CS3191 Section 3

Medium Games

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The idea for this section is to employ computer power to help us find good strategies for a game. To some extent we will be building on our theory of games developed in Sections 1 and 2, since the notion of game tree will still be present.

Algorithmic point of view

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Beyond small games

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$$2^{m+1} - 1$$

decision points, that is their number grows exponentially.

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2	7	2	4	6
3	15	8	4	12
4	31	8	64	72
5	63	128	64	192
6	127	128	16384	16512
7	255	32768	16384	49152
8	511	32768	1073741824	1073774592

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The aim for this section is to find good strategies without creating all strategies first.

For that we assume that the game tree is given in a computer.

However, our algorithms will not require that the game is present in memory at all times—it is sufficient if it can be created on demand.

The minimax algorithm

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Note that the player has to assume that the others will do their worst in order for the guarantee to work.

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We can deal with elements of chance if we stick to the expected pay-off as before.

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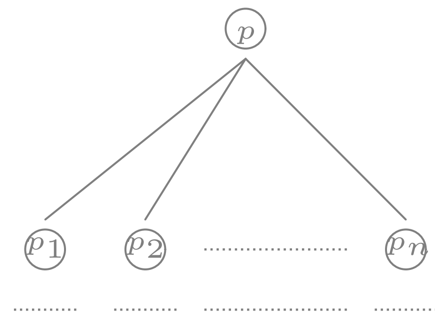

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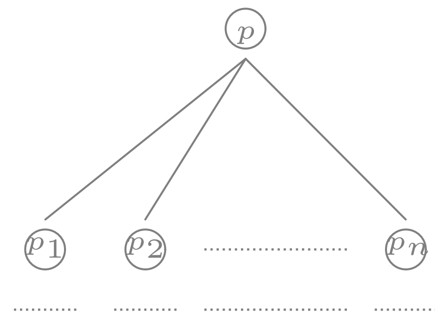

Proposition 3.1 *Let p be a position with p_1, p_2, ..., p_m the positions which can be reached from p with one move (see the figure). Then the value for player X of p, $v_X(p)$, is*

$$v_X(p) = \begin{cases} p_X(p) & p \text{ is final} \\ \max_{1\leq i\leq m} v_X(p_i) & X \text{ to move at } p \\ \min_{1\leq i\leq m} v_X(p_i) & Y \neq X \text{ to move at } p \\ q_1 v_X(p_1) + q_2 v_X(p_2) + \cdots + q_n v_X(p_n) & \text{chance move at } p; \\ & q_i\text{: probability for } p_i \end{cases}$$

Proof

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This is really obvious from the definition:

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If it is a chance move then the minimal expected pay-off from p is

$$q_1 v_X(p_1) + q_2 v_X(p_2) + \cdots + q_m v_X(p_m).$$

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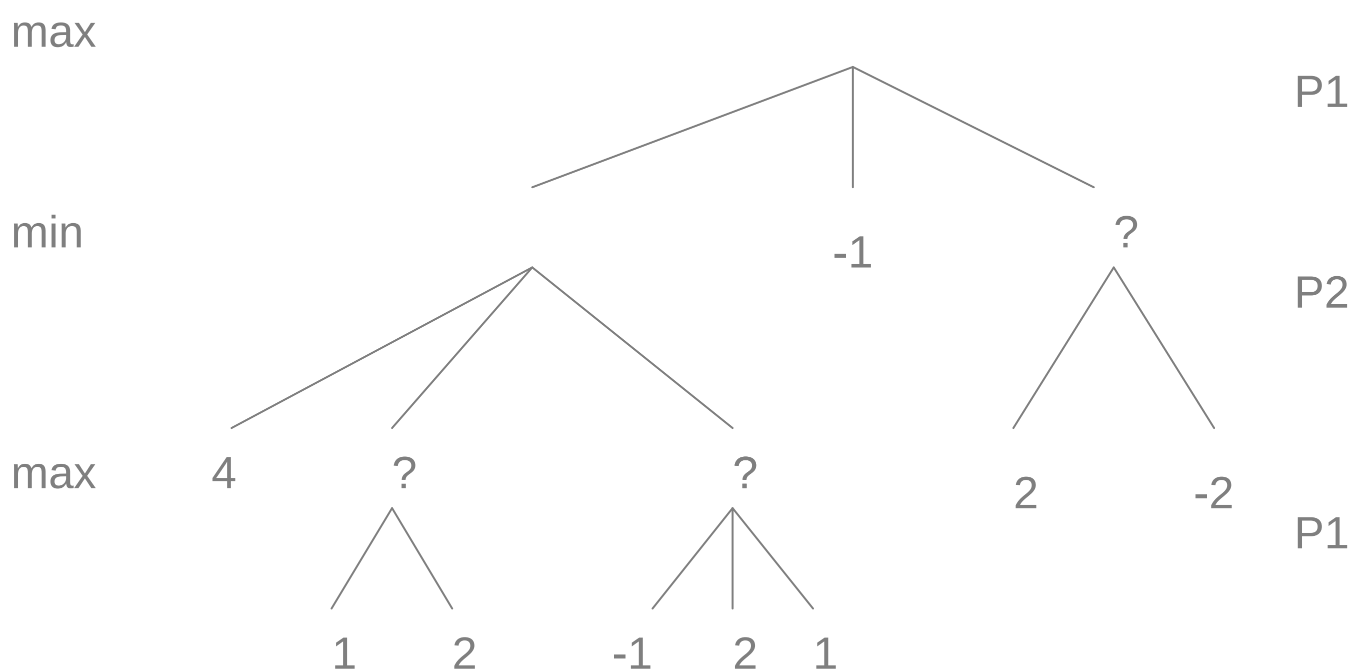
We have given a recursive algorithm—in order to find the value (for a player) of some node we first have to know the value (for that player) of all its children.

How to find the value–bottom-up



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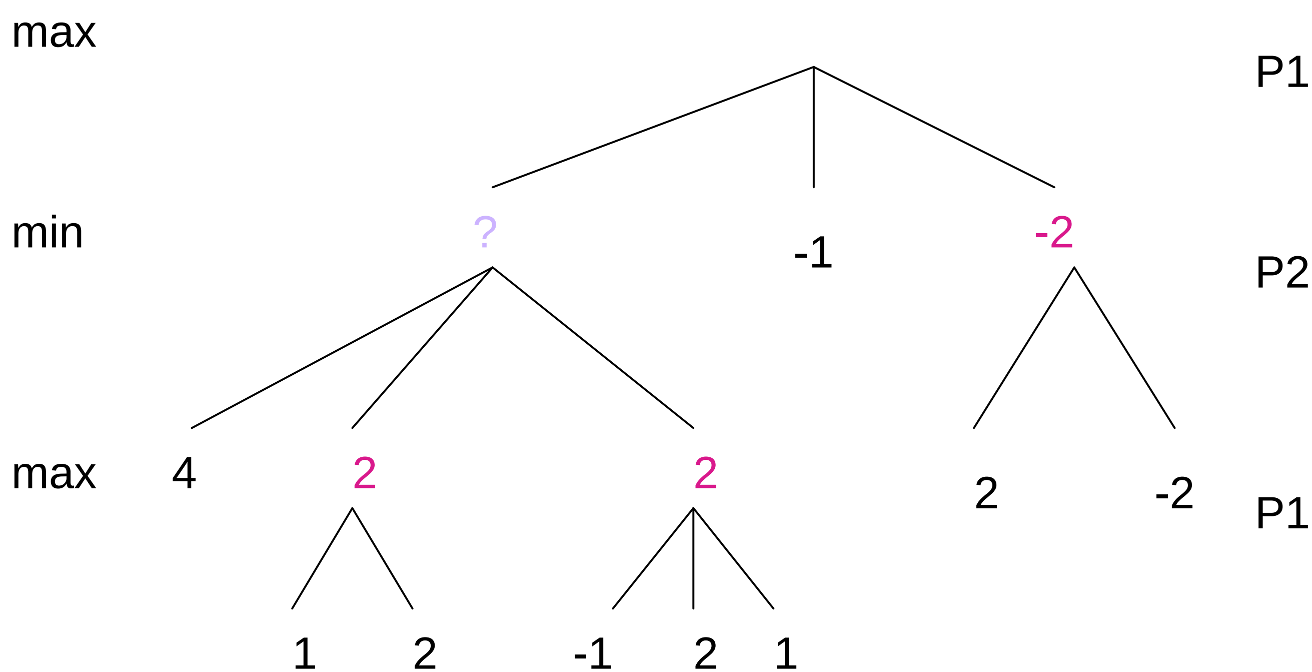

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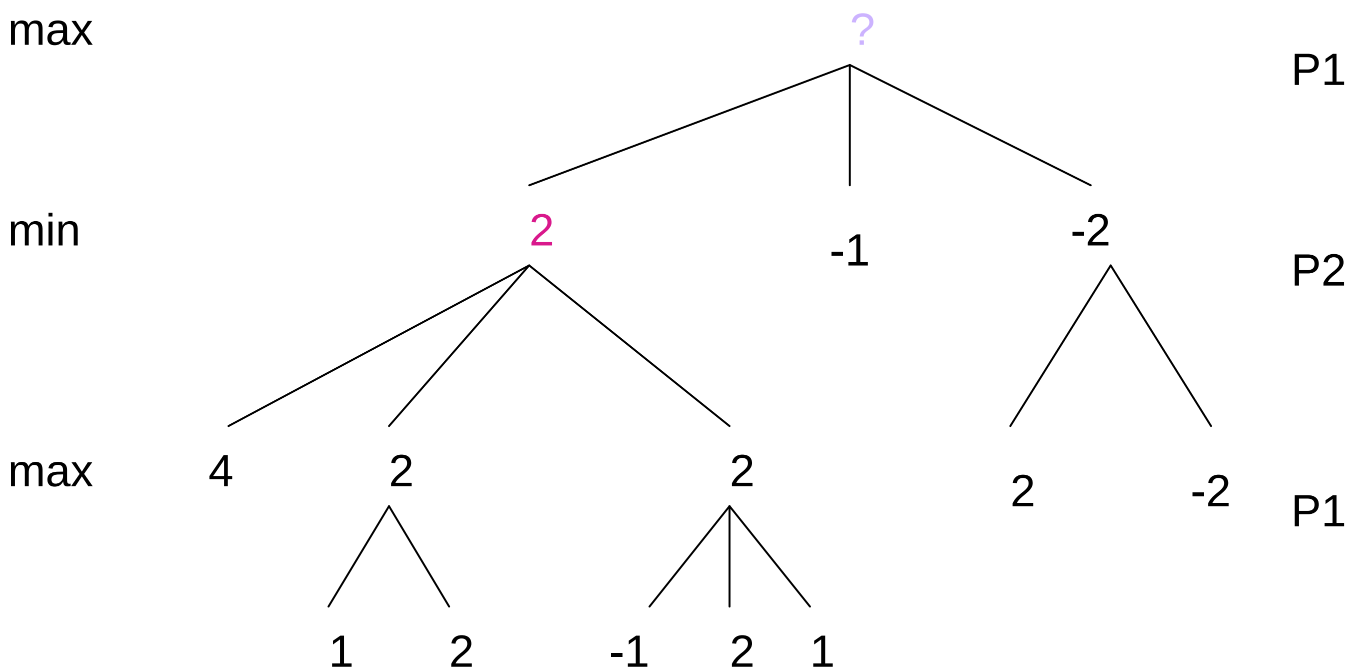

