Exam Performance Feedback Form

COMP11120, Semester 1

2019/2020

First some general remarks. 220 students sat the exam. The median mark was 36 out of 60 (or 60%), which was also the average. This was not as high as we would have liked. The main reason for this is the poor performance on Question 1. See the discussion of each question below for more detail on this.

Thirty-five students had a failing mark, that is a mark of less than 24 out of 60, or a mark of less than 40%. Of these, twenty-two students had a mark below 18 (which is below 30%), with a lowest mark of 4. Fifteen students had a third class, forty-two a lower second class and thirty-six an upper second class mark. On the top end, eighty-two students achieved first class marks, thirty-eight students a mark of more than 80%, with a top mark of 59, which is 98%.

Statistical analysis of individual questions:

Question 1. The average for Question 1 was 9.6, which corresponds to a mark of 48%. Seventy-seven students had a failing and forty students a first class mark, three achieving full marks, and sixteen students on high marks of 18–20.

Question 2. This had an average mark of 13.6 (68%), with 39 students on a failing mark and 139 on a first class mark. Thirteen students achieved full marks. Sixty-five(!) students got a very high mark of at least 18.

Question 3. Overall average: 64%. 42 students got 18–20/20 (9 got 20/20).

On this question the distribution was positively skewed with just under 50% of students achieving a first class level mark (14—20/20), about 25% second class level marks (10–13/20), 12% third class marks (8–9/20) and 15% fails (0–7/20).

General note about marking: As a general rule when a mistake is made in a part of a question we continue marking that part and also the parts that depend on it. This means that if a mistake was made early on it was still possible to get close to full marks if this was the only mistake. (This note is particularly relevant to Question 3a) in which the parts depended on each other.)

Question 1. Overall there was a great range of quality in answers, in particular for parts a) and c).

a) One may write any natural number n into $100k + l$, by Fact 2 (applied to 100), with $k \in \mathbb{N}$ and $0 \leq l \leq 99$, so l is the number given by the last two digits of n. It is then not hard to use statements about divisibility (by 4) to show the claimed result.

   Marks were typically lost for not providing a rigorous argument (looking at examples, trying to argue about the progression of the last two digits for multiples of 4), or using statements not established. Also, quite a few students only proved one direction for this ‘if and only if’ statement.

b) Almost all students were successful in providing a counterexample to commutativity, with the example provided in the question being the most popular choice.

   The given operation is associative. The proof of this requires working out the ith symbol in the result of calculating $(s \circledast s') \circledast s''$ and $s \circledast (s' \circledast s'')$, for both i even and odd. The majority of students were able to do this, either by determining the corresponding symbol from the input or by pointing out that the calculation of the ith bit only requires the ith bits from each input, and giving a table that covered all the combinations.

Marks were lost by not giving a proof, doing a sample calculation (possibly one with letters, but not one indicating how it applied to strings consisting of 1024 symbols). There was a lot of sloppy writing here—the given operation is only defined for binary strings of length 1024! Some students did not seem to understand the meaning of 'binary' and used symbols such as 2 in their work.

c) There was clearly a lot of confusion about the remainder operation when the inputs are negative. The remainder in integer division is always a positive number (described in Fact 4), so the only ones possible in the question are 0, 1 and 2. Fact 4 also gives the equation that has to be satisfied, so for example $-1 = -1 \cdot 3 + 2$, so $-1 \bmod 3 = 2$. This means that the value of the given function at -1 is 2.

   Many students miscalculated the values of the function they were asked to consider, and they then correspondingly gave an incorrect answer regarding injectivity and surjectivity.

   The given function is both injective and surjective, and the easiest solution is to give the inverse function and prove that it is the inverse function. Where purported inverse functions were given these were often incorrect.

   Many of the injectivity arguments made either went made a jump from $x + (x \bmod 3) = x' + (x' \bmod 3)$ to $x = x'$ without justification, or they made some case distinction that did not cover all cases. There are three correct approaches:

   - One can argue that if the given equality holds then x and x' must have the same remainder when dividing by 3, but that needs to be carefully established. Once one has this the remaining proof is easy.
   - One can consider all nine possible cases of the remainders x and x' may have when dividing by 3, in which case 6 lead to a contradiction, leaving the three cases where x and x' have the same remainder, which give the required $x = x'$.
   - One can express x and x' in the form of $3n+k$ and $3n'+k'$ respectively (using Fact 4), in which case $fx = fx'$ implies $3(n - n') = 2(k' - k)$, and use that to make an argument that $n = n'$ and $k = k'$ must follow.

   For surjectivity, given $y \in \mathbb{Z}$ we must find $x \in \mathbb{Z}$ with $fx = y$. There were very few answers that tried to do this. A number of them made a case distinction based on the remainder x leaves when divided by 3, but that is invalid since x is unknown. The correct answer requires one to do this based on $y \bmod 3$, that is the remainder y leaves when divided by 3, qand if one does this correctly one has a description of the inverse function.

