

Forty-five minutes

UNIVERSITY OF MANCHESTER
SCHOOL OF COMPUTER SCIENCE

Mathematical Techniques for Computer Science

04/11/14

Time: 12.00

This is a CLOSED book examination

The use of electronic calculators is not permitted.

1. a) Consider the following function:

$$f: \mathbb{Z} \longrightarrow \mathbb{N}$$

$$x \longmapsto \begin{cases} 3x & x \geq 0 \\ -3x + 1 & \text{else.} \end{cases}$$

Is this function injective? Is it surjective? Justify your answers. (5 marks)

b) Consider the following binary operation on \mathbb{C} : We set

$$z \otimes z' = z \cdot z' + z + z'.$$

Is this operation associative? Is it commutative? Justify your answers. (5 marks)

2. a) Show

$$(P \rightarrow Q) \equiv (\neg Q \rightarrow \neg P)$$

in the Boolean semantics by using truth tables. (3 marks)

b) Give a brief explanation of **one** of the following. (2 marks)

- i. subformula
- ii. Boolean valuation
- iii. substitution for a propositional variable

c) Consider the following propositional formula. (5 marks)

$$(\neg P \vee Q) \rightarrow (P \rightarrow (P \rightarrow Q))$$

- i) Give a conjunctive normal form for the formula. Simplify as far as possible.
- ii) Give a disjunctive normal form for the formula.