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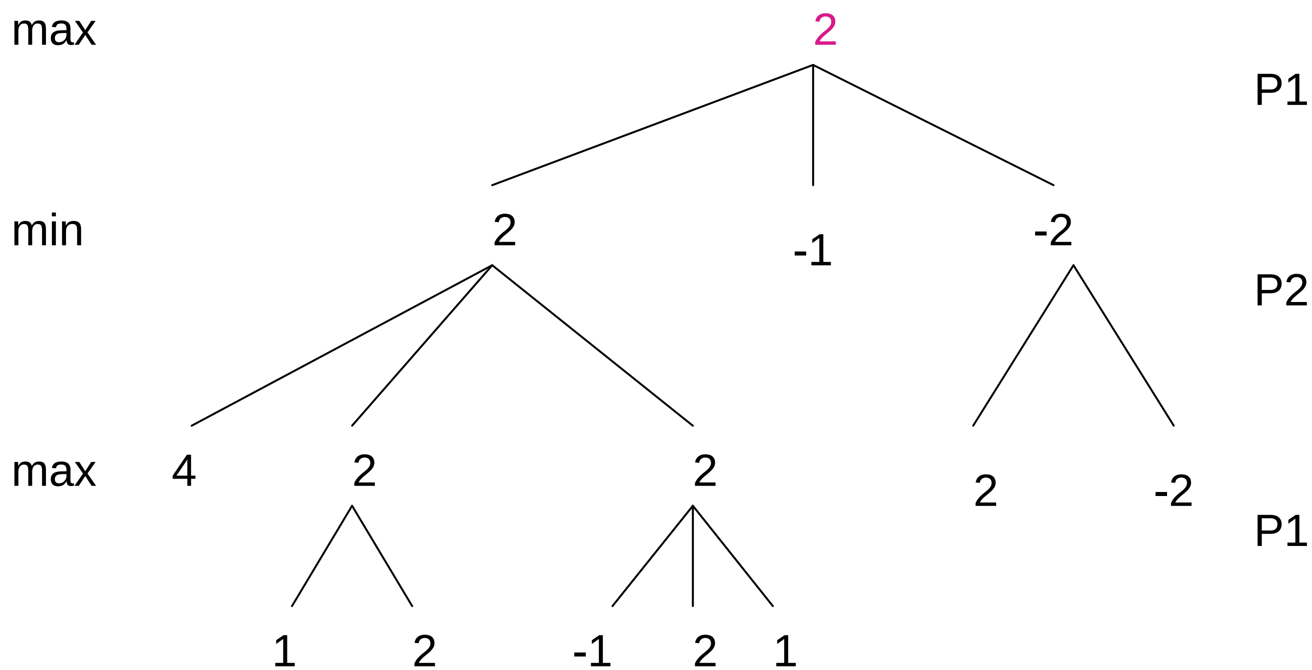

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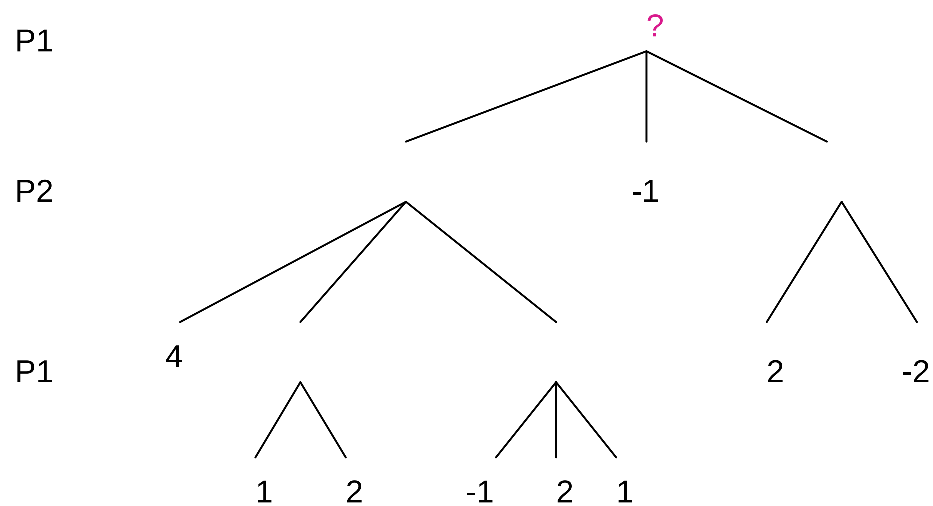
It is much better to traverse the tree depth-first. We will study this idea next.

Depth-first algorithm



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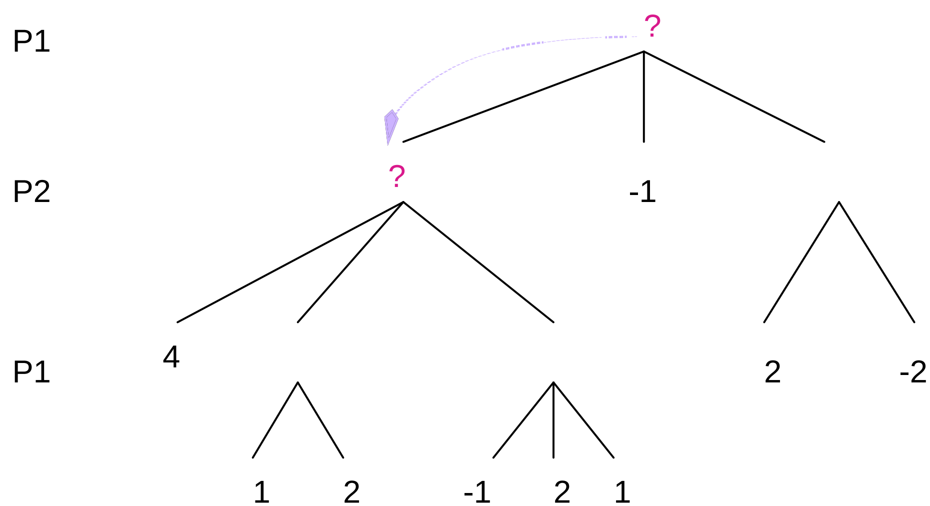

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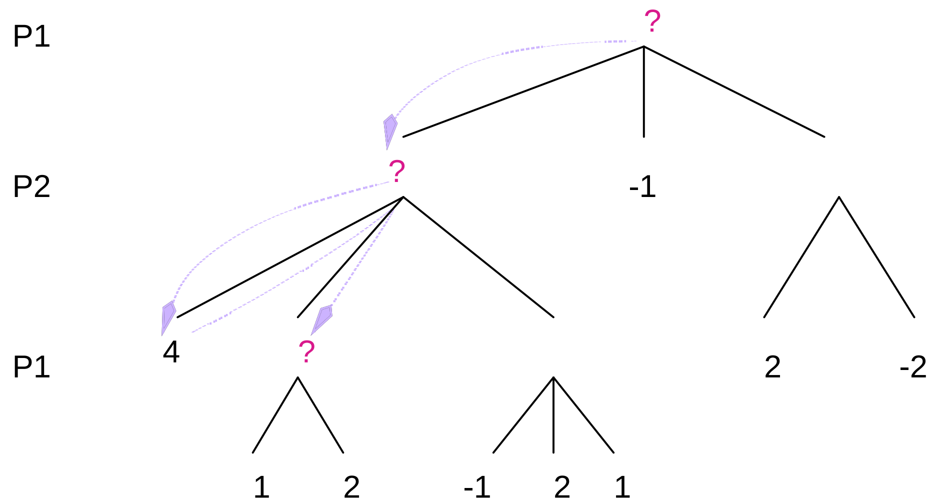

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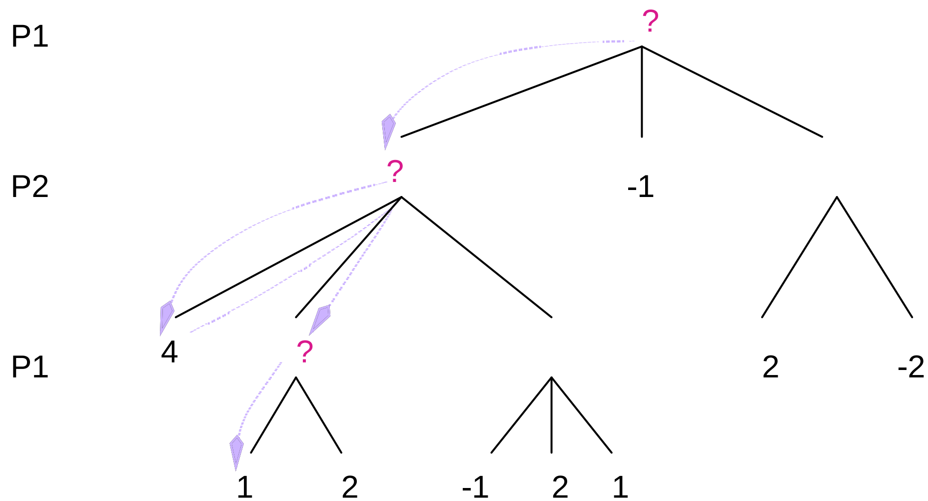

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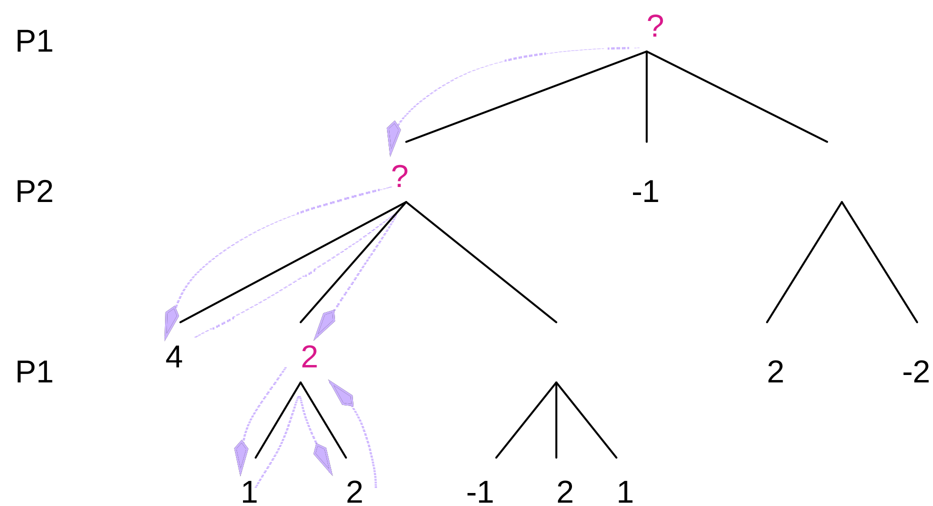