   I saw one or two correct arguments that did not quite fit into the above patterns.

Question 2. There were a lot of good to very good answers for this question.

a) The sample set has only two elements, 0 and 1. Marks were typically lost because some component of describing a probability space were missed. A correct answer needs a statement $\mathcal{E} = \ldots$, or 'the set of events is ...' Writing $\mathcal{E} \subseteq \mathcal{P}\,S$ only repeats one of the specifications of $\mathcal{E}$ but it doesn't define it. Similarly, writing $P\colon \mathcal{E} \longrightarrow [0, 1]$ repeats a specification, but does not tell us which probability assignment is required here. Some students wrote the set of outcomes as $[0,1]$, which is the set of *all real numbers from 0 to 1*, while other wrote $\{0,0,0,1\}$, which just confuses matters—a set cannot have several copies of the same element.

b) The answers are 7/16, 3/4 and 15/16 for the three tetrahedra in rising numbers of surfaces marked 1. A number of students only calculated one of these, but many students carried out this task correctly. The easiest way of calculating these is to add the probabilities that we get 0, and that we get 1, and subtract those from 1.

c) The answer is 7/4. Some students did not seem to realize that an expected value for a random variable consists of just one number. Some students computed this for all tetrahedra, when it was asked only for the one from part a). Some students computed this removing the cases where the random variable is not allowed to take the values 0 and 1. Presumably this was on their mind based on the previous part, but that does not give a well-defined random variable. There were also a couple of clever solutions noting that the given random variable X could be expressed as $Y+2Y+4Y$, where Y is the random variable given by throwing the tetrahedron just once, but note the weights required here.

d) Some students did not really know what they were asked for her. We need a pmf for the set of outcomes from the numbers 0 to 7 (000 to 111 in binary), provided we already know that the first throw is 0. Some students changed the available outcomes, but that is not a valid move—the random variable itself does not change, just the probability distribution under consideration. The outcomes 4 to 7 have the probability 0.

e) The overwhelming reason why the average for this question is high is that the majority of students could carry out Bayesian updating correctly. The final probabilities are 9/20, 8/20 and 3/20. For full marks the final answer needed to include a statement that the first of these are close together, and so the result is not very reliable at this point.

Question 3. Everybody attempted a). Some skipped question b). Question c), or parts of c), were most often skipped; perhaps an indication of running time at the end of the exam or insufficient revision.

a) Average mark: 74% overall (87% part i), 52% part ii), 62% part iii), 74% part iv), 76% part v)). 53 students got full marks.

   As the average indicates this question was answered very well. Parts i), iv) and v) posed few problems and answers were overwhelming good also for the other parts. Most mistakes were made in ii) and iii). Common mistakes for which marks where lost were:

   * Mistakes in the calculations, e.g. in the application of De Morgan's law: (not changing $\wedge$ to $\vee$ or vice versa, or forgetting to negate the second subformula).
   * Too few brackets in a formula with $\wedge$ and $\vee$.
   * Not providing justifications or mistakes in justifications, e.g. wrong name of laws.
   * In ii): not noticing that the DNF from i) had four disjuncts (flattening helps making this explicit) and mistakes in the inevitably longer calculation when not using the general distributivity law (Step 4 in our CNF algorithm).

  * In iv) and v): not explaining the answer. Just writing the definition of a tautology or satisfiability did not earn any marks. I expected in each case a valuation of the propositional variables to be given.

  Note that we have not proved that if a formula cannot be simplified to $\top$ then the formula is not a tautology. We do know though that if a formula can be simplified to $\top$ using our semantic equivalences then it is a tautology.

  Because the parts of this question depended on each other please see the general note above about marking.

b) Average mark: 62%. 77 students got full marks.

  Most students identified that in lines 4 and lines 6 $\rightarrow$ Introduction was applied wrongly and got 2/4. Alternatively lines 3 and 5 could have been corrected, but this would have required correcting the justification as well, which some missed. Most noticed that lines 1 and 2 had no justifications, but they are not obtained by the Axiom rule (which some thought). The most frequent mistake was not being able to work out what the correct application of the Axiom rule was. There is an obvious correction that adds an extra line at the beginning, but correct proofs with two applications of the axiom rule were also accepted.

  A variety of corrected proofs were given and all correct ones received full marks even if they were more complicated than needed. Note a full corrected proof did not need to be given; adequately and correctly describing the corrections would have also received full marks.

c) Average mark: 48%. 30 students got full marks.

  Parts i), ii) and iii) were generally answered well. They required to work out if the given formula is true or false in the given interpretation. Some students did not read the question for this properly and only wrote down the translations of the formulas (more than expected), and therefore lost easy marks. A translation did not need to be given, but I expected the answer to be correctly explained.

  Part iv) proved to be the hardest question, but 46 students got 2/2 and the same number got 1/2.