Exam Performance Feedback Form

COMP11120, Semester 1

2017/2018

First some general remarks. 196 students sat the exam. The median mark was 32 out of 60 (or 53.6%), and the average also 32.5 (54%). This was a slightly better performance than last year, and we felt overall that students knew the material better but made a number of sometimes silly mistakes that prevented a higher set of marks. See below regarding the mistakes that were typically made.

This year's Semester 1 exam left us with very mixed feelings. Some quite easy questions were answered largely wrong, in particular for Question 1. On the other hand quite a large number of students could do most of Questions 2 and 3 correctly.

Forty students had a failing mark, that is, a mark of less than 24 out of 60, or a mark of less than 40%. Of these, seventeen students had a mark below 18 (which is below 30%), with a lowest mark of 12. At the top end, forty students achieved first class marks with a top mark of 56, which is 93%.

Statistical analysis of individual questions:

Question 1. The average for Question 1 was 41%. Ninety-eight students had a failing and fifteen students a first class mark. One student achieved full marks. This is much lower than expected, given that part a) is a wrong statement with lots of counterexamples, part b) is a very straightforward proof, and part e) is a bit of information from the notes/lectures.

Question 2. This had an average mark of 60%, with fifty-seven students on a failing, and ninety-six students on a first class mark, of which seventeen had full marks. Only forty-three students are between those two groups—either students could do most of the question, or not much of it.

Question 3. Overall average: 62.3%. Fourteen students got 18-20/20 (2 got 20/20).

Question 1.

a) The statement as given is wrong. It is very easy to find counterexamples—there are plenty. For example, 4 divides $2 \cdot 2$, 6 divides $2 \cdot 3$, but neither divides either of the given factors. The vast majority of students wrote something they claimed as a proof, gaining no marks. One mark was given for saying the statement was wrong, one for providing a valid counterexample.

b) This is a very short proof that only requires knowing how the absolute $|z|$ of a complex number is defined. Nonetheless many students struggled with this. Some students lost a mark because they used r when they should have used $|r|$, making some of their chain of reasoning wrong. There is no sensible way of proving this by case distinction—some students wrote statements like 'if $z > 0$', which makes no sense for complex numbers.

c) Most students could answer part (i) correctly. The operation is not commutative and there are plenty of counterexamples. One mark was awarded for the correct statement, one for the counterexample. The operation is associative, and many students could write down a correct argument, but quite a few students were confused about what they needed to show, or how the operation works. There are quite a lot of sensible approaches to this proof involving some kind of case distinctions. The shortest one is to observe that if m is even then both, $(m \circledast n) \circledast k$ and $m \circledast (n \circledast k)$, evaluate to 0, and if m is odd they both evaluate to $n \circledast k$. There's nothing wrong with making further case distinctions. However, some students got a bit carried away and also looked at whether k is even or odd, which does not matter at all, and sometimes they made mistakes in their claims regarding what the two expressions evaluate to, losing a mark or two. Some students lost marks because they made unsubstantiated claims about calculating with mod2.

d) This part was meant to include the hardest bits of this question, and so it proved. Using the real and imaginary part of a complex number isn't very helpful for the given function. Valid arguments may be derived either by thinking of such numbers via their absolute and argument, or thinking more about what the function does and finding its inverse. Some students wrote expressions where they divided by a complex number (usually $\overline{z}$) without any explanation, despite the warning given in the notes, and they lost at least one mark. Some students gave an inverse but got it wrong, assuming the inverse function to be one that maps z to $|z|^2 z$. Instead, the function f is its own inverse. What it does is to map each complex number to a real multiple of itself, and that multiple ensures that if the absolute of z is above 1, it gets mapped to a number whose absolute is below 1 and vice versa. Carrying the same process out again reverses it. Alternatively one can argue for injectivity that if fz and fz' are equal then their absolutes must be equal, and from there on it is possible to solve the problem in the usual way, but nobody made that argument. Typically students ended up with systems of equations which they could not solve, gaining only one or two marks for the proof part for each property. Many students were able to correctly determine that the given function is both, injective and surjective.

e) This was bookwork—the reason why this concept is interesting for computer scientists is explained in the notes, and it was also in the relevant lecture. I was looking for a statement that indicated that this is interesting in the context of comparing how two different algorithms perform as the size of their inputs increase. The function that is eventually dominated will describe the algorithm that is the better choice for large inputs.