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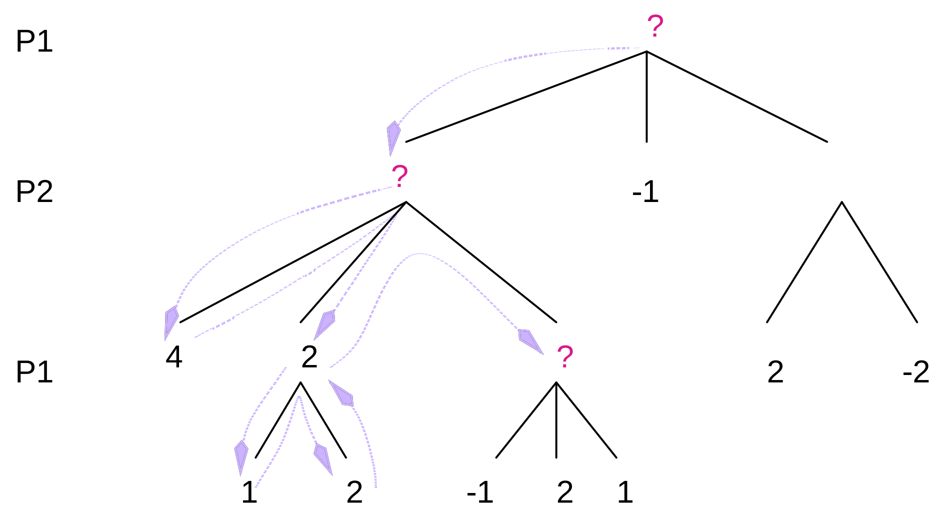

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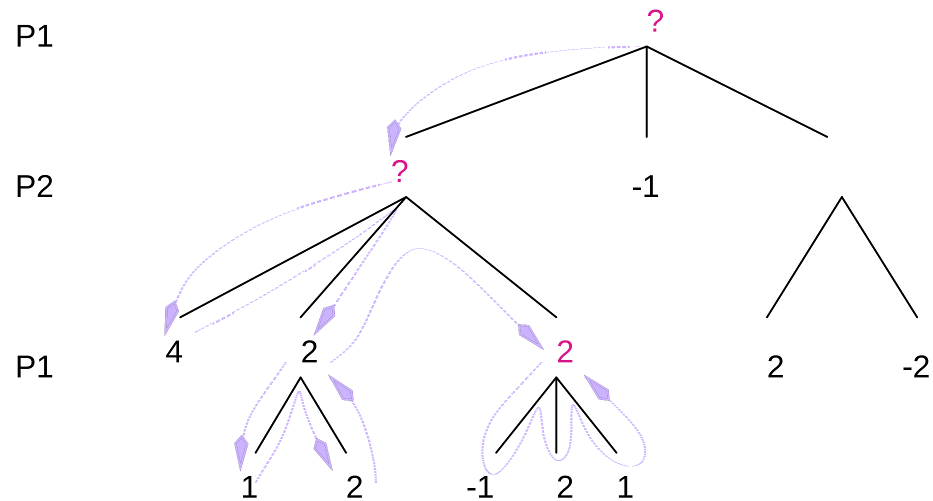

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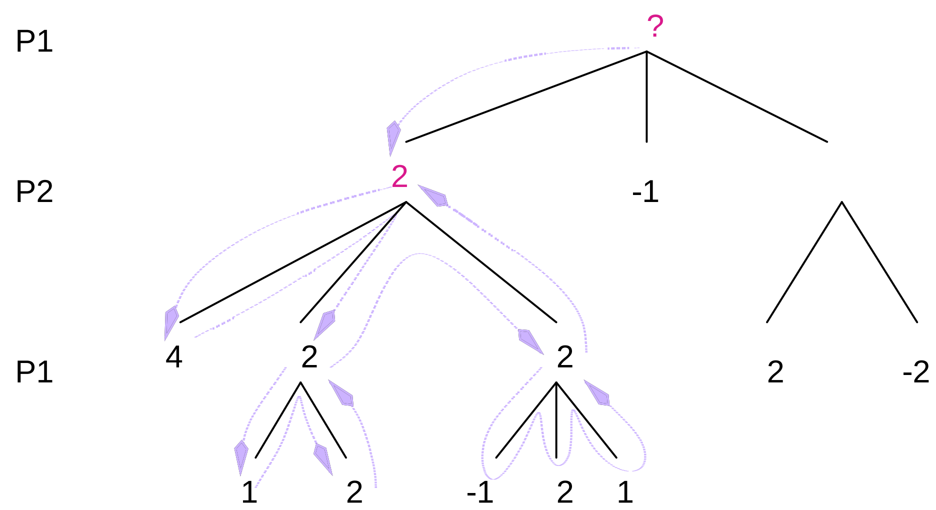

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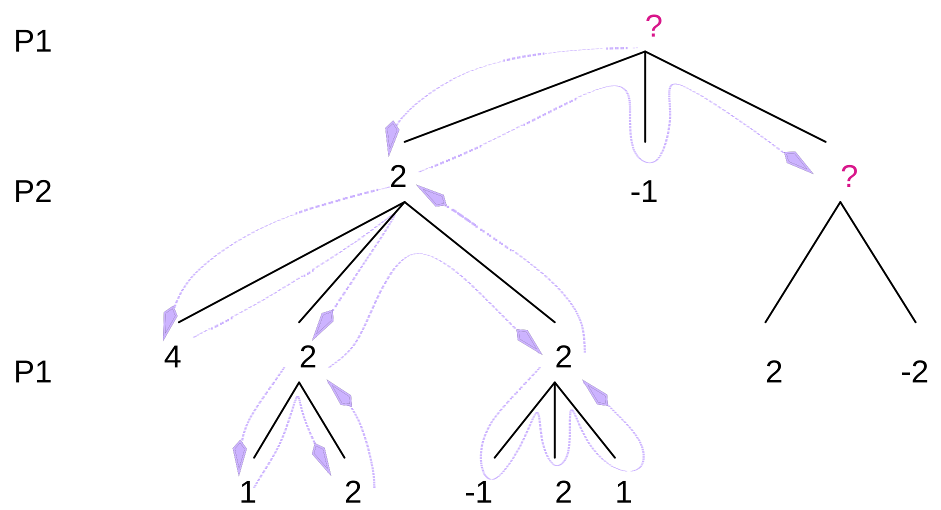

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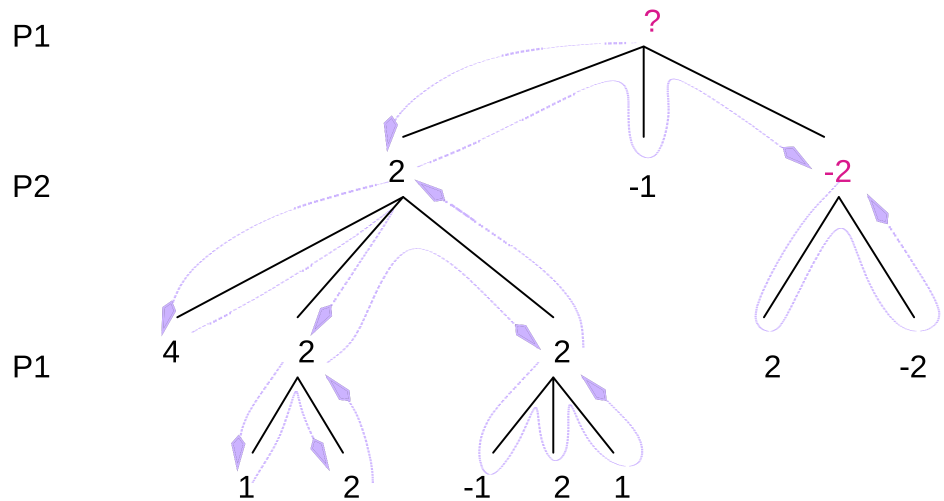

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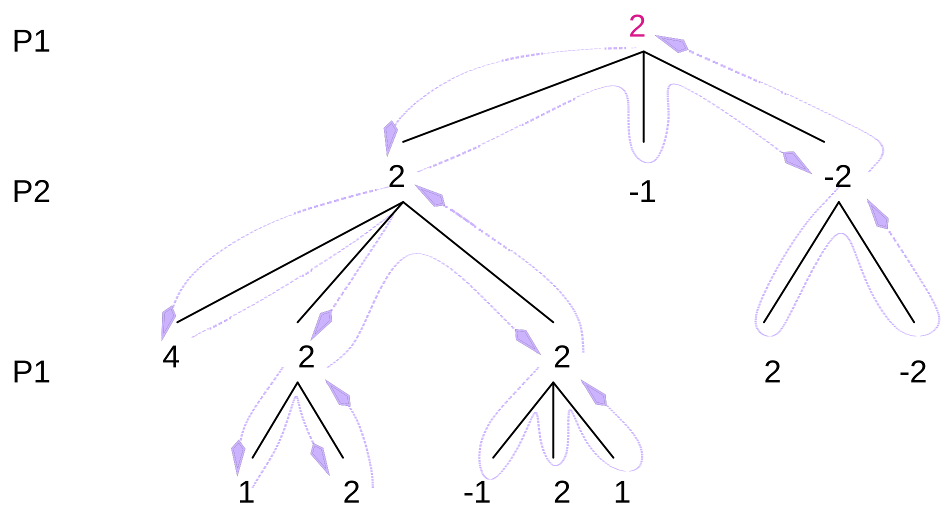

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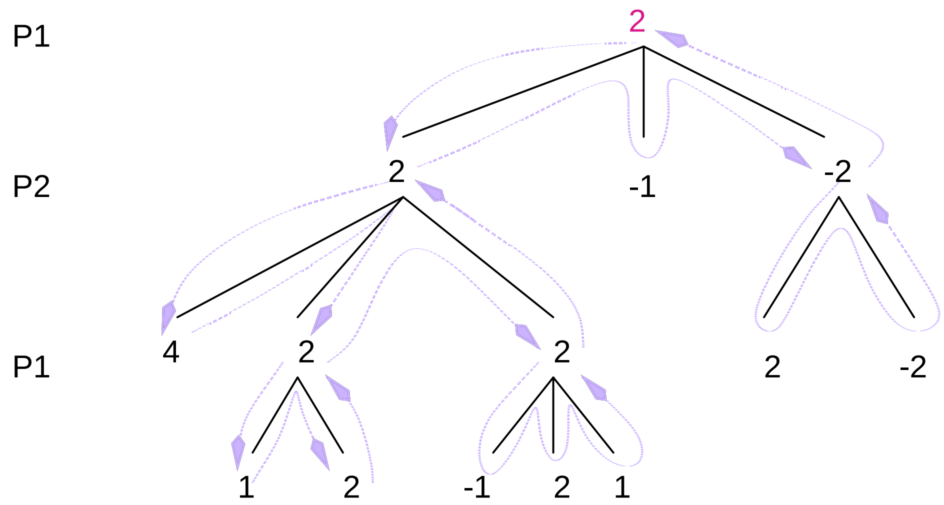

This is known as the minimax-algorithm.

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If the algorithm remembers how to achieve the calculated value it calculates a strategy for guaranteeing it.

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Once again, we get a less than desirable solution to our problem, but it is still the best we can do.

The problem with the idea of trying to take into account what other players might really do is that when this is done by all the players then this results in a process which usually goes around in circles.

More improvements?

