 0 & 4 & 2 \\ -4 & -2 & -2 & 0 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = -6x + 4 \qquad \text{and} \qquad y = 2x + 2.$$

We get $x = 1/4$. The value of the game is the corresponding y-value, $5/2$. For Player 2 the intersection is at $x = 3/4$.

Hence the unique equilibrium point of the original game is $((0, 3/4, 1/4, 0), (1/4, 0, 3/4, 0))$.

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 16 & 14 & 6 & 11 \\ -14 & 4 & -10 & -8 \\ 0 & -2 & 12 & -6 \\ 22 & -12 & 6 & 10 \end{vmatrix}$$

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 16 & 14 & 6 & 11 \\ -14 & 4 & -10 & -8 \\ 0 & -2 & 12 & -6 \\ 22 & -12 & 6 & 10 \end{vmatrix}$$

Player 1's strategy 1 dominates his strategy 2.

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 16 & 14 & 6 & 11 \\ 0 & -2 & 12 & -6 \\ 22 & -12 & 6 & 10 \end{vmatrix}$$

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 16 & 14 & 6 & 11 \\ 0 & -2 & 12 & -6 \\ 22 & -12 & 6 & 10 \end{vmatrix}$$

Player 2's strategy 4 dominates her strategy 1.

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 14 & 6 & 11 \\ -2 & 12 & -6 \\ -12 & 6 & 10 \end{vmatrix}$$

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 14 & 6 & 11 \\ -2 & 12 & -6 \\ -12 & 6 & 10 \end{vmatrix}$$

Player 1's first strategy dominates his third one.

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 14 & 6 & 11 \\ -2 & 12 & -6 \end{vmatrix}$$

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 14 & 6 & 11 \\ -2 & 12 & -6 \end{vmatrix}$$

Now Player 2's strategy 3 dominates her strategy 1.

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 6 & 11 \\ 12 & -6 \end{vmatrix}$$

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 6 & 11 \\ 12 & -6 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 6 & 11 \\ 12 & -6 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = 6x + 6 \qquad \text{and} \qquad y = -17x + 11.$$

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 6 & 11 \\ 12 & -6 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = 6x + 6 \qquad \text{and} \qquad y = -17x + 11.$$

We set

$$6x + 6 = -17x + 11,$$

which is equivalent to

$$23x = 5 \qquad \text{or} \qquad x = \frac{5}{23}.$$

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 6 & 11 \\ 12 & -6 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = 6x + 6 \qquad \text{and} \qquad y = -17x + 11.$$

We get $x = 5/23$. The value of the game is the corresponding y-value,

$$6 \times \frac{5}{23} + 6 = 168/23.$$

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 6 & 11 \\ 12 & -6 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = 6x + 6 \qquad \text{and} \qquad y = -17x + 11.$$

We get $x = 5/23$. The value of the game is the corresponding y-value, $168/23$. For Player 2 we have to intersect the two lines given by

$$y = 5x + 6 \qquad \text{and} \qquad y = -18x + 12,$$

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 6 & 11 \\ 12 & -6 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = 6x + 6 \qquad \text{and} \qquad y = -17x + 11.$$

We get $x = 5/23$. The value of the game is the corresponding y-value, $168/23$. which happens at $x = 6/23$.

Solving matrix games

Exercise 16 (a): Solving matrix games.

$$\begin{vmatrix} 6 & 11 \\ 12 & -6 \end{vmatrix}$$

To find Player 1's equilibrium point strategy we have to find the intersection of the two lines given by

$$y = 6x + 6 \quad \text{and} \quad y = -17x + 11.$$

We get $x = 5/23$. The value of the game is the corresponding y-value, $168/23$. which happens at $x = 6/23$.

$$\begin{vmatrix} 16 & 14 & 6 & 11 \\ -14 & 4 & -10 & -8 \\ 0 & -2 & 12 & -6 \\ 22 & -12 & 6 & 10 \end{vmatrix}$$

Hence the unique equilibrium point of the original game is $((18/23, 0, 5/23, 0), (0, 0, 17/23, 6/23))$.

Dominance of mixed strategies

Exercise 17 (a): Dominance of mixed strategies.

$$\begin{vmatrix} 2 & 1 & 0 \\ 2 & 0 & 3 \\ -1 & 3 & -3 \end{vmatrix}$$

Dominance of mixed strategies

Exercise 17 (a): Dominance of mixed strategies.

$$\begin{vmatrix} 2 & 1 & 0 \\ 2 & 0 & 3 \\ -1 & 3 & -3 \end{vmatrix}$$

Checking for candidates for elimination we settle on strategy 1 for Player 2.

Dominance of mixed strategies

Exercise 17 (a): Dominance of mixed strategies.