   To gain both marks students had to write something about performance of algorithms, and also about large inputs. Quite a few wrote only about 'dominate' but not 'eventually dominate', and very few wrote anything about the size of the input. It was a rarity for anybody to get both marks, suggesting that most students had not tried to retain this kind of information.

Question 2. Overall this was quite well answered. Clearly whether or not one could perform Bayesian updating made a huge difference to the final mark. Some students seemed to be completely unaware of the definition of an expected value of a random variable. Stating that a given outcome was the one that occurs with the highest probability of all the possible outcomes was not asked, and did not get any marks.[1]

a) I asked specifically for a probability space. This consists of three components, and many students lost marks because they only give one or two of these. It may be worth pointing out that $\mathcal{E} \subseteq PS$ is a requirement for the set of events, but does not define it, and similarly for $P\colon \mathcal{E} \longrightarrow [0,1]$. There are plenty of examples in the notes for how one may describe a probability

[1] I wrote exactly the same thing last year.

space, and I have asked for for a probability space in every exam so far—I can't really understand why students aren't better prepared for this.

b) There were plenty of correct answers for this question. One mark was awarded for giving the correct answer and one for giving the correct probabilities. The simplest way of answering this is to work out that the sum of two numbers is even if they're both even or if they're both odd, and then just calculating those probabilities (the probability that a number is odd is 5/8—for this question it's irrelevant which number it is). Many students used a tree to find all the possibilities, and there's nothing wrong with that, but it does cost time and finding all the corresponding probabilities and adding the up correctly is error prone.

c) Much the same comments as for the previous part apply.

d) The simple way of answering this is to calculate the expected value of the random variable of turning the wheel once, which is 13/8, and arguing that we need twice that value since $E(Y + Z) = EY + EZ$ for random variables Y and Z. Many students instead calculated the distribution of the given random variable X, which was quite error prone, and they lost the mark available.

e) With careful reasoning one can express the desired number as the sum of two different random variables, and that leads to the correct answer. For full marks the reasoning needs to be correct. It is not hardto calculate the probability distribution of the new random variable which is obtained by conditioning. The answer is 121/40.

f) Bayesian updating is explained in great detail in the notes. What seemed to cause the most problems is the question of what we are trying to find a distribution for. The tip on page 183 of the printed notes makes it very clear that we first have to determine which possibilities we are trying to distinguish between. These are whether we have the first wheel, or the second wheel, so the correct thing is an initial distribution that assigns 1/2 to either possibility. Many students thought we were trying to distinguish between the possible values on the wheel. They then plugged numbers into calculations, but those numbers did not make any sense. (What does it mean that the chances of getting 3 is some probability given we have the number 1?) Most students who started with the correct initial distribution were able to carry out the correct calculation. After the first update the probability of having the right hand wheel is 4/5, which doesn't change after the second update (getting the number 2 does not help us to distinguish between the two wheels), and it changes to 16/17 in the third update.

Question 3. There was an unfortunate typo in Question 3 e). In the formula F the arguments of $G(x, j)$ should have been swapped. Many students noticed the error and actually accommodated it well in their answers. Nevertheless, as compensation, since no meaningful translation could be given for the formula, everyone was awarded full marks for i).

a) Average mark: 72%. 105 students got full marks.

   A common mistake was to interpret the symbol $\top$ as a propositional variable, but otherwise the question posed no problems. $\top$ is true for any valuation, which means that the truth table column for $\top$ is a list of all 1s.

b) Average mark: 62%. 27 students got full marks.