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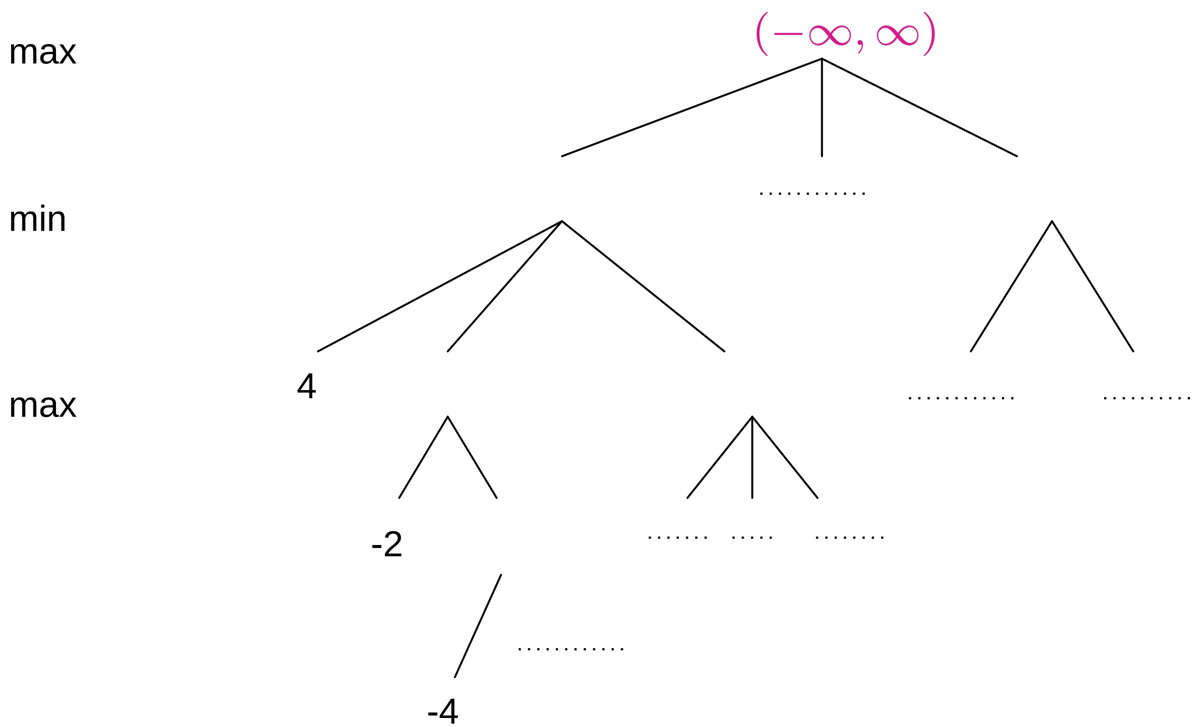

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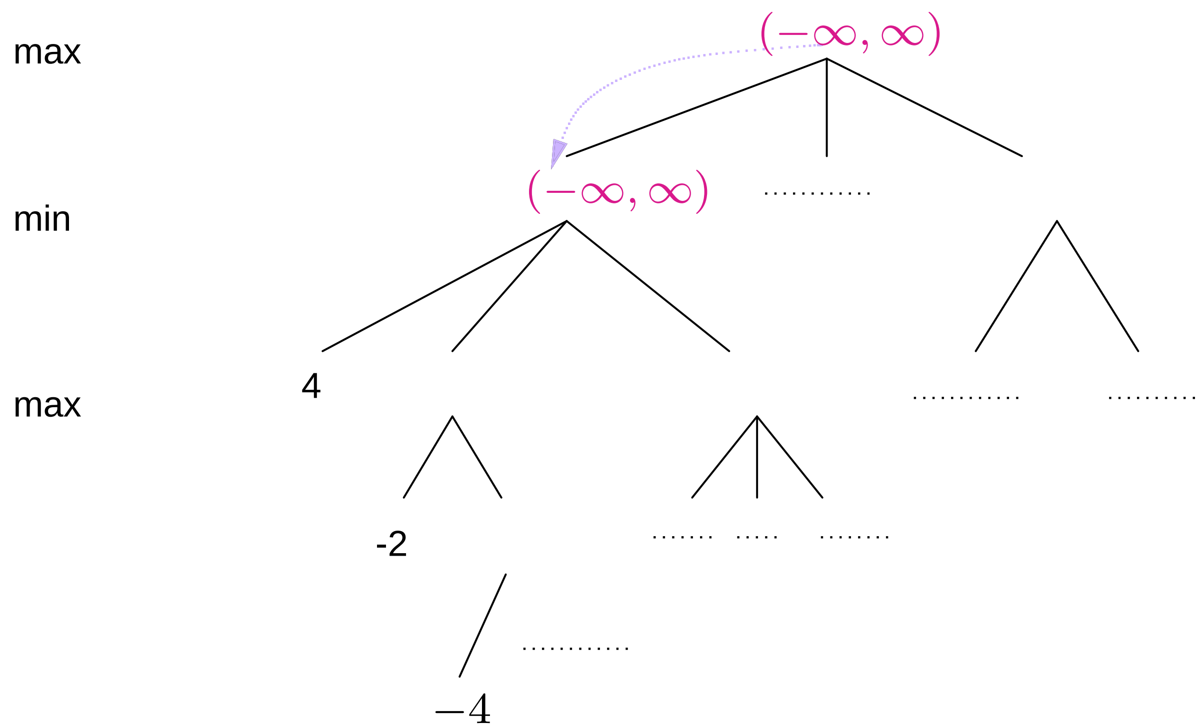

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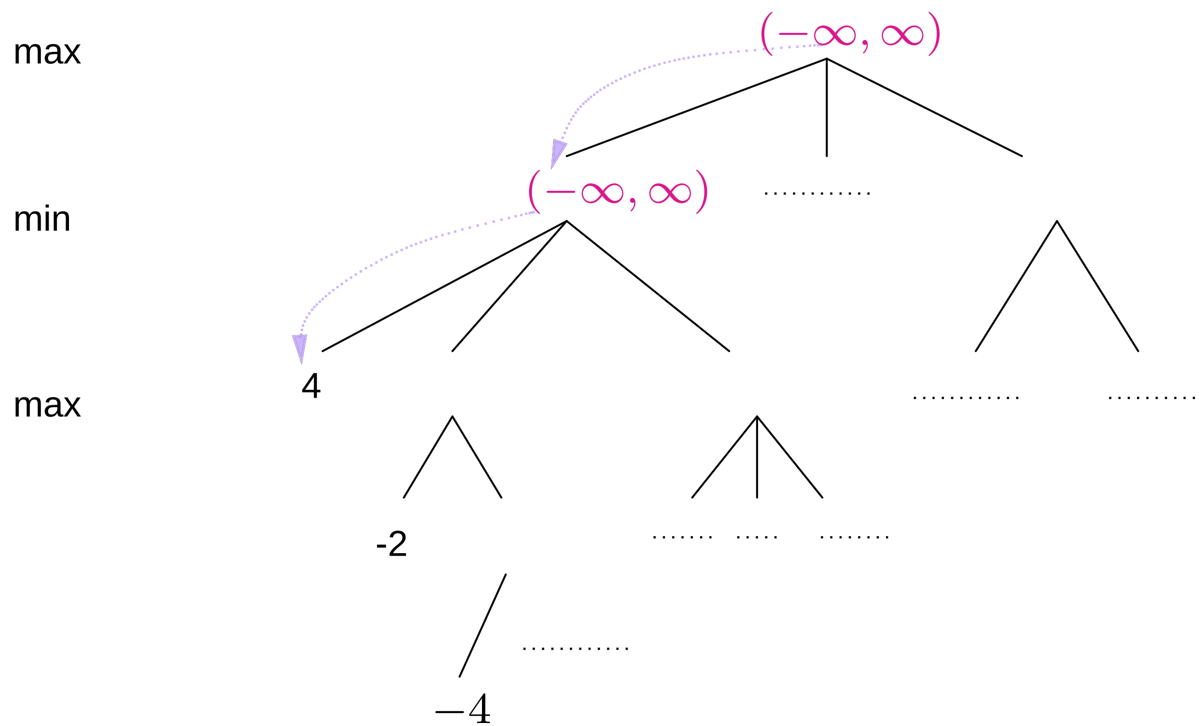

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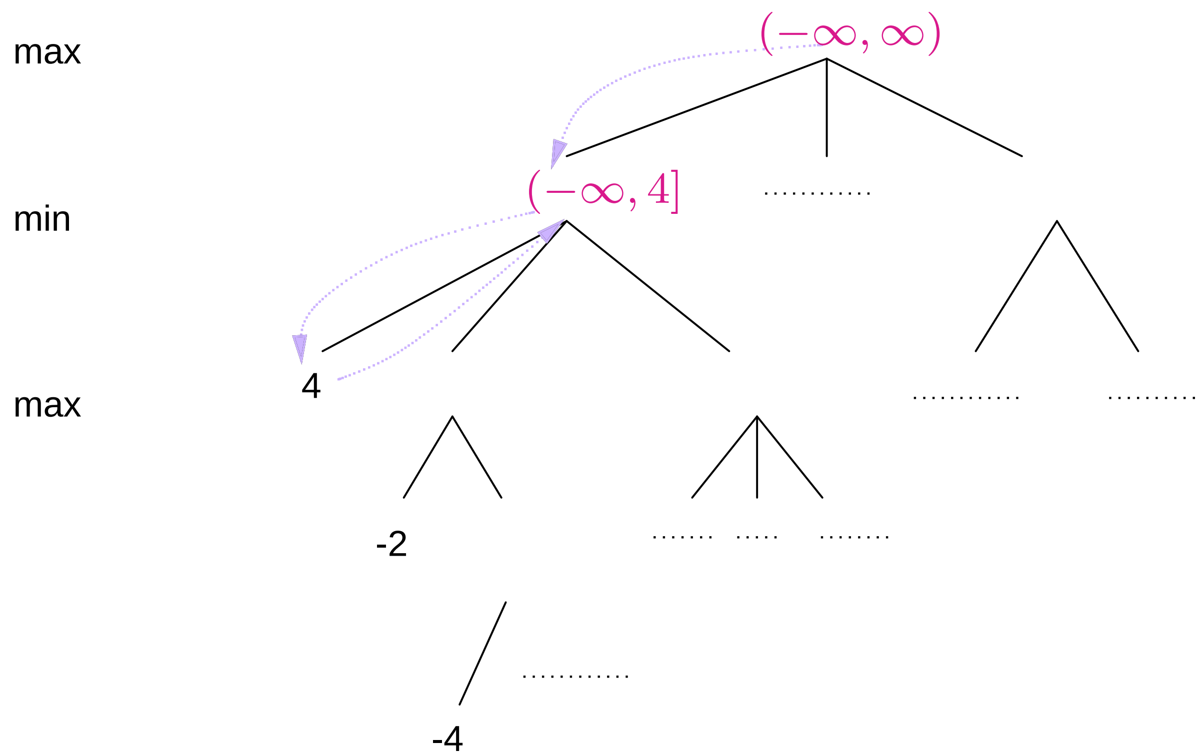

