

Forty-five minutes

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Mathematical Techniques for Computer Science

11/11/18

Time: 14.00

Please answer all TWO Questions

This is a CLOSED book examination

The use of electronic calculators is not permitted.

1. a) Consider the following function:

$$f: \mathbb{Z} \longrightarrow \{n \in \mathbb{Z} \mid n \text{ is odd}\} \cup \{0\}$$

$$x \longmapsto x \cdot (x \bmod 2).$$

Is this function injective? Is it surjective? Justify your answers and indicate which properties you are using. (5 marks)

- b) Recall the max operation which returns the largest number in a finite set, so for example $\max\{1, e, \pi, 10, \sqrt{2}\} = 10$. Consider the binary operation on complex numbers:

$$(a + bi) \otimes (a' + b'i) = \max\{a, b\} + \max\{a', b'\}i$$

For example we have

$$(1 + 2i) \otimes (\pi + 4i) = 2 + 4i.$$

Is this operation associative? Is it commutative? (5 marks)

2. a) Show

$$P \wedge (\neg P \vee Q) \equiv P \wedge Q$$

in the Boolean semantics by using truth tables. (3 marks)

- b) Give a brief explanation of **one** of the following. (2 marks)

- i) contradiction
- ii) contrapositive law

- c) Consider the following propositional formula.

$$(P \vee Q) \rightarrow (\neg(P \rightarrow Q) \vee \neg P)$$

- i) Use our CNF algorithm to transform the formula into conjunctive normal form. (2 marks)
- ii) Simplify the conjunctive normal form as far as possible using our fundamental semantic equivalences. (3 marks)

Justify all the steps in your derivations and simplifications.