This was a bookwork question where two concepts needed to be explained. The quality of the answers varied from perfect and very good to weak. Answers needed to be to the point and use correct terminology. No marks were awarded if the answer was too vague.

Most answered i) and mostly correctly. One mark was lost if it wasn't said that a truth tables gives the interpretation of a propositional formula for *all possible* valuations of the propositional variables.

In ii) a common mistake was not to make clear the restrictions on negation in formulae in DNF. Most correctly said that a formula in DNF is a disjunction of conjunctions, but did not say that it is a disjunction of conjunctions of propositional variables or negated propositional variables (or $\top$ and $\bot$).

Few fully correct answers were given for iii). Many mistook Boolean functions for Boolean valuations, and therefore got the source of the functions wrong. Not making clear what the source of the function is was another common mistake.

Some students did not notice there was a choice in the question and answered all three parts.

c) Average mark: 65%. 57 students got full marks.

Overall this question was answered well. Marks were most commonly lost for:

- mistakes in replacing $\rightarrow$ (not often)
- mistakes in applying De Morgan's law when moving negation inwards over disjunction or conjunction. For example, $\neg(P \vee Q) \not\equiv (P \wedge \neg Q)$.
- wrong application of Step 4/general distributivity: this transforms a disjunction of conjunctions into a conjunction of disjunctions, basically by multiplying out. It is useful to notice that each conjunct in the result is a disjunction of as many parts as there are disjuncts in the given disjunction. In this case it was three. This step is a short cut for successively applying the distributive laws; because it's only one step it is easier to apply and leads to fewer mistakes and marks lost (but one has to remember how to do it).
- giving a DNF as an answer to i)
- mistakes in the use of laws which should have been simple: $\top \vee R \not\equiv R$.
- not justifying every step in the derivation; this means saying what laws have been applied.
- wrong names of laws given; if you don't remember the name of a law you can write down the law you are using.
- omitting brackets, e.g., writing $A \vee B \wedge C$ and consequently getting confused and continuing in the wrong way.

d) Average mark: 55%. 72 students got full marks.

The quality of the answers to this question was considerably better than the natural deduction question last year. This was very pleasing to see!

What makes natural deduction proofs tricky is to know with which assumptions to start in the application of the Axiom rule. Generally there is no need to guess and one can get hint from the judgement that we want to prove. If this has assumptions then these are assumptions that we should try.

When the judgement we aim to prove has no assumptions like in this case, it is a good idea to use a backward reasoning strategy and determine a subgoal that would produce the sought judgement as the last line of the proof. This means we need to work out what rule applied to which judgement gives the sought judgement. That judgement is then a subgoal and our aim is to find a ND proof for this subgoal. In this case the assumption of this subgoal was sufficient to construct a proof.

The rules that were expected to be used were given at the end of the exam paper. I expected a justification to be given in every line in the proof. This means it needs to said which rule is applied *to which premises.*

e) Average mark: 59% (before the adjustment 40%). 46 students got full marks (27 before the adjustment)

This was the least well answered question, probably due to the unfortunate typo, but very likely also because for many too little time was left at the end of the exam for this, the last question.

Most students handled the presence of the typo in the formula quite well (85 students got 2/2 for i), without the adjustment), but to give benefit of doubt I decided to award every student 2/2 for part i) anyway. It was required to express the formula in idiomatic English, which means *no variables may be used in translation*! It is ok though, and will help, to write the literal translation with variables first, and then try to express the information in proper English.

For ii) the answers were either very good and correct, or disappointing. The question had some similarity with a core question in the exercises. It was required to negate the given formula and simplify the formula as much as possible. I expected the outer negation to be pushed inwards as much as possible. 54 students got 2/2 for the question.

iii) was worth 1 mark for translating to English the *simplified* formula. This means the result of the transformation in part ii) needed to be expressed in English. 54 students got 1/1 for the question.