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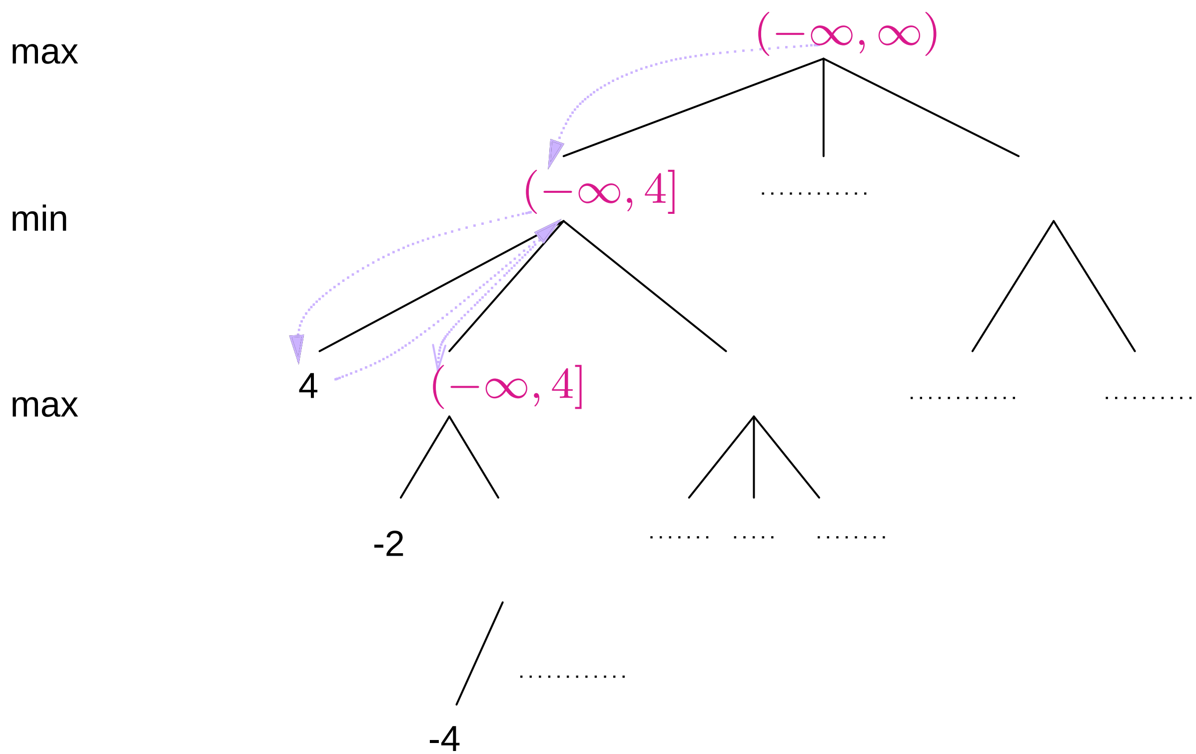

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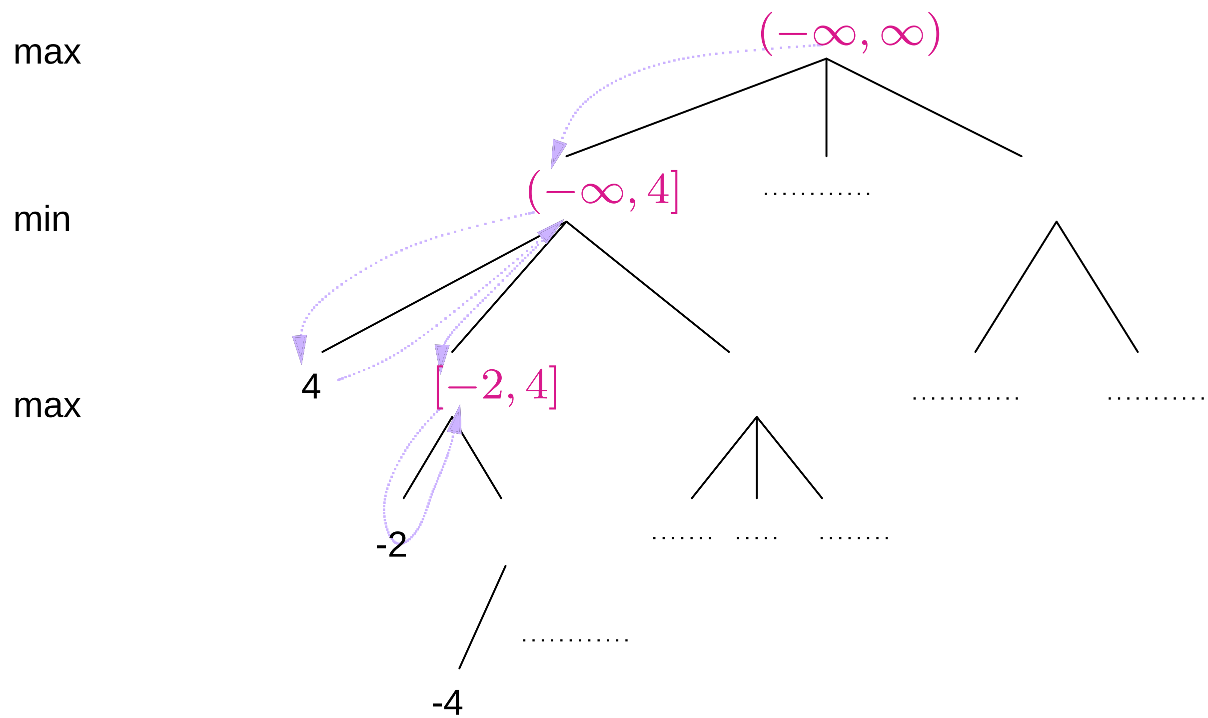

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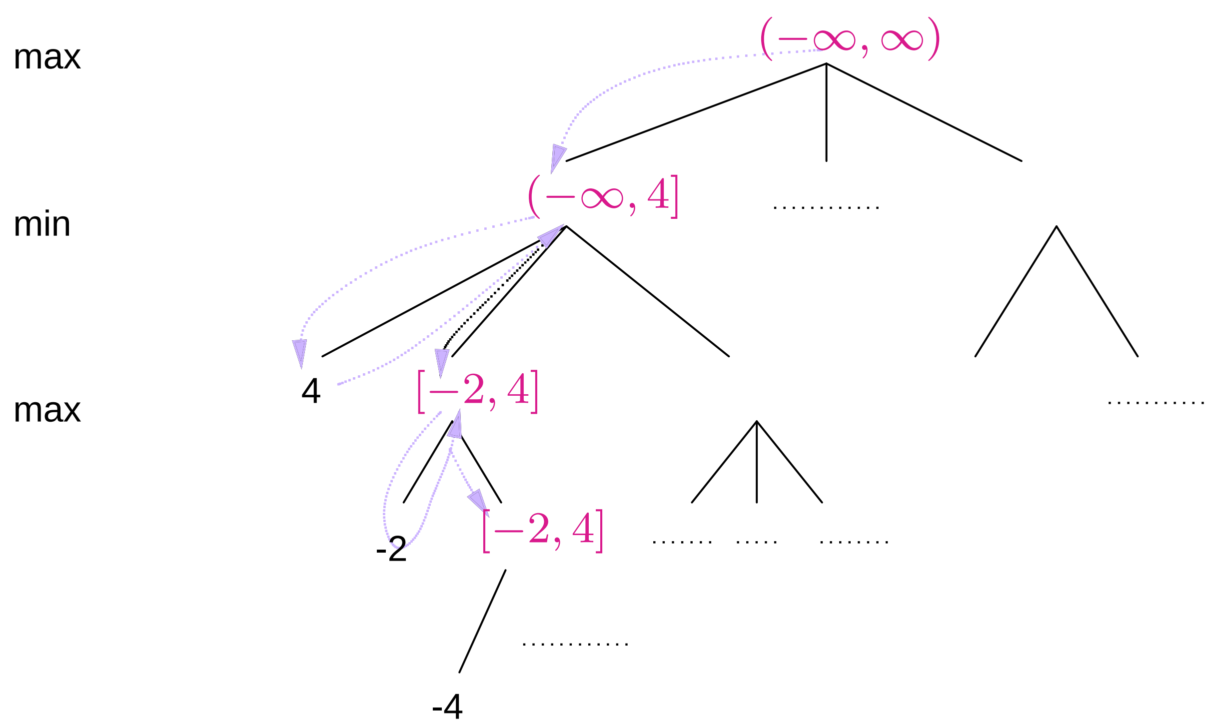

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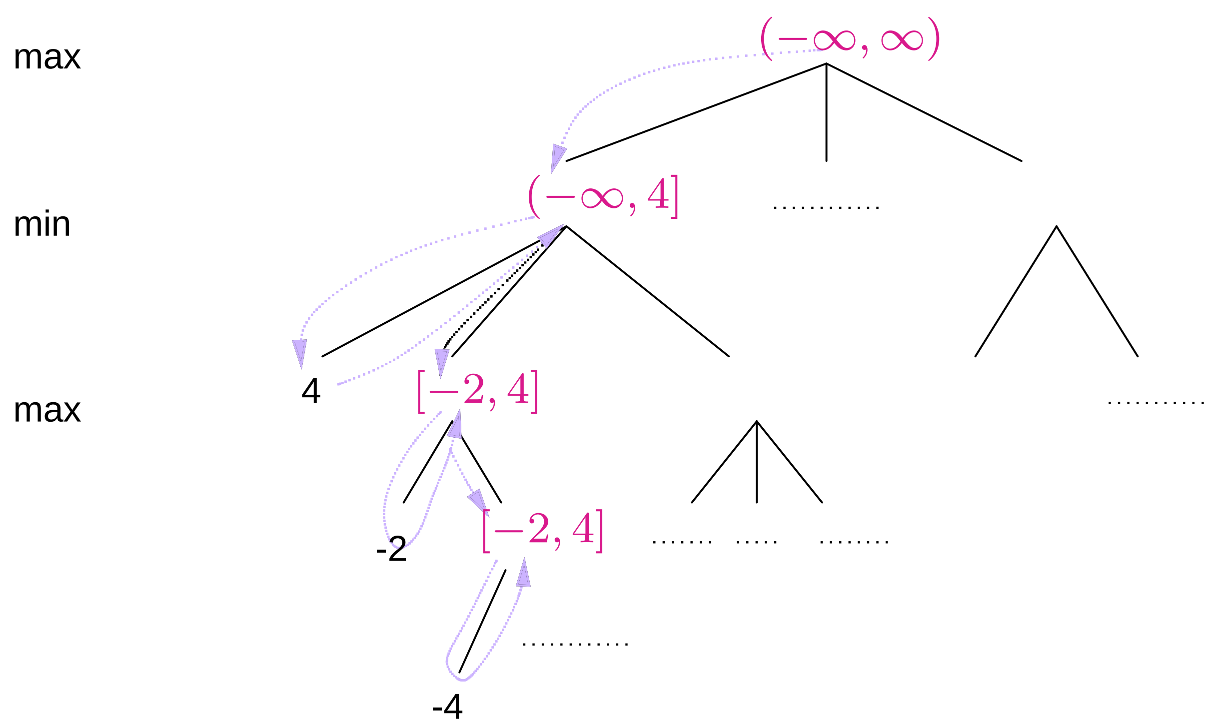