Checking for candidates for elimination we settle on strategy 1 for Player 2. We need $0 \leq \lambda \leq 1$ such that

$$\begin{vmatrix} 2 & 1 & 0 \\ 2 & 0 & 3 \\ -1 & 3 & -3 \end{vmatrix}$$

$$\begin{aligned} 2 &\geq \lambda \\ 2 &\geq 3(1-\lambda) = 3 - 3\lambda \\ -1 &\geq 3\lambda - 3(1-\lambda) = 6\lambda - 3 \end{aligned}$$

Dominance of mixed strategies

Exercise 17 (a): Dominance of mixed strategies.

Checking for candidates for elimination we settle on strategy 1 for Player 2. We need $0 \leq \lambda \leq 1$ such that

$$\begin{vmatrix} 2 & 1 & 0 \\ 2 & 0 & 3 \\ -1 & 3 & -3 \end{vmatrix}$$

$$\begin{aligned} 2 &\geq \lambda & \text{always} \\ 2 &\geq 3(1-\lambda) = 3 - 3\lambda \\ -1 &\geq 3\lambda - 3(1-\lambda) = 6\lambda - 3 \end{aligned}$$

Dominance of mixed strategies

Exercise 17 (a): Dominance of mixed strategies.

Checking for candidates for elimination we settle on strategy 1 for Player 2. We need $0 \leq \lambda \leq 1$ such that

$$\begin{vmatrix} 2 & 1 & 0 \\ 2 & 0 & 3 \\ -1 & 3 & -3 \end{vmatrix}$$

$$\begin{aligned} 2 &\geq \lambda && \text{always} \\ 2 &\geq 3(1-\lambda) = 3 - 3\lambda && \text{iff } \lambda \geq 1/3 \\ -1 &\geq 3\lambda - 3(1-\lambda) = 6\lambda - 3 && \end{aligned}$$

Dominance of mixed strategies

Exercise 17 (a): Dominance of mixed strategies.

Checking for candidates for elimination we settle on strategy 1 for Player 2. We need $0 \le \lambda \le 1$ such that

$$\begin{vmatrix} 2 & 1 & 0 \\ 2 & 0 & 3 \\ -1 & 3 & -3 \end{vmatrix}$$

$$\begin{aligned} 2 &\geq \lambda && \text{always} \\ 2 &\geq 3(1-\lambda) = 3 - 3\lambda && \text{iff } \lambda \geq 1/3 \\ -1 &\geq 3\lambda - 3(1-\lambda) = 6\lambda - 3 && \text{iff } \lambda \leq 1/3 \end{aligned}$$

Dominance of mixed strategies

Exercise 17 (a): Dominance of mixed strategies.

Checking for candidates for elimination we settle on strategy 1 for Player 2. We need $0 \leq \lambda \leq 1$ such that

$$\begin{vmatrix} 2 & 1 & 0 \\ 2 & 0 & 3 \\ -1 & 3 & -3 \end{vmatrix}$$

$$\begin{aligned} 2 &\geq \lambda && \text{always} \\ 2 &\geq 3(1-\lambda) = 3 - 3\lambda && \text{iff } \lambda \geq 1/3 \\ -1 &\geq 3\lambda - 3(1-\lambda) = 6\lambda - 3 && \text{iff } \lambda \leq 1/3 \end{aligned}$$

Hence $\lambda = 1/3$ will do.

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$$\begin{vmatrix} 1 & 0 \\ 0 & 3 \\ 3 & -3 \end{vmatrix}$$

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The only candidate for a dominated strategy is Player 1's strategy 1.

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The only candidate for a dominated strategy is Player 1's strategy 1. We have to find $0 \le \lambda \le 1$ such that

$$\begin{aligned} 1 \le& \quad 3(1-\lambda) = 3 - 3\lambda \\ 0 \le& 3\lambda - 3(1-\lambda) = 6\lambda - 3 \end{aligned}$$

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The only candidate for a dominated strategy is Player 1's strategy 1. We have to find $0 \leq \lambda \leq 1$ such that

$$1 \leq \quad 3(1-\lambda) = 3 - 3\lambda \qquad \text{iff } \lambda \leq 2/3$$

$$0 \leq 3\lambda - 3(1-\lambda) = 6\lambda - 3$$

Dominance of mixed strategies

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The only candidate for a dominated strategy is Player 1's strategy 1. We have to find $0 \le \lambda \le 1$ such that

$$1 \le 3(1-\lambda) = 3 - 3\lambda \quad \text{iff } \lambda \le 2/3$$

$$0 \le 3\lambda - 3(1-\lambda) = 6\lambda - 3 \quad \text{iff } \lambda \ge 1/2$$

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The only candidate for a dominated strategy is Player 1's strategy 1. We have to find $0 \leq \lambda \leq 1$ such that

$$1 \leq 3(1-\lambda) = 3 - 3\lambda \qquad \text{iff } \lambda \leq 2/3$$

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There are plenty of λs (from $1/2$ to $2/3$) to choose from.

Dominance of mixed strategies

Exercise 17 (a): Dominance of mixed strategies.

$$\begin{vmatrix} 0 & 3 \\ 3 & -3 \end{vmatrix}$$

We cannot reduce this matrix any further, but we know how to solve (2×2)-matrices.