Forty-five minutes

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Mathematical Techniques for Computer Science

13/11/17

Time: 12.00

Please answer all TWO Questions

This is a CLOSED book examination

The use of electronic calculators is not permitted.

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1. a) Consider the following function:

$$f\colon \mathbb{N} \longrightarrow \{k \in \mathbb{Z} \mid k \text{ is even}\}$$
$$n \longmapsto \begin{cases} n & n \text{ even} \\ -2n & \text{else.} \end{cases}$$

Is this function injective? Is it surjective? Justify your answers. (5 marks)

b) Consider the binary operation on the set

$$\{0,1,2\}$$

given by the assignment

$$m \circledast n = (mn+1) \bmod 3.$$

Is this operation associative? Is it commutative? Justify your answers. (5 marks)

2. a) Let A be the following propositional formula.

$$P_1 \leftrightarrow (P_2 \rightarrow P_1)$$

i) Construct a truth table for the formula. (2 marks)
ii) Describe in a sentence for which valuations the formula is true. (1 mark)

b) Give a brief explanation of **one** of the following. (2 marks)

i) tautology
ii) atomic formula (in this case, also give an example)
iii) Boolean function

c) Consider this propositional formula.

$$\neg(\neg R \vee P) \vee (P \wedge R).$$

i) Use our CNF algorithm to transform the formula into conjunctive normal form. (2 marks)
ii) Simplify your answer as much as possible. (3 marks)

END OF EXAMINATION