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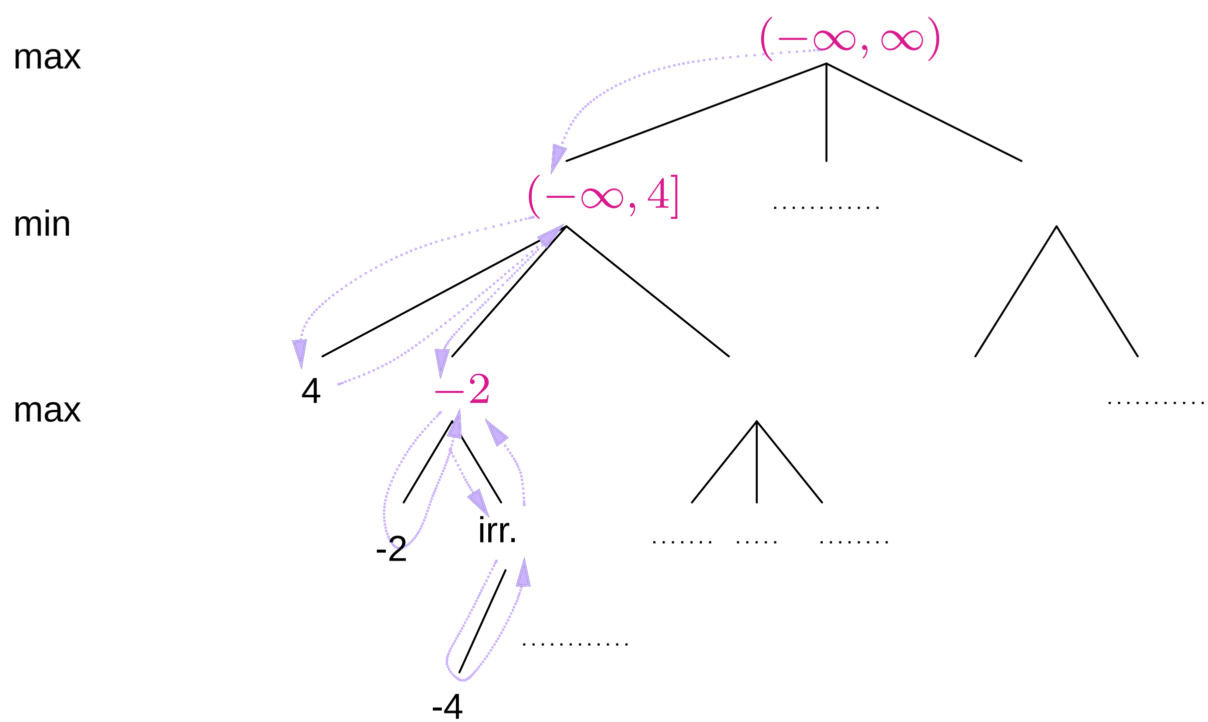

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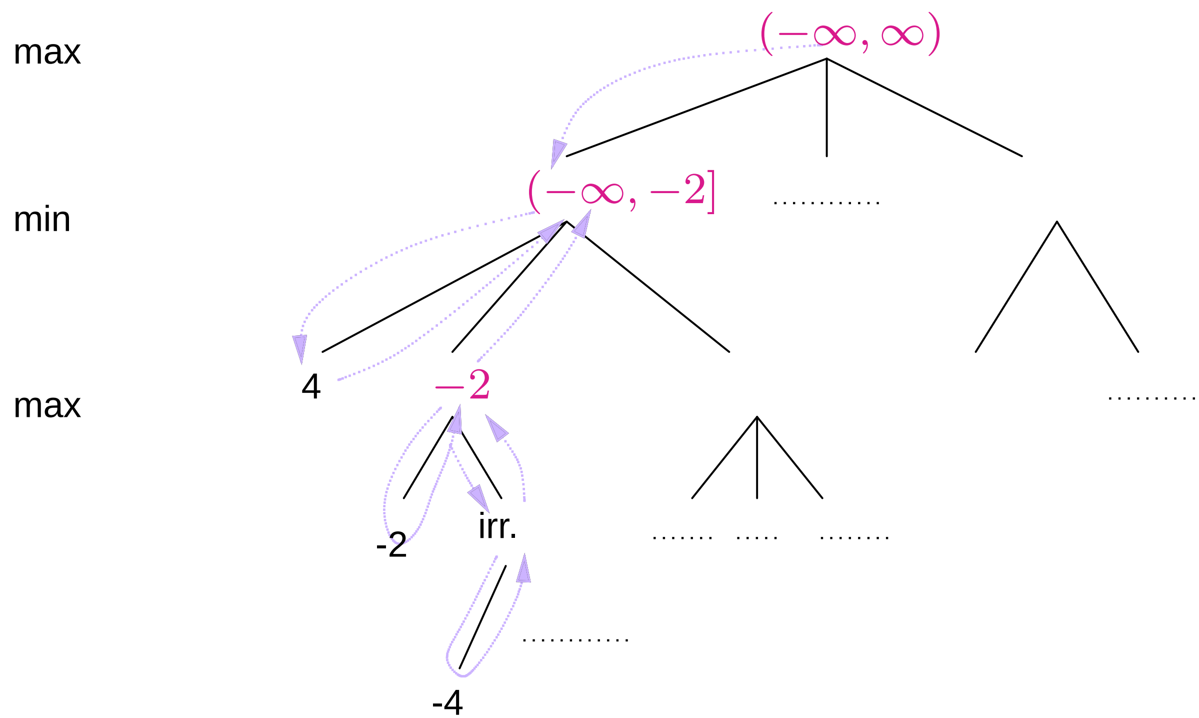

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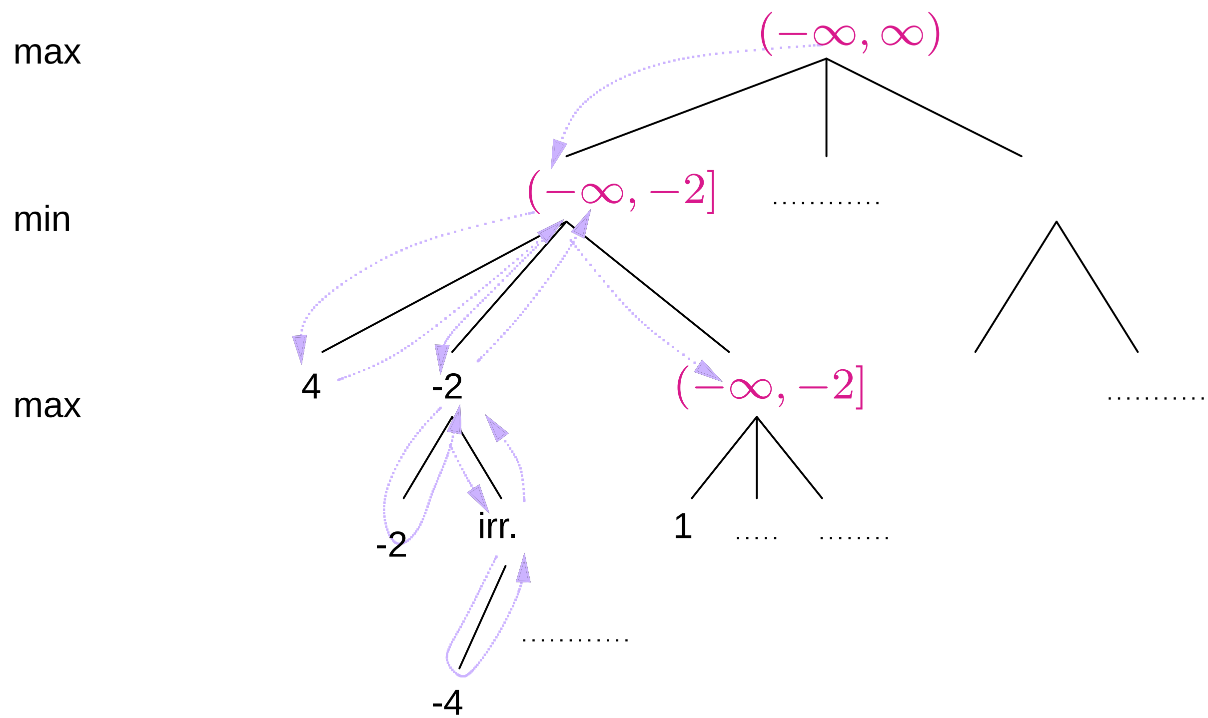

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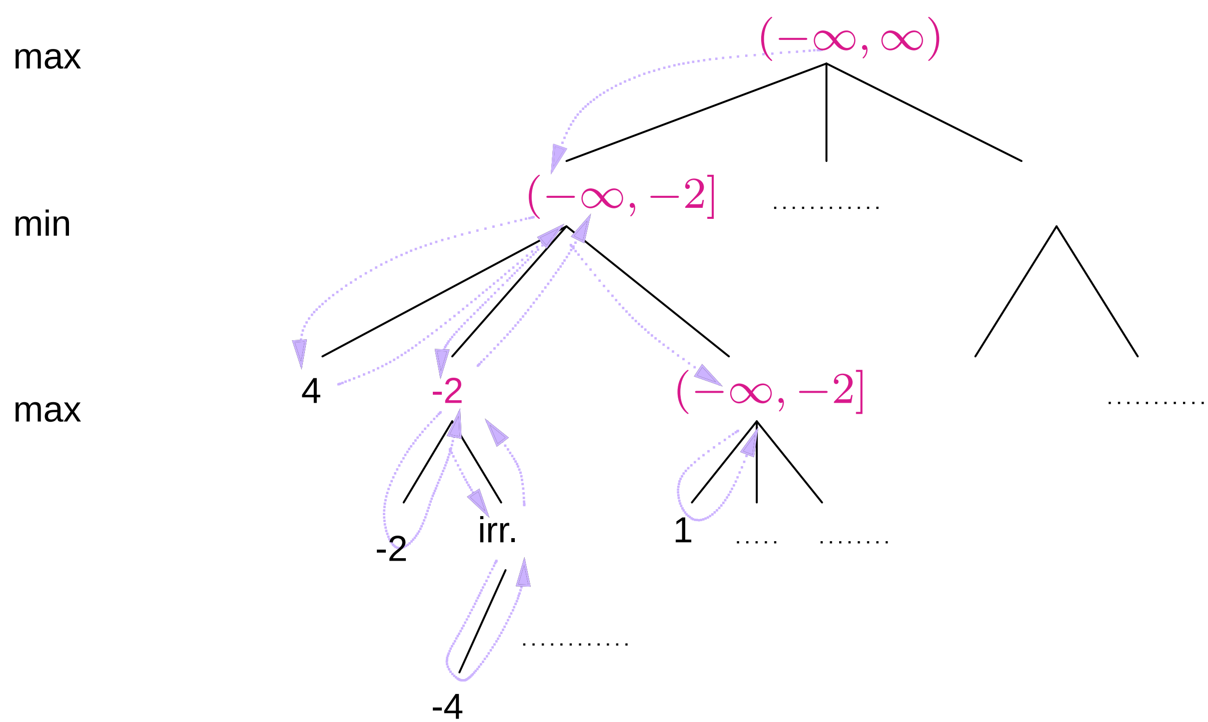

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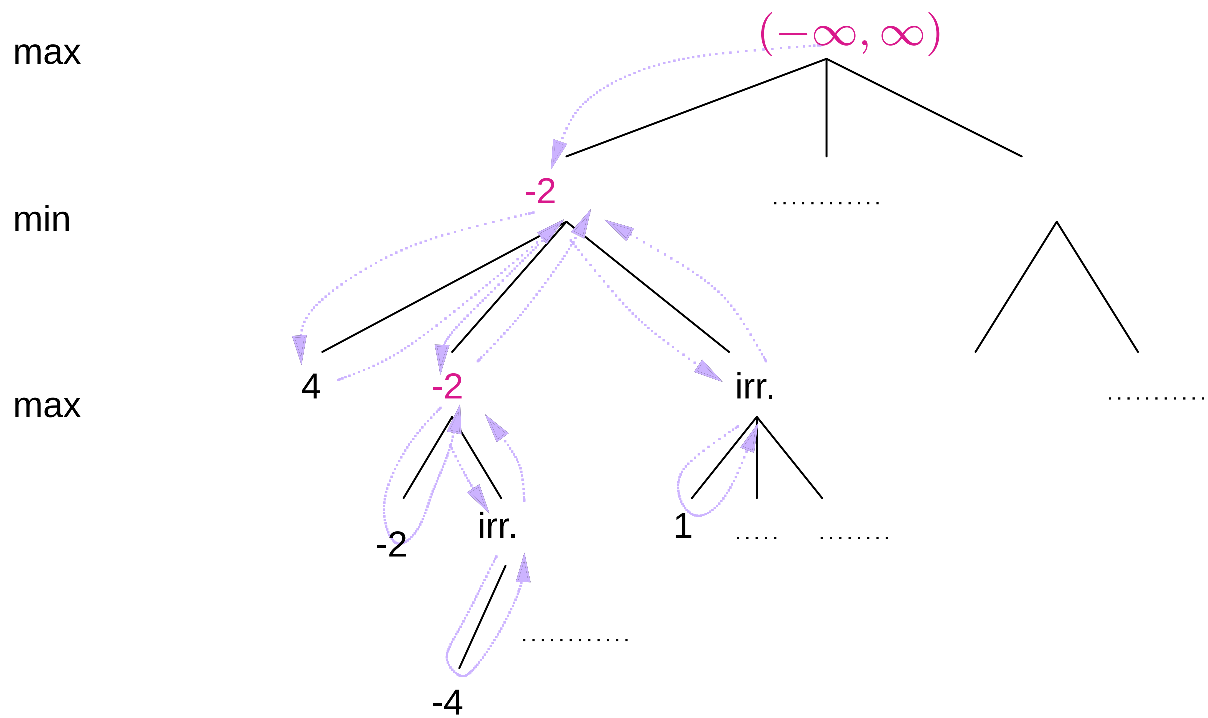

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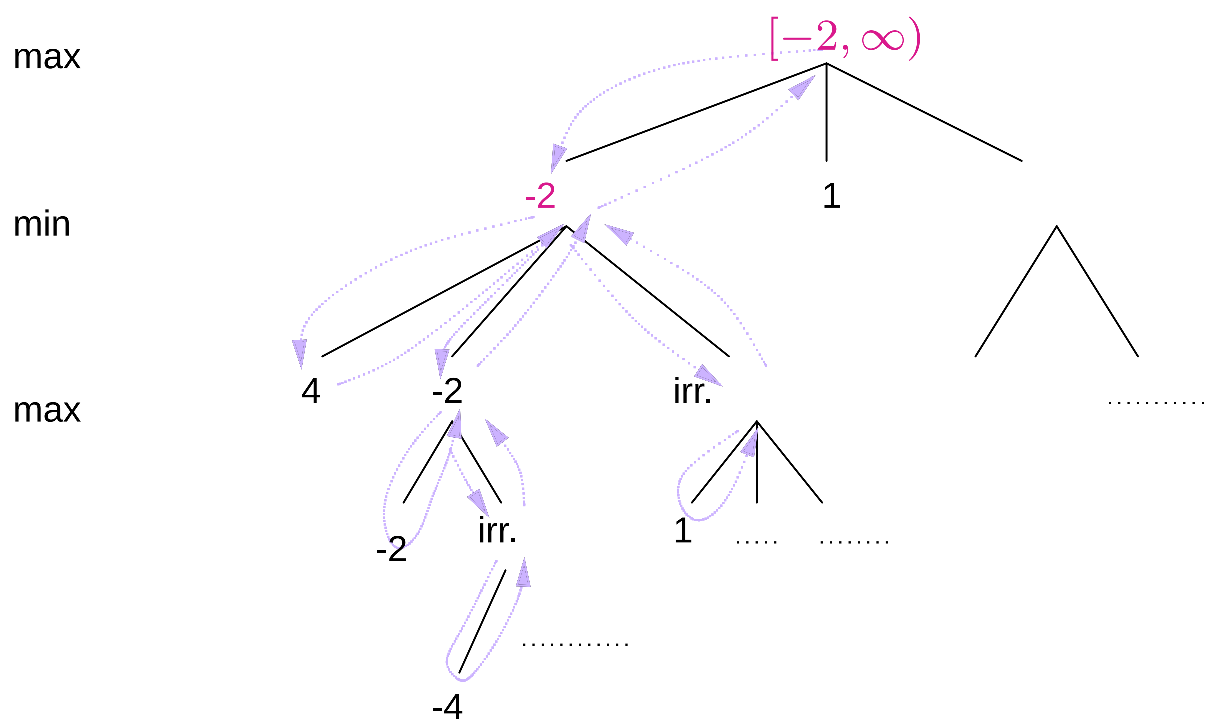

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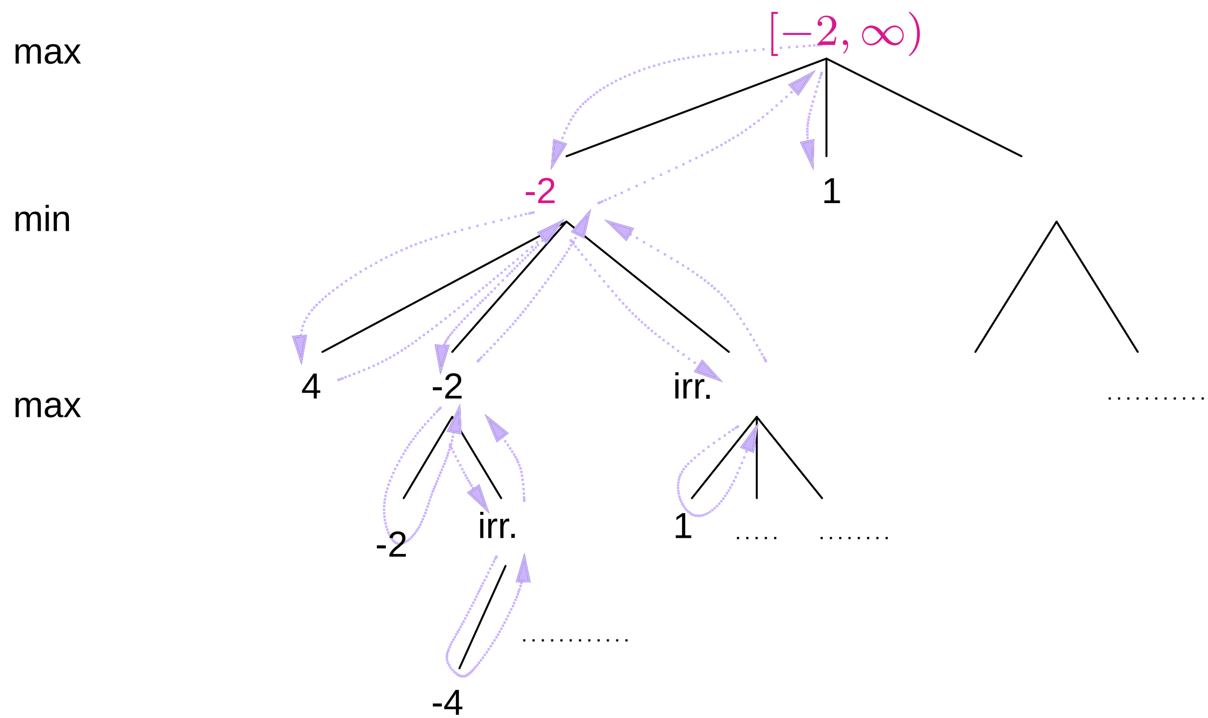

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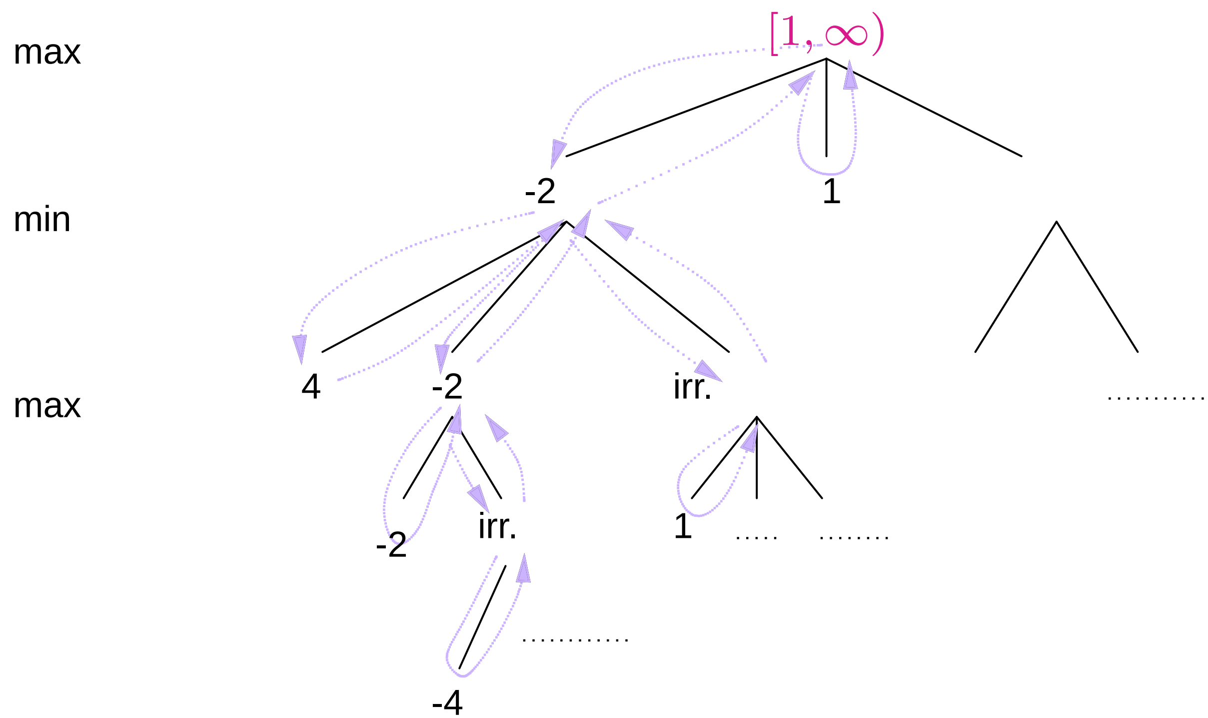

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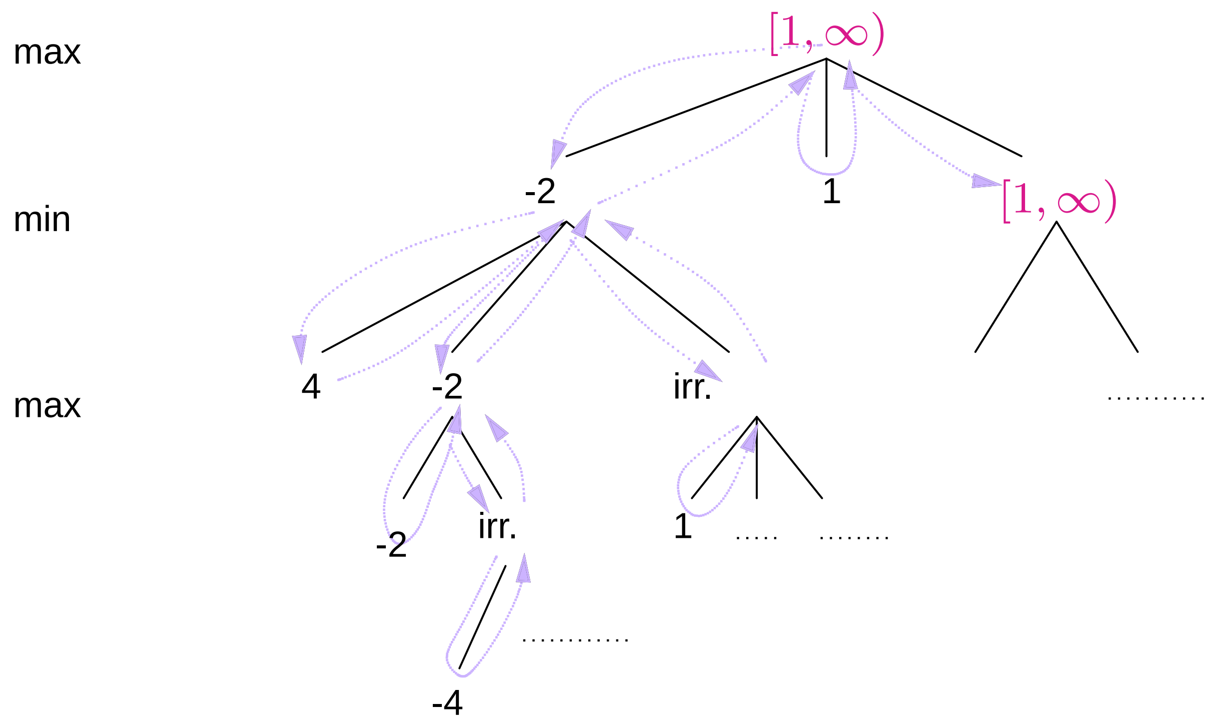

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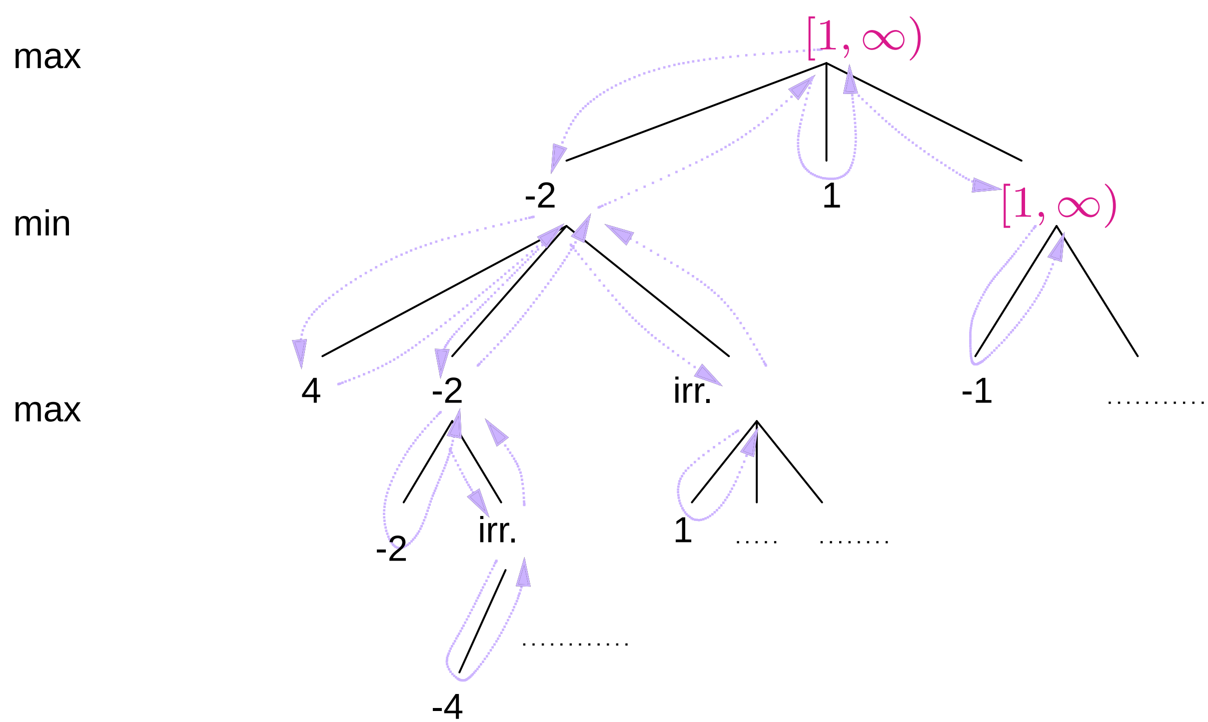

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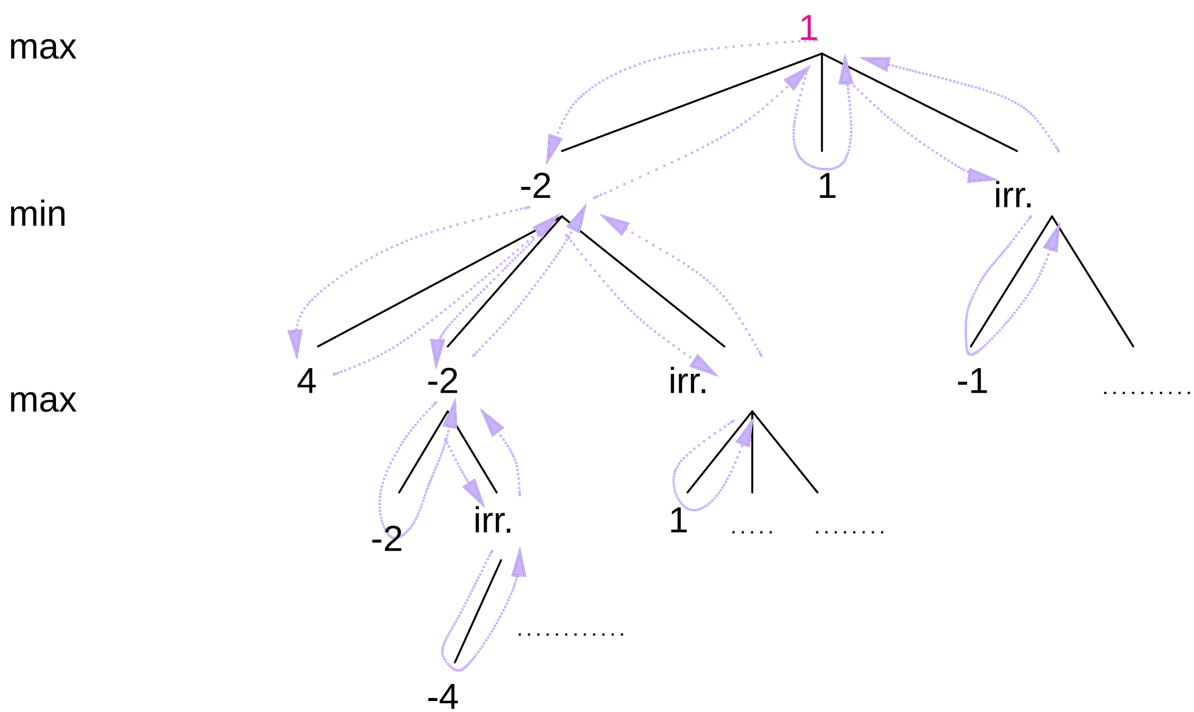

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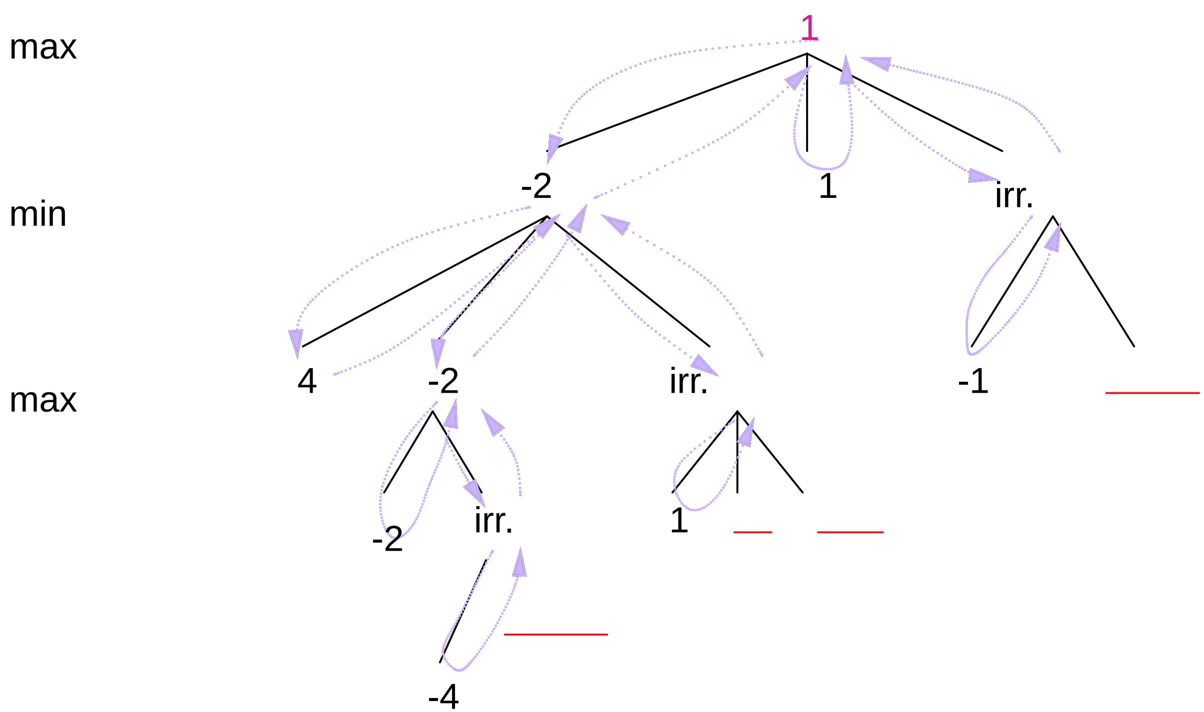

There are parts of the tree we have not searched at all!

How does this work?

On each recursive call of the procedure that determines the value, two parameters

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decrease upper bound β when value found is smaller		$\checkmark$
Stop search and return to parent when value found is		

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decrease upper bound β when value found is smaller		$\checkmark$
Stop search and return to parent when value found is	smaller than α	

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decrease upper bound β when value found is smaller		$\checkmark$
Stop search and return to parent when value found is	smaller than α	greater than β

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Deciding which available moves are likely to be good for a game like Chess, Go, Othello, or the like, is hard!

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- This algorithm does not require the whole game tree to be in memory as it runs, it merely has to build it locally in order to visit all nodes in a depth-first search.
- For non-zero sum games, the outcome of this procedure is to be viewed with a grain of salt.
- This algorithm only works for games of perfect information.
- We can improve on this algorithm to get one which does not have to visit all nodes in the tree; this is known as alpha-beta pruning or alpha-beta search. Again we can use this algorithm to determine the best move, namely one that leads